$k_\perp$ factorization
and
quark production from the CGC

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Outline

- Introduction
- Quark production in pA collisions in the CGC framework
  - Numerical estimate on violation of \( k_t \) factorization
    - Single Quark production
    - Quark Pair production
- Conclusions
  - HF, Gelis, Venugopalan, hep-ph/0504047 and in progress
Introduction

- **Collinear factorization**: one large scale \( \sqrt{s} \sim p_\perp \gg \Lambda_{\text{QCD}} \)

\[
\frac{d\sigma}{d^2x} = \int \hat{s} x_1 G(x_1) x_2 G(x_2)
\]

- **\(k_\perp\) factorization**: two large scales \( \sqrt{s} \gg p_\perp \gg \Lambda_{\text{QCD}} \)

Collins Ellis 1991, Catani Ciafaloni Hautmann 1991
Levin Ryskin Shabelsky Shuvaev 1991

\[
\frac{d\sigma}{d^2x} = \int \frac{|\mathcal{M}|^2}{k_1^2 \cdot k_2^2} \phi_1(x_1, k_1\perp) \phi_2(x_2, k_2\perp) \delta(k_1\perp + k_2\perp - q\perp)
\]

- Useful devise to include the intrinsic \(k_\perp\)
- Extension to include rescattering corrections in pA?
  \(\Rightarrow\) CGC provides a natural framework
Introduction

Particle production from the CGC in pA collisions

- Nucleus at high energy
Introduction

Particle production from the CGC in pA collisions

- Nucleus at high energy = dense system of small-x gluons
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  - gluon production
  - quark production
Introduction

- **Aim of this talk:**
  - to provide a quantitative estimate for violation of $k_T$ factorization in quark production in pA

- **Framework:**
  - McLerran-Venugopalan (Gaussian) model
  - Without $x$-evolution
Classical Color Field

- Small-$x$ gluons are described as classical field generated by charge source $\rho_{p,A}$ on the light-front (McLerran Venugopalan 1994)

- Yang-Mills equations:

$$[D_\mu, F^{\mu\nu}] = J^\nu , \quad [D_\nu, J^\nu] = 0 , \quad \partial_\mu A^\mu = 0$$

$$J^\nu|_{LO} = \delta^\nu + \delta(x^-)\rho_p(x_\perp) + \delta^\nu - \delta(x^+)\rho_A(x_\perp)$$

- Gluon and Quark production amplitudes are known to first order in $\rho_p$ and to all orders in $\rho_A$ (Kovchegov Muller 1998, Blaizot Gelis Venugopalan 2004)
Gluon production

- (Skechy) interpretation of solution:

- $k_{\perp}$-factorization is valid for gluon production

$$\frac{d\sigma}{d^2qdy_q} \sim \frac{\alpha_sN}{\pi^4 d_A q_{\perp}^2} \int_{k_{\perp}} \phi_{p\,g}(k_{\perp}) \phi_{A\,g}^g(q_{\perp} - k_{\perp})$$

- $\phi_{A\,g}^g = FT\langle U(x_{\perp}) U^\dagger(0) \rangle$
  - obtained analytically in MV model
  - characterized by charge density $\mu_{\perp}^2 \sim Q_s^2$ upto log
Quark cross section

- **Total amplitude** (after some cancellation):

\[
\mathcal{M}_F = g^2 \int \frac{\rho_{p,a}(\vec{k}_1)}{k_{1\perp}^2} \int e^{i\vec{k}_1 \cdot \vec{x}_\perp} e^{i(\vec{p}_\perp + \vec{q}_\perp - \vec{q}_\perp - \vec{k}_1) \cdot \vec{y}_\perp} \times \bar{u}(\vec{q}) \left\{ [\tilde{U}(\vec{x}_\perp) t^a \tilde{U}^\dagger(\vec{y}_\perp)] T_{q\bar{q}}(\vec{k}_\perp) + [t^b U_{ba}(\vec{x}_\perp)] \tilde{L} \right\} v(\vec{p})
\]

with

\[
T_{q\bar{q}}(\vec{k}_\perp) \equiv \frac{\gamma^+(\vec{q} - \vec{k} + m) \gamma^-(\vec{q} - \vec{k} - \vec{k}_1 + m)}{2p^+[(\vec{q}_\perp - \vec{k}_\perp)^2 + m^2] + 2q^+[(\vec{q}_\perp - \vec{k}_\perp - \vec{k}_1\perp)^2 + m^2]}
\]

and \(\tilde{L}\) Lipatov’s vertex

- **(Skechy) interpretation of solution:**

Pair production cross section:

\[ \frac{d\sigma_{q\bar{q}}}{d^2p_\perp d^2q_\perp dy_p dy_q} = \frac{\alpha_s^2 N}{8\pi^4 d_A} \int d\vec{k}_1 \cdot \vec{k}_2 \frac{\delta(\vec{p}_\perp + \vec{q}_\perp - \vec{k}_1 \perp - \vec{k}_2 \perp)}{\vec{k}_1 \perp \cdot \vec{k}_2 \perp} \]

\[ \times \left\{ \int_{\vec{k}_1 \perp, \vec{k}'_1 \perp} \text{tr} \left[ (\phi + m) T_{q\bar{q}}(\vec{k}_1 \perp) (\not{\rho} - m) T^*_{q\bar{q}}(\vec{k}'_1 \perp) \right] \phi_{A}^{q\bar{q}, q\bar{q}}(\vec{k}_2 \perp | \vec{k}_1 \perp, \vec{k}'_1 \perp) \right. \]

\[ + \int_{\vec{k}_1 \perp} \text{tr} \left[ (\phi + m) T_{q\bar{q}}(\vec{k}_1 \perp) (\not{\rho} - m) \not{\lambda}^* + \text{h.c.} \right] \phi_{A}^{q\bar{q}, g}(\vec{k}_2 \perp | \vec{k}_1 \perp) \]

\[ + \text{tr} \left[ (\phi + m) \not{\lambda} (\not{\rho} - m) \not{\lambda}^* \right] \phi_{A}^{g, g}(\vec{k}_2 \perp) \left. \right\} \varphi_p(\vec{k}_1 \perp) \]

\[ k_\perp \text{-factorization is violated for the nucleus: } \]

one needs three different “distributions” to describe the nucleus

\[ k_\perp \text{-factorization is recovered in the leading twist approximation } \]

Gelis Venugopalan 2004
**Quark cross section**

- **Interpretation:**

\[
\phi_A^{g,g} \sim \text{FT tr}\langle U U^{\dagger} \rangle \propto
\]

\[
\phi_A^{q\bar{q},g} \sim \text{FT tr}\langle \tilde{U} t^a \tilde{U}^{\dagger} t^b U_{ba} \rangle \propto
\]

\[
\phi_A^{q\bar{q},q\bar{q}} \sim \text{FT tr}\langle \tilde{U} t^a \tilde{U}^{\dagger} \tilde{U} t^a \tilde{U} \rangle \propto
\]
Introduction

Classical Color Field

Quark Cross Section

kT factorization

Single quark production

Pair production

Conclusions

Quark cross section

- Properties of $\phi_A$’s:
  - Sum rule
    \[
    \int_{\vec{k}_\perp, \vec{k}_\perp'} \phi_A^{q\bar{q}, q\bar{q}}(\vec{k}_2\perp | \vec{k}_\perp, \vec{k}_\perp') = \int \phi_A^{q\bar{q}, g}(\vec{k}_2\perp | \vec{k}_\perp) = \phi_A^{g,g}(\vec{k}_2\perp)
    \]
  - $k_\perp$-factorization is recovered if one can neglect $k_\perp$ in $T_{q\bar{q}}(k_\perp)$

- In large $N$ limit, 4-, 3-pt fns become a product of 2-pt fns
  \[
  \phi_A^{q\bar{q}, q\bar{q}}(\vec{k}_2\perp | \vec{k}_\perp, \vec{k}_\perp') \xrightarrow{\text{large } N} #C(k_\perp)C(k_2\perp - k_\perp)(2\pi)^2 \delta(\vec{k}_\perp - \vec{k}_\perp')
  \]
  where $C(k_\perp) = FT\langle \tilde{U}(x_\perp)\tilde{U}^\dagger(0) \rangle$ expresses multiple scattering of a quark in the nucleus

- Large $N$ approximation is rather good: we present the large $N$ results (mainly)
**Quark cross section**

- **Single quark cross section:**

\[
\frac{d\sigma_q}{d^2 \vec{q}_\perp dy_q} = \frac{\alpha_s^2 N}{8\pi^4 d_A} \int dy_p \int \frac{1}{\vec{k}_{1\perp} \cdot \vec{k}_{2\perp}} \nabla_{\vec{q}}^2 \nabla_{\vec{q}}^2 \psi_{\vec{q}}^2
\]

\[
\times \left\{ \text{tr} \left[ (\vec{q} + \vec{m}) T_{qq} (\vec{k}_{\perp}) (\vec{p} - \vec{m}) T_{qq}^* (\vec{k}'_{\perp}) \right] \frac{C_F}{N} \phi_A^{q,q} (\vec{k}_{2\perp}) \right. \\
+ \int_{\vec{k}_{\perp}} \text{tr} \left[ (\vec{q} + \vec{m}) T_{qq} (\vec{k}_{\perp}) (\vec{p} - \vec{m}) \nabla^* + \text{h.c.} \right] \phi_A^{qq,g} (\vec{k}_{2\perp} | \vec{k}_{\perp}) \\
+ \text{tr} \left[ (\vec{q} + \vec{m}) \nabla (\vec{p} - \vec{m}) \nabla^* \right] \phi_A^{g,g} (\vec{k}_{2\perp}) \left\} \psi_p (\vec{k}_{1\perp})
\]

- **A simplification:** 4-point fn does not appear
- **\( k_{\perp} \)-factorization is still violated for the nucleus**
Numerical evaluation of violation

- $k_{\perp}$-factorization is recovered if

$$
\phi^{q\bar{q},g}_{A}(k_{2\perp}|k) = \frac{1}{2}(2\pi)^2 [\delta(k_{\perp}) + \delta(k_{\perp} - k_{2\perp})] \phi^{g,g}_{A}(k_{2\perp})
$$

This means either of $Q$ or $\bar{Q}$ exchanges all the momentum

- Is the ratio $\phi^{q\bar{q},g}_{A}/\phi^{g,g}_{A}$ close to two $\delta$-fns?

HF, Gelis, Venugopalan 2005
- Exact 3-point function in the MV model:

\[ \frac{\phi^{qq,\bar{q}}(k_{2\perp}; k_{\perp})}{\phi^{gg}(k_{2\perp})} \text{ for } Q_s^2=2 \text{ GeV}^2 \] (thin lines: large N)

- For \( k_{2\perp} \gg Q_s \) two peaks with width \( \sim Q_s \) separated by \( k_{2\perp} \)
- For \( k_{2\perp} \lesssim Q_s \) two peaks merge into one

- \( k_{\perp} \)-factorization holds if \( \sqrt{s} \gg m_{p_{\perp}} \gg Q_s \)
  - typical \( k_{2\perp} \) is large since \( p_{\perp} + q_{\perp} = k_{1\perp} + k_{2\perp} \)
  - multiple scattering \( \sim Q_s \) cannot see pair structure \( \sim m_{p_{\perp}} \)
Single quark production

- Numerical estimate of 'exact / $k_\perp$-fact' for a light quark with several values of $Q_s$ in the MV model

![Graph showing numerical estimate of 'exact / $k_\perp$-fact' for a light quark with several values of $Q_s$ in the MV model.]

- Please don’t confuse this with the Cronin peak
Single quark production

- charm quark

**Graph:**

- y-axis: exact / $k_T$-factorized (m = 1.5 GeV)
- x-axis: $q_T$ (GeV)
- Markers for $Q_s^2$: 1 GeV$^2$, 4 GeV$^2$, 15 GeV$^2$, 25 GeV$^2$

Equations:

- $Q_s^2 = 1$ GeV$^2$
- $Q_s^2 = 4$ GeV$^2$
- $Q_s^2 = 15$ GeV$^2$
- $Q_s^2 = 25$ GeV$^2$
Single quark production

- bottom quark

\[ q_{\perp} \text{ (GeV)} \]

exact / \( k_{\perp} \)-factorized (\( m = 4.5 \text{ GeV} \))

- \( Q_s^2 = 1 \text{ GeV}^2 \)
- \( Q_s^2 = 4 \text{ GeV}^2 \)
- \( Q_s^2 = 15 \text{ GeV}^2 \)
- \( Q_s^2 = 25 \text{ GeV}^2 \)
Asymptotically small-$x$ regime — another extreme:

Charge distribution becomes a non-local gaussian

\[ W_x[\rho_A] = \exp \left[ - \int_{x_\perp, y_\perp} \frac{\rho_{A,a}(x_\perp) \rho_{A,a}(y_\perp)}{2 \mu_A^2(x, x_\perp - y_\perp)} \right] \]

with

\[ \mu_A^2(x, k_\perp) = \frac{2}{\gamma c} k_\perp^2 \ln \left( 1 + \left( \frac{Q_s^2(x)}{k_\perp^2} \right) \right) \]
Single quark production

- charm quark with $\phi_A$'s of non-local gaussian

![Graph](image-url)

**exact / $k_\perp$-factorized (m = 1.5 GeV)**

- $Q_s^2 = 1 \text{ GeV}^2$
- $Q_s^2 = 4 \text{ GeV}^2$
- $Q_s^2 = 15 \text{ GeV}^2$
- $Q_s^2 = 25 \text{ GeV}^2$
Single quark production

General trends:

- Violation is more significant for larger $Q_s$ and smaller $m$.
- Violation has a maximum around $q_\perp \sim Q_s$ and is recovered as $q_\perp \to \infty$.
- If $Q_s < m$, $q_\perp$, the violation enhances the cross section.
- If $m, q_\perp \ll Q_s$, the violation reduces the cross section: quark is likely to be produced with a transverse mass $\gtrsim Q_s$. 
Pair production

- Pair spectrum for total mom $P_\perp$ with a fixed invariant mass $M$

- Bump around $Q_s$ in $k_\perp$-factorization approach $\phi_{A}^{g,g}(k_{2\perp})$
- Full calculation smears out the bump
Pair production

- Ratio of 'exact / $k_T$-fact' for pair production

\[
\text{ratio of pair Xsec, exact/fact, } Q_s^2 = 2\text{GeV}^2, m = 1.5\text{GeV}
\]

- $k_T$ factorization is recovered at large $P_T$

- Violation of $k_T$ enhances small-$P_T$ cross section:
  \[
  \Rightarrow \text{Opposite to the single quark case}
  \]
Pair production

- invariant mass spectrum of the pair
- binary scaling holds at large $M$
- suppression at smaller $M$
Pair production

- Ratio of the invariant mass spectrum of full & $k_T$-fact results
- Size and trend are similar to single quark case
Conclusions

- **Violation of $k_\perp$-factorization in pA collisions:**
  - one needs 2-, 3-, 4-pnt fns to describe the nucleus side
  - quantitative estimate is shown in the MV model and in the saturated limit

- **Single quark production:**
  - violation can be significant when $Q_s \gtrsim m_{p_\perp}$
    - small effect for charm quark with realistic $Q_s$ at RHIC
    - significant with large $Q_s$ at LHC and at very forward rapidities

- **Pair production:**
  - violation of $k_\perp$ factorization significantly depends on the pair momentum $P_\perp$
    - Bump in $P_\perp$ is seen in $k_\perp$ factorization approximation, but smeared out in full result due to multiple scattering

- **Outlook:**
  - more realistic $\phi_{p,A}$’s for phenomenology
  - $x$-evolution and rapidity dependence