Expanding Color Flux Tubes and Instabilities

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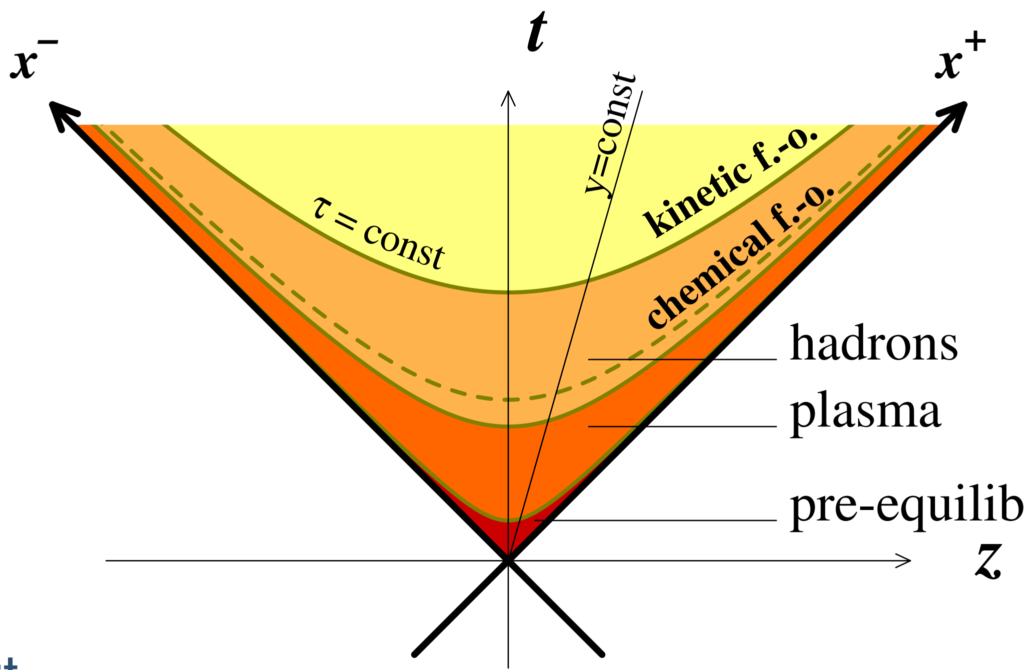
based on arXiv:0803.0410 [hep-ph]

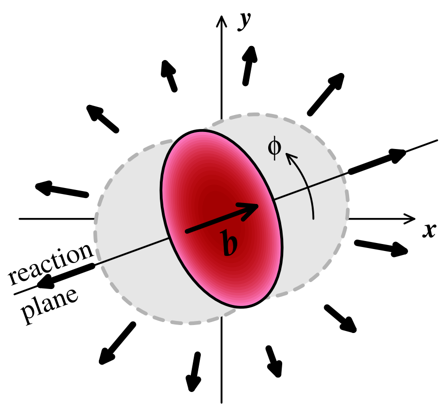
Introduction



- Freeze-out
- Hadronic transport
- Hadronization
- (*Locally thermalized?*) system of quarks and gluons
- Pre-equilibrium stage
- in-coming nuclei at nearly the velocity of light = CGC !

Introduction

Strong collectivities observed at RHIC = “QCD matter” !?



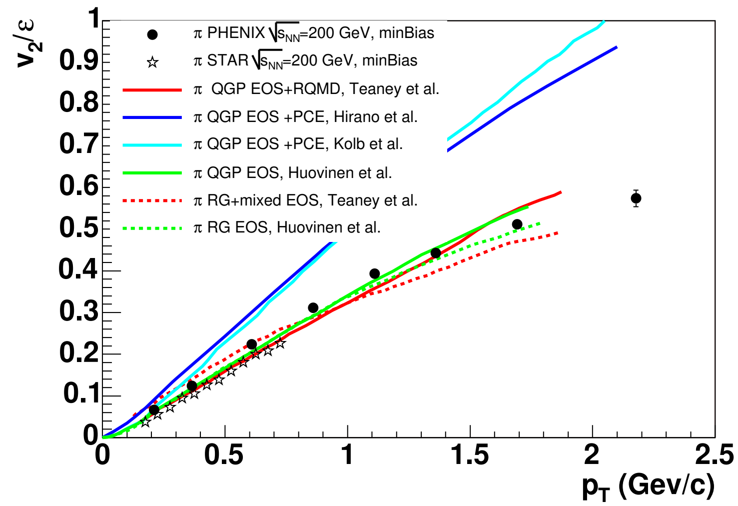


Figure 1: Cartoon, and pion v_2 taken from PHENIX WP

- v2 at RHIC is comparable to ideal-hydro ‘prediction’

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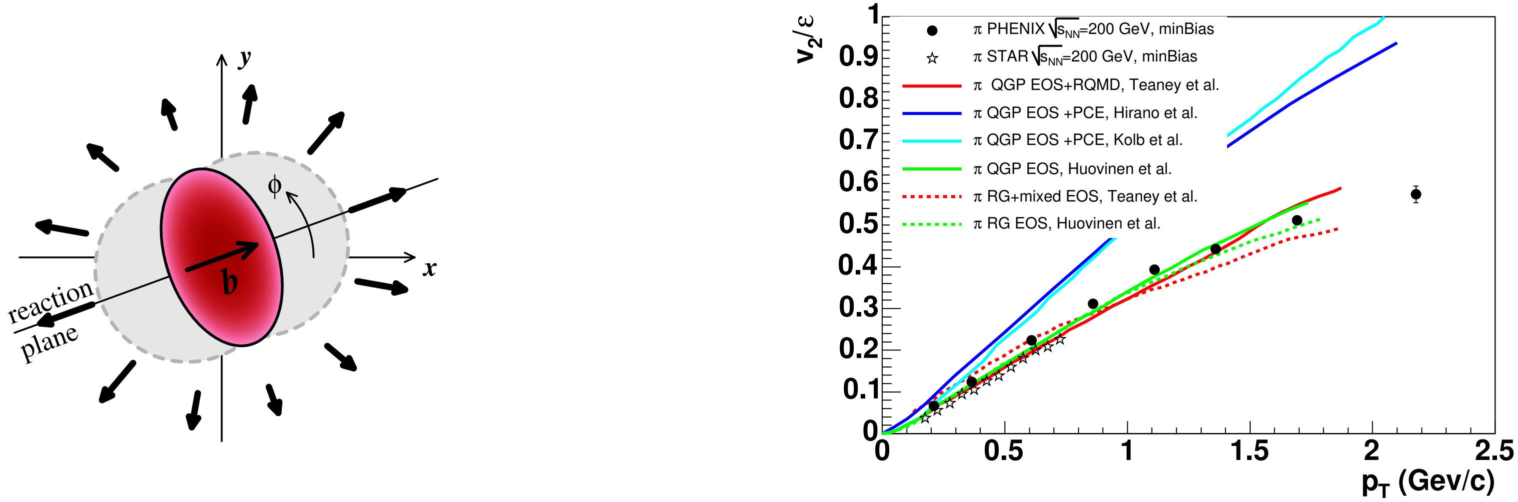

Figure 1: Cartoon, and pion v_2 taken from PHENIX WP

- v2 at RHIC is comparable to ideal-hydro ‘prediction’
- *Accidental coincidence to ideal hydro?*

  Cf. PCE gives too large v2

  Ideal-QGP+ had-cascade = best modeling to date

  v2 value depends on initial profile as well as τ_0
- Anyway, $\tau_0 \lesssim 1$ fm/c is discussed with RHIC data

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in what mechanism does the system thermalize within 1 fm/c?

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theoretical/numerical efforts to understand early-time evolution

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  C. Greiner et al.

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- Instability inherent to the initial configuration
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- Anomalous viscosity due to random strong gauge fields
  Asakawa-Bass-Muller

Objective and Outline

Objective = Analytic study of ...

- Early-time evolution of the coherent color field (= Glasma[a])
- Unstable modes in the Glasma configuration

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[a] Glasma is a jargon meaning the transient state between initial CGC and thermalized states.

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1. Introduction
2. CGC-CGC model for high-energy A-A collsions
3. Classical field evolution
4. Fluctuations - linear analysis
5. Concluding remarks and open issues

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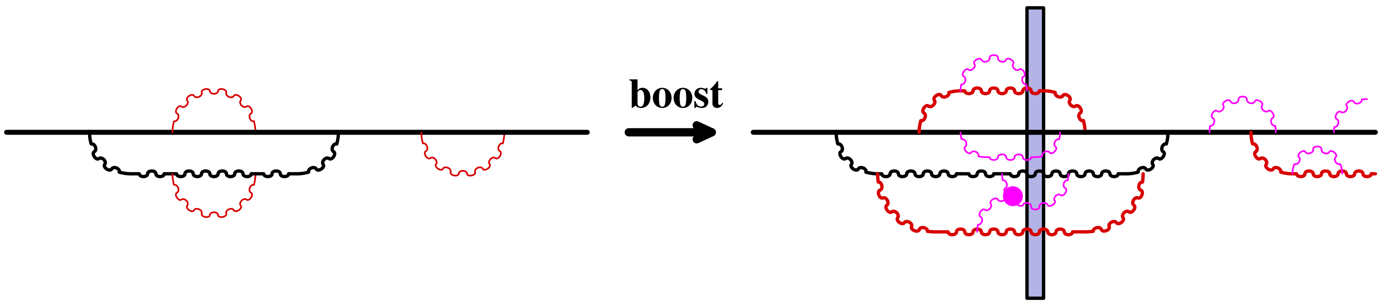
Small-x wavefunction



▷ in a high energy reaction we take a snap-shot of these fluctuations generated from the frozen source

▷ frozen source may be treated statistically, since the moment of collision is arbitrary and we only observe limited correlations

MV model = simplest CGC picture

McLerran, Venugopalan (1994), Iancu, Leonidov, McLerran (2001)

- Small x modes = classical color field
- Large x modes = "frozen" color sources ρ_a
- The classical field obeys Yang-Mills equations:

$$[D_\nu, F^{\nu\mu}]_a = \delta^{\mu +}\delta(x^-)\rho_a(\vec{\boldsymbol{x}}_\perp)$$

w/ "sol" $\mathcal{A}^\mu_{aCOV}(\vec{x}) = \delta^{\mu +}\alpha_a(\vec{x})$ s.t.

$$-\nabla^2_\perp \alpha_a(\vec{x}) = \rho_{aCOV}(\vec{x})$$

- gauge transform to the LC gauge :

$\mathcal{A}^i(\vec{x}) \sim \theta(x^-)\frac{i}{g}V(\partial^i V^\dagger), \quad V^\dagger(x_\perp) = \mathrm{Pexp}\left\{ig\int_{-\infty}^{\infty} dz^- \alpha(z^-, x_\perp)\right\}$

- The color sources ρ_a has random distribution $W_{x_0}[\rho]$ [a]
- Typical scale: $\langle\rho^2\rangle \sim Q_s^2(x_0)$ saturation

[a] Observables are calculated in the presence of the classical field, and then averaged over the configurations of the sources ρ_a: $\langle\mathcal{O}\rangle = \int[D\rho_a]\, W_{x_0}[\rho_a]\, \mathcal{O}[\rho_a]$

CGC-CGC collision

- Two-current problem:

$$[D_\mu, F^{\mu\nu}] = J^\nu \quad \text{w/ } J^\nu = \delta^{\nu+}\delta(x^-)\rho_1(x_\perp) + \delta^{\nu-}\delta(x^+)\rho_2(x_\perp)$$

- Boost invariance ⇒ YM eq. in $\tau - \eta$ coordinate:

$$\tau[D_\tau, \frac{1}{\tau}F_{\tau\eta}] - [D_i, F_{i\eta}] = 0\,, \tag{1}$$

$$[D_\eta, \frac{1}{\tau}F_{\tau\eta}] - [D_i, \tau F_{i\tau}] = 0\,, \tag{2}$$

$$\frac{1}{\tau}[D_\tau, \tau F_{i\tau}] - \frac{1}{\tau^2}[D_\eta, F_{i\eta}] + [D_j, F_{ji}] = 0 \tag{3}$$

- w/ Fock-Schwinger gauge: $A_\tau = \frac{1}{\tau}(x^+A^- + x^-A^+) = 0\,.$
- Boudary condition on the light-cone: Kovner-McLerran-Weigert (1995)

$$\alpha(\tau = 0, x_\perp) = -\frac{ig}{2}[\alpha_1^i(x_\perp), \alpha_2^i(x_\perp)]\,, \tag{4}$$

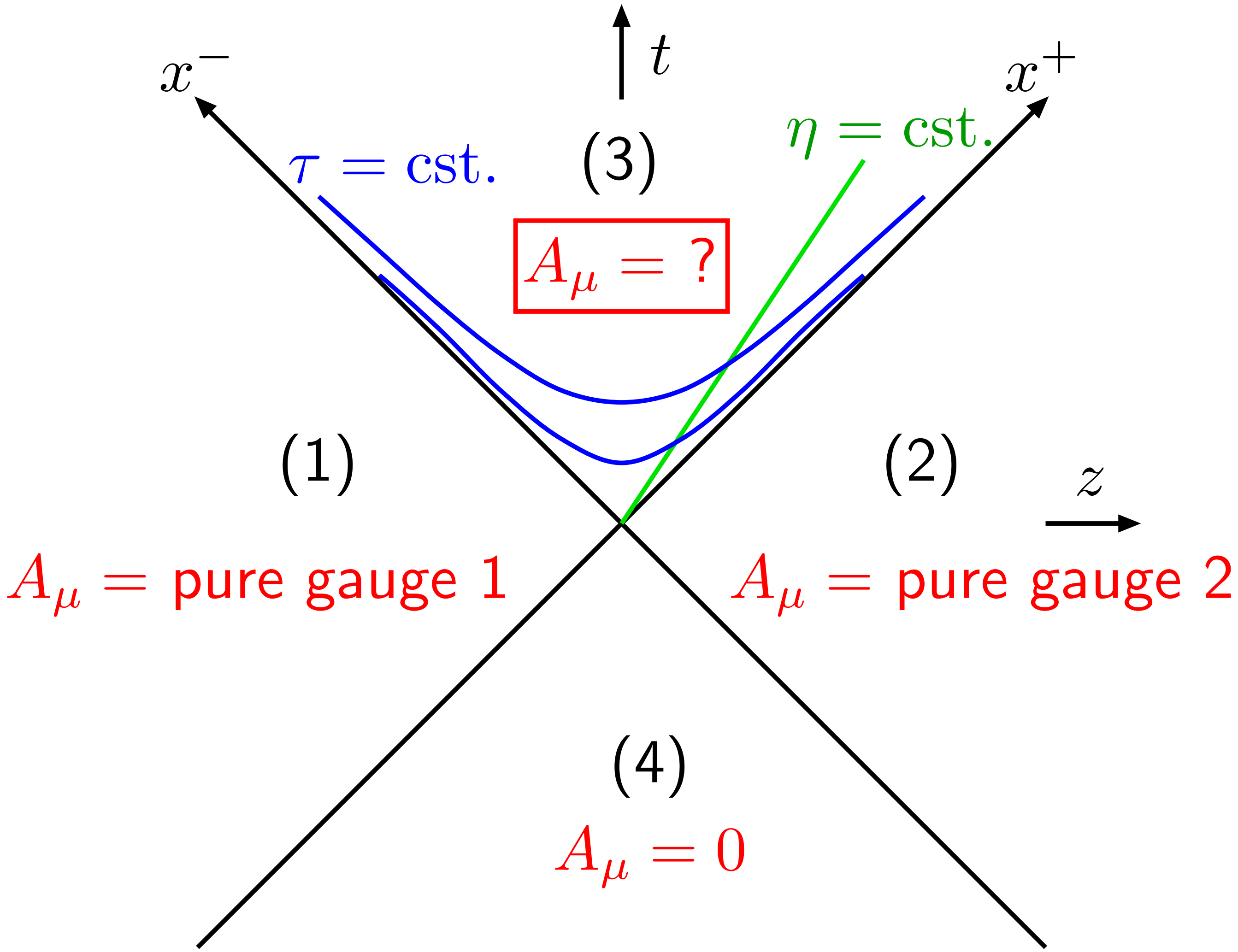
$$\alpha^i(\tau = 0, x_\perp) = \alpha_1^i(x_\perp) + \alpha_2^i(x_\perp)\,, \tag{5}$$

for

$$A_\eta = \mathcal{A}_\eta = -\tau^2\alpha(\tau, x_\perp)\,, \quad A_i = \mathcal{A}_i = \alpha_i(\tau, x_\perp). \tag{6}$$

CGC-CGC collision

T. Lappi's fig.



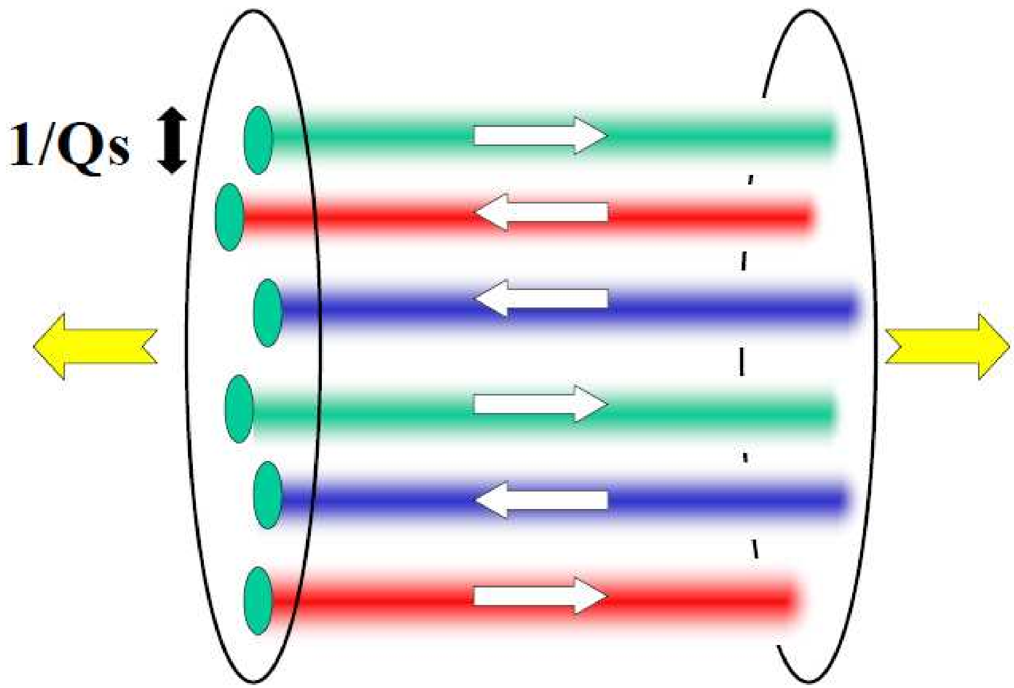
Strong longitudinal field

- Purely longitudinal configuration just after the collision:

$$E^z|_{\tau=0^+} = -ig[\alpha_1^i, \alpha_2^i], \quad (7)$$

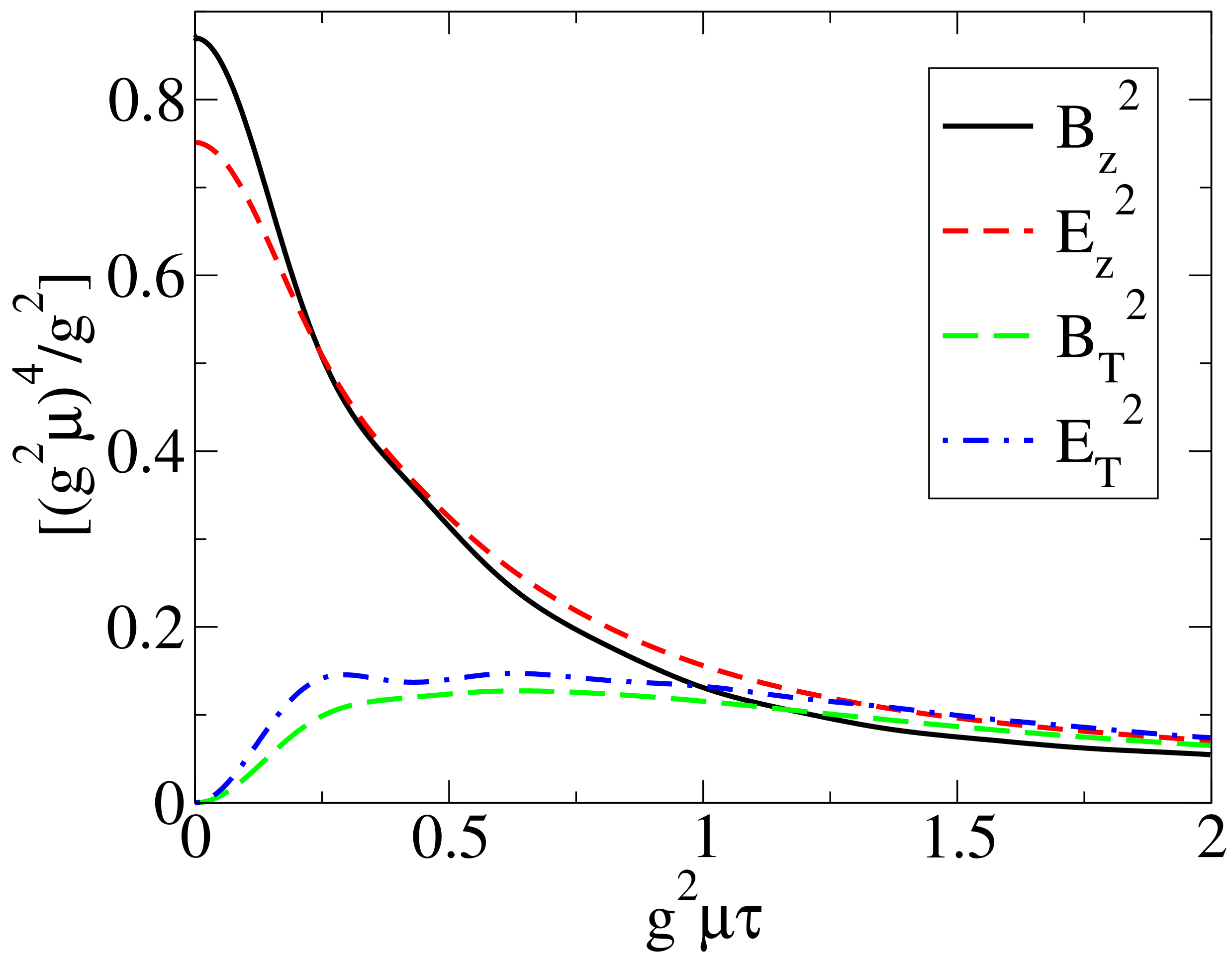
$$B^z|_{\tau=0^+} = ig\epsilon_{ij}[\alpha_1^i, \alpha_2^j] \quad (8)$$

- Our picture of the initial boost-invariant cofiguration



Numerical simulation

T. Lappi (after ensemble average)



Our approach/assumption/expectation

- One event corresponds to one fixed initial color configuration
- Every configuration evolves rapidly to thermalized state

Avoid ensemble average

- Each flux tube will have associated unstable modes

- Work with SU(2)$_c$

Classical field evolution

Pure magnetic classical field

- Assume (1) initial configuration

$$\alpha = 0, \qquad \alpha_i \neq 0$$
$$E^z = 0\,,\ B^z = -F_{12}\,,\ E^i = -F_{i\tau}\cosh\eta\,,\ B^i = \epsilon^{ij}F_{j\tau}\sinh\eta.$$

($\eta = 0$ in the following)

- Assume (2) $B^z \parallel \hat{3} \quad \Rightarrow$ Abelian dynamics

- Writing $\tilde{\alpha}_i = k_i C_{\parallel} + \epsilon_{ij}k_j C_{\perp}$, we find $C_{\parallel}$ is unphysical gauge freedom, and

$$\partial_\tau^2 C_\perp + \frac{1}{\tau}\partial_\tau C_\perp + k_\perp^2 C_\perp = 0$$

with solution:

$$\alpha_i(\tau, \boldsymbol{x}) = \int \frac{d^2k_\perp}{(2\pi)^2}\, \mathrm{e}^{ik_\perp \boldsymbol{x}} J_0(|k_\perp|\tau)\ \tilde{\alpha}_i^{\mathrm{init}}(k_\perp)$$

Simple example: Gaussian

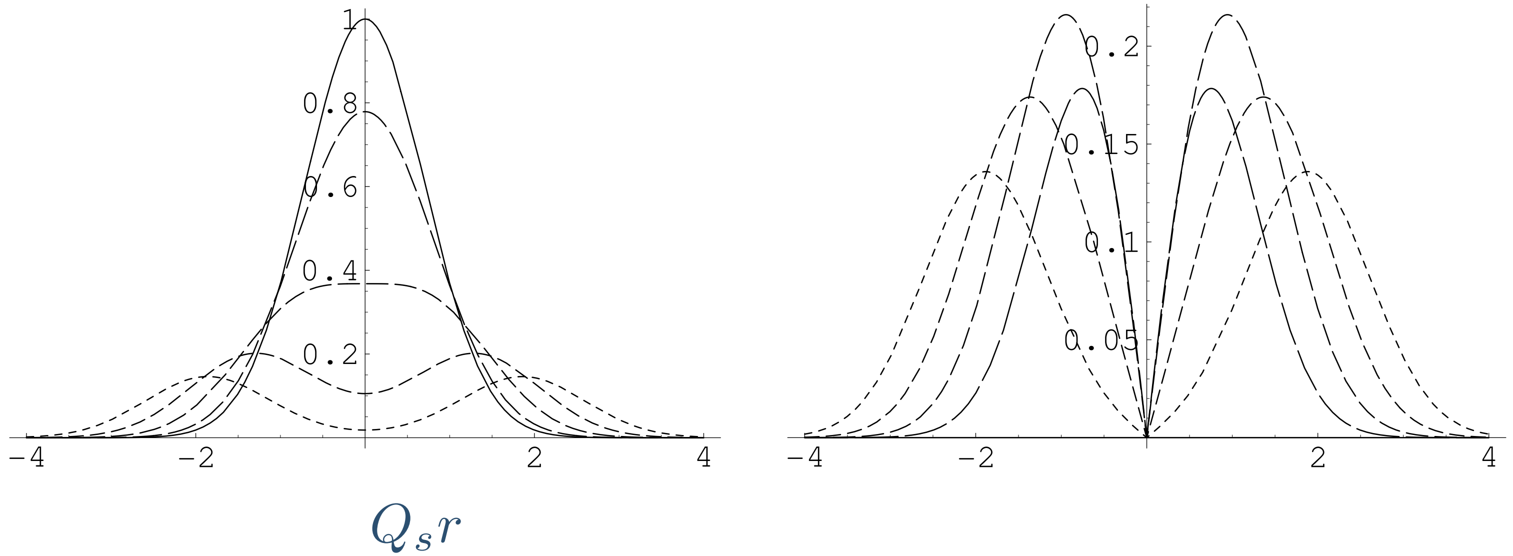
- For initial configuration,

$$\tilde{\alpha}_i^{\rm init}(k_\perp) \propto -i \frac{\epsilon_{ij} k_j}{k_\perp^2}\, {\rm e}^{-\frac{k_\perp^2}{4Q_s^2}}$$

$$B^z(\tau, r) = B_0\, {\rm e}^{-Q_s^2 r^2} {\rm e}^{-Q_s^2 \tau^2} I_0(2Q_s^2 r\tau)$$

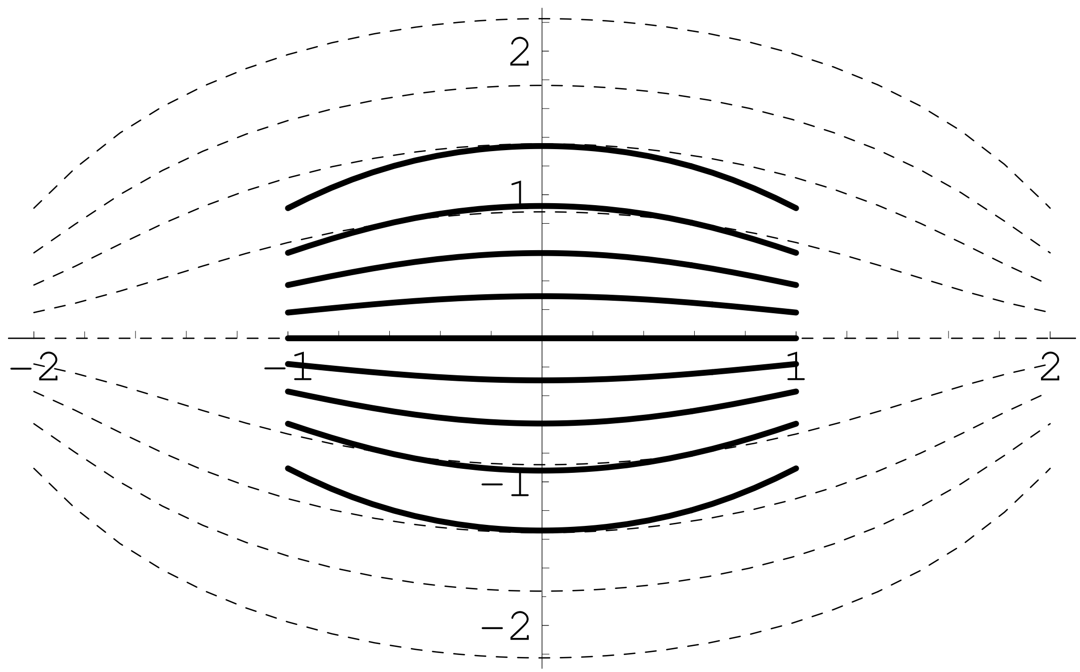
$$E^T(\tau, r) = B_0\, {\rm e}^{-Q_s^2 r^2} {\rm e}^{-Q_s^2 \tau^2} I_1(2Q_s^2 r\tau)$$

- $Q_s\tau$=0, 0.5, 1.0, 1.5, 2.0



Simple example: Gaussian

At constant time, $Q_s t = 1, 2$



Field expands outwards inducing electric field component, and weakens.

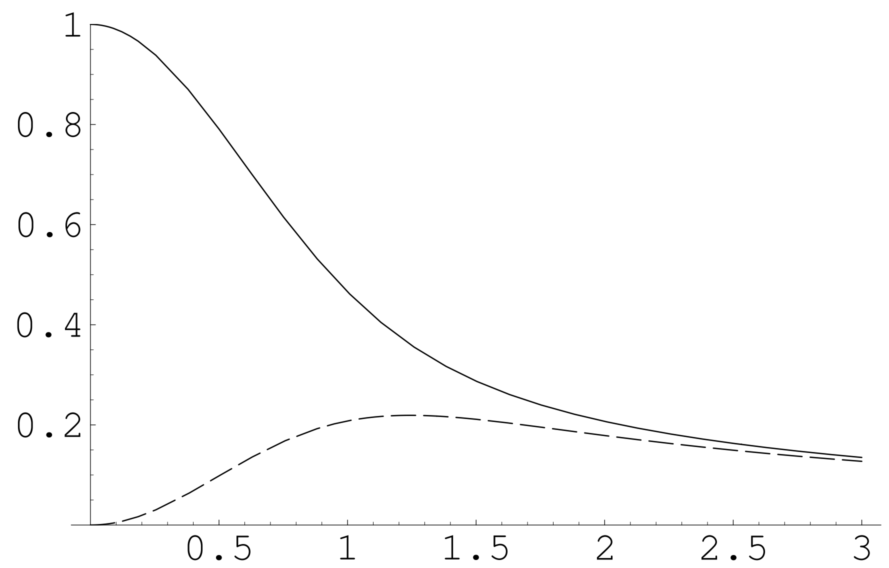
Remember an electric capacitor
- our model is essentially Abelian

Simple example: Gaussian

Evolution of the average energy density:

$$\overline{(B^z)^2} \equiv \int d^2x_\perp (B^z)^2 = \frac{\pi}{2}\frac{B_0^2}{Q_s^2}\,\mathrm{e}^{-Q_s^2\tau^2} I_0(Q_s^2\tau^2)\,,$$

$$\overline{(E^T)^2} \equiv \int d^2x_\perp (E^T)^2 = \frac{\pi}{2}\frac{B_0^2}{Q_s^2}\,\mathrm{e}^{-Q_s^2\tau^2} I_1(Q_s^2\tau^2)\,.$$



Quite similar to Lappi's: important dynamics is Abelian!??

Long. pressure is negative: $(\overline{(B^z)^2} - \overline{(E^T)^2})/2$

Pure electric classical field

- Assume (1) initial configuration

$$\alpha \neq 0, \qquad \alpha_i = 0$$

$$E^i = 0\,,\ E^z = \frac{1}{\tau}\partial_\tau(-\tau^2\alpha)\,,\ B^i = \tau\epsilon_{ij}\partial_j\alpha\,,\ B^z = 0$$

$(\eta = 0)$

- Assume (2) $E^z \parallel \hat{3} \quad \Rightarrow$ Abelian dynamics

- EoM and solution:

$$\frac{1}{\tau^3}\partial_\tau(\tau^3\partial_\tau\alpha) - \partial_i^2\alpha = 0$$

$$\alpha(\tau, x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2}\,\mathrm{e}^{ik_\perp x_\perp} J_0(|k_\perp|\tau)\,\tilde{\alpha}_i^{\mathrm{init}}(k_\perp)$$

- The same analysis as magnetic case with $E \leftrightarrow B$.

Fluctuations

- Fluctuations are essential to thermalization (isotopic pressure)

$$A_i = \mathcal{A}_i + a_i(\tau, \eta, x_\perp)\,, \quad A_\eta = \mathcal{A}_\eta + a_\eta(\tau, \eta, x_\perp)$$

- Look for unstable modes associated with the tube config.
- For simplisity, assume a constant field over a size of $1/Q_s$
- Test of stability
  = Linear analysis around a constant field
  = Abelian background + charged/neutral fluctuations

Fluctuation on magnetic background

- Assume a constant magnetic field in $\hat{3}$ direction:

$$\alpha = 0, \qquad \alpha_i^a = \delta^{a3}\frac{B}{2}\left(y\delta_{i1} - x\delta_{i2}\right),$$

- Case of $a_\eta \neq 0$ and $a_i = 0$ is stable.
  Choose $a_\eta = 0$ and $a_i \neq 0$.

$$[\mathcal{D}_i, \partial_\eta a_i] = 0\,,$$
$$[\mathcal{D}_i, \tau\partial_\tau a_i] = 0\,,$$
$$-\frac{1}{\tau}\partial_\tau(\tau\partial_\tau a_i) + \frac{1}{\tau^2}\partial_\eta^2 a_i + [\mathcal{D}_j, \delta\mathcal{F}_{ji}] - ig[a_j, \mathcal{F}_{ji}] = 0$$

where $\delta\mathcal{F}_{ji} = [\mathcal{D}_j, a_i] - [\mathcal{D}_i, a_j]$

- Setting $[\mathcal{D}_i, a_i] = 0$ (Gauss law), we have

$$-\frac{1}{\tau}\partial_\tau(\tau\partial_\tau\tilde{a}_i^a) - \frac{\nu^2}{\tau^2}\tilde{a}_i^a + \partial_\perp^2\tilde{a}_i^a + gB\epsilon^{3ab}\partial_\theta\tilde{a}_i^b$$
$$+2gB\epsilon^{3ab}\left(\tilde{a}_2^b\delta_{i1} - \tilde{a}_1^b\delta_{i2}\right) + \frac{g^2B^2r^2}{4}\epsilon^{3cb}\epsilon^{3ba}\tilde{a}_i^c = 0\,,$$

- “neutral” a_i^3 field is free.

Fluctuation on magnetic background

- Fourier modes in the rapidity:

$$a_i(\tau,\eta,\boldsymbol{x}) = \int \frac{d\nu}{2\pi}\, \mathrm{e}^{i\nu\eta}\, \tilde{a}_i(\tau,\nu,\boldsymbol{x})$$

- Cylindrical coordinate and angular momentum $\partial_\theta \to im$:

$$x = r\cos\theta, \qquad y = r\sin\theta$$
$$\tilde{a}_i^a \propto \mathrm{e}^{im\theta}$$

- Diagonalize the eqn. introducing

$$\tilde{a}_i^{(\pm)} = \tilde{a}_i^1 \pm i\,\tilde{a}_i^2\,, \qquad \tilde{a}_\pm^{(\cdot)} = \tilde{a}_1^{(\cdot)} \pm i\,\tilde{a}_2^{(\cdot)}\,.$$

- EoM:

$$\frac{1}{\tau}\partial_\tau(\tau\partial_\tau\tilde{a}_+^{(\pm)}) + \left(\frac{\nu^2}{\tau^2} \mp mgB \pm 2gB\right)\tilde{a}_+^{(\pm)} + \left(-\partial_\perp^2 + \frac{g^2B^2r^2}{4}\right)\tilde{a}_+^{(\pm)} = 0\,,$$
$$\frac{1}{\tau}\partial_\tau(\tau\partial_\tau\tilde{a}_-^{(\pm)}) + \left(\frac{\nu^2}{\tau^2} \mp mgB \mp 2gB\right)\tilde{a}_-^{(\pm)} + \left(-\partial_\perp^2 + \frac{g^2B^2r^2}{4}\right)\tilde{a}_-^{(\pm)} = 0\,.$$

Charged fluctuation with anomalous mag. momemt in a mag. field

Fluctuation on magnetic background

- Introducing the HO wavefunc. of 2dim with eigenvalue, $E_n = (2n + |m| + 1)gB$, we find

$$\frac{1}{\tau}\partial_\tau(\tau\partial_\tau \tilde{a}_+^{(\pm)}) + \left(E_n \mp mgB \pm 2gB + \frac{\nu^2}{\tau^2}\right)\tilde{a}_+^{(\pm)} = 0\,,$$

$$\frac{1}{\tau}\partial_\tau(\tau\partial_\tau \tilde{a}_-^{(\pm)}) + \left(E_n \mp mgB \mp 2gB + \frac{\nu^2}{\tau^2}\right)\tilde{a}_-^{(\pm)} = 0$$

- Spring cnst. $E_n \mp mgB \pm 2gB$ or $E_n \mp mgB \mp 2gB$ is crucial
- Solution is again given by Bessel fn.
- EoM and sol. of unstable modes for n=0

$$\partial_\tau^2 f + \frac{1}{\tau}\partial_\tau f + \left(-gB + \frac{\nu^2}{\tau^2}\right) f = 0$$

$$\tilde{a}_+^{(-)},\ \tilde{a}_-^{(+)} \propto \mathrm{e}^{im\theta}\, r^{|m|}\, \mathrm{e}^{-\frac{gBr^2}{4}}\, I_{i\nu}\left(\sqrt{gB}\,\tau\right) \rightarrow \frac{\mathrm{e}^{\sqrt{gB}\tau}}{\sqrt{2\pi\sqrt{gB}\,\tau}}$$

Fluctuation on magnetic background

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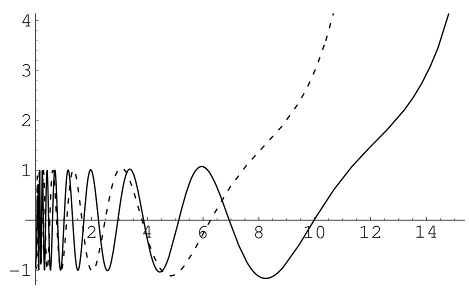
- Spring cnst. $E_n \mp mgB \pm 2gB$ or $E_n \mp mgB \mp 2gB$ is crucial
- Instability manifestly shows up when (...) vanishes
- The larger-ν^2 modes participate in the instability later

$$Q_s(\tau_{\text{wait}} + \tau_{\text{grow}}) \sim \nu + 1$$

with $\sqrt{gB} \sim Q_s$.

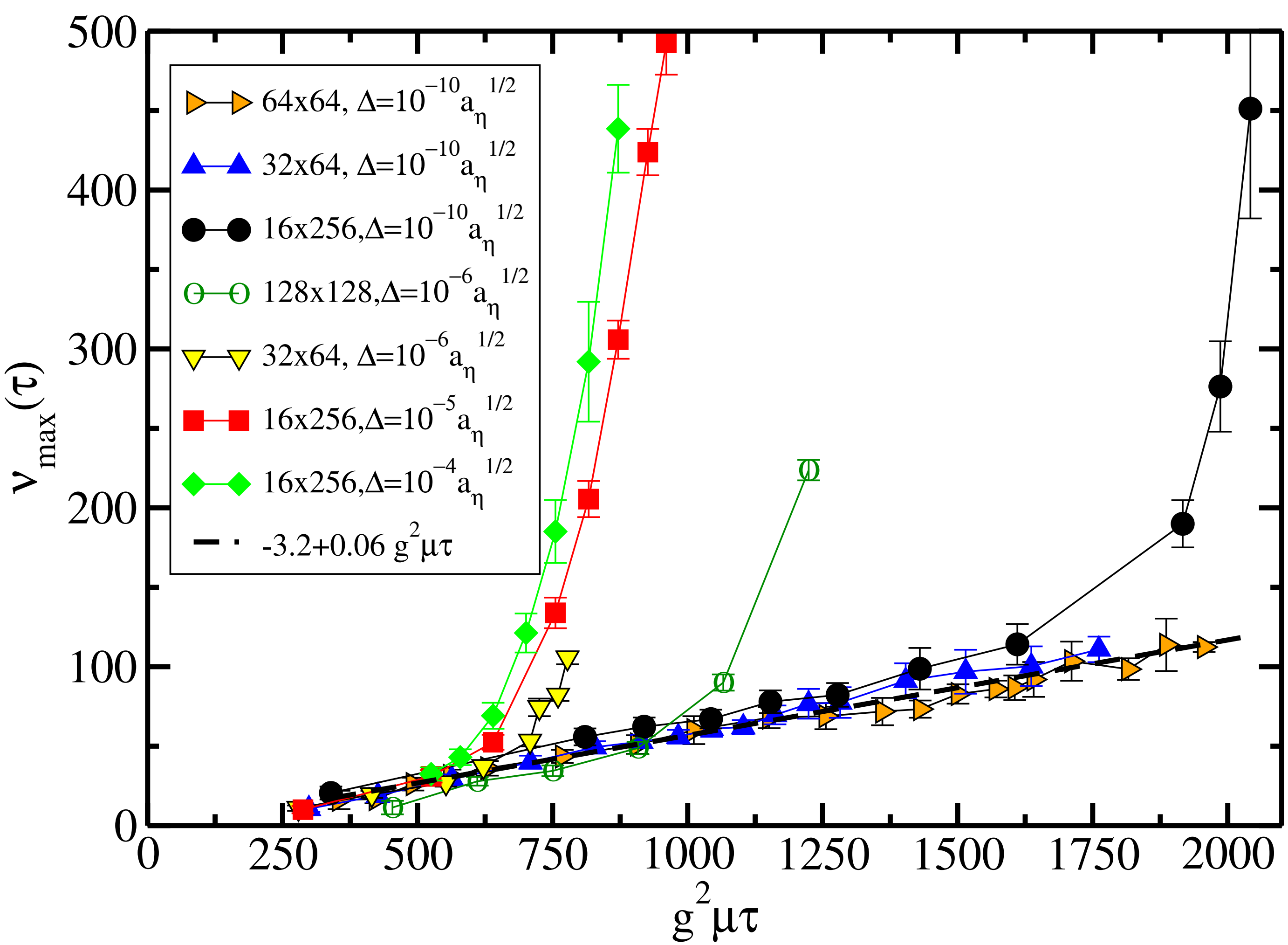


$\nu = 8, 12$

Fluctuation on magnetic background

- $\nu_{\max} \propto \tau$ is observed

Romatschke-Venugolapan



Fluctuation on electric background

- Assume a constant electric field in $\hat{3}$ direction:

$$\alpha = -\frac{1}{2}E\hat{3}, \qquad \alpha_i^a = 0\,,$$

- Again, case of $a_\eta \neq 0$ and $a_i = 0$ is stable.
  Choose $a_\eta = 0$ and $a_i \neq 0$

$$\partial_i[\mathcal{D}_\eta, a_i] = 0\,,$$
$$\partial_i(\tau\partial_\tau a_i) = 0\,,$$
$$-\frac{1}{\tau}\partial_\tau(\tau\partial_\tau a_i) + \frac{1}{\tau^2}[\mathcal{D}_\eta, [\mathcal{D}_\eta, a_i]] + \partial_j(\partial_j a_i - \partial_i a_j) = 0\,.$$

- Introducing the Fourier transform:

$$a_i(\tau,\eta,\boldsymbol{x}) = \int \frac{d\nu d^2k_\perp}{(2\pi)^3}\, \mathrm{e}^{i\nu\eta}\, \mathrm{e}^{ik_\perp \boldsymbol{x}} \left(\epsilon_{ij}k_j\, \tilde{b}(\tau,\nu,k_\perp)\right)$$

- 1st, 2nd eqns. are OK since (...) $\perp k_i$. The 3rd:

$$\frac{1}{\tau}\partial_\tau(\tau\partial_\tau \tilde{b}^a) + \frac{1}{\tau^2}\left(\nu^2\tilde{b}^a - i\nu gE\tau^2\epsilon^{3ba}\tilde{b}^b + \frac{g^2E^2\tau^4}{4}\epsilon^{3ca}\epsilon^{3cb}\tilde{b}^b\right) + k_\perp^2\tilde{b}^a = 0\,.$$

Fluctuation on electric background

- Diagonalize the eqn. with charged fields

$$\tilde{b}^{(\pm)} = \tilde{b}^1 \pm i\,\tilde{b}^2$$

as

$$\left(\partial_\tau^2 + \frac{1}{\tau}\partial_\tau\right)\tilde{b}^{(\pm)} + \left\{k_\perp^2 + \frac{1}{\tau^2}\left(\nu^2 \pm \nu g E \tau^2 + \frac{g^2 E^2}{4}\tau^4\right)\right\}\tilde{b}^{(\pm)} = 0\,.$$

- In small τ region, one might be able to neglect τ^4 in (...). Then instability might exist for $\nu^2 - \nu g E \tau^2 < 0$
- wrong! because τ^4 non-negligible when $gE\tau^2 \sim \nu$. Indeed,

$$(\ldots) = (\nu \pm gE\tau^2/2)^2 \geq 0$$

- No instability in the electric back-ground
- But, the Schwinger mechanism in electric field is a kind of instability...
any importance to our discussion?

Remarks and open issues

- We have studied clasical field evolution of a color flux tube with fixed color direction w/o ensemble average
- Energy density profile is very similar to simulation result
- Is Abelian dynamics essential??
  Any room for non-Abelian effects??

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- We have performed stability analysis of constant color config.
- We have found an instability on the magnetic constant field
- The max value of rapidity-mom. of unstable modes, $\nu_{\rm max} \propto \tau$; similar behavior observed in simulation
- Electric background field has no apparent instability - how about the Schwinger mechanism?

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- Electric background field has no apparent instability - how about the Schwinger mechanism?
- Physical picture - we want more vivid picture for our result
- Back-reactions, which must be important when instability has grown up
- Relation to Weibel instability, Schwinger mechanism, etc
- Duality - Larry repeatedly told us 'think about duality of E&B'