December 9th 2009

Random matrix models for chiral phase transition

Takashi Sano (University of Tokyo, Komaba),

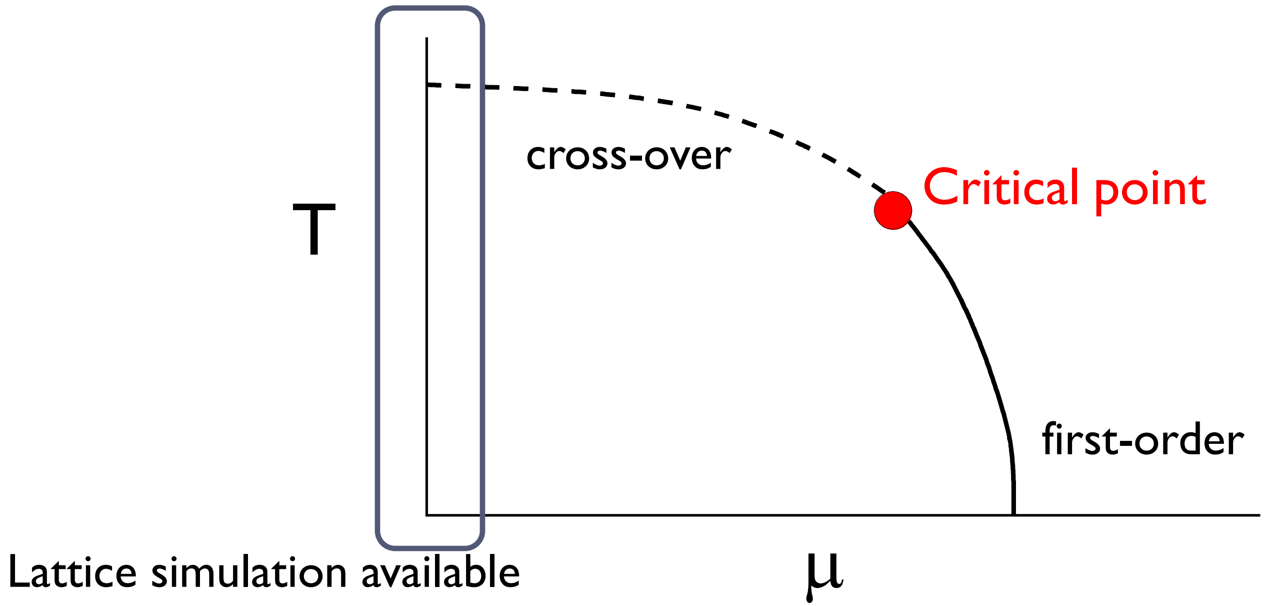
with H. Fujii, and M. Ohtani

- UA(1) breaking and phase transition in chiral random matrix model
  Phys. Rev. D **80**, 034007 (2009) TS, H. Fujii & M. Ohtani
- Work in progress , H. Fujii & TS

QCD phase diagram

- Expected phase diagram

Asakawa & Yazaki (1989)



- smooth cross-over at μ=0 ← lattice QCD simulations
- 1st-oder phase transition at low T ← NLJ model

→ Critical point may exist on phase diagram

QCD critical point

QCD critical point Stephanov et. al. (1998)

→ concrete evidence of chiral restoration

Existence of Critical Point (CP) : sensitive to …

⇓ cf. strange-quark d.o.f.

Chiral random matrix models as schematic models for QCD

Halasz et. al. (1998) (2 flavors)

Problem: Phase transition does not depend on # of flavors

→ 2nd-order even if Nf=3

→ cannot examine strange quark effects

Resolution:

We propose a ChRM model with a U(1) breaking determinant term

Summary: What we do

1. Extend ChRM model to show Nf-dependent phase transition (including anomaly effect)
2. Analyze phase structure of the model in the 2+1 flavor (ud & s- quark) case

Outline

- ✓ Introduction
- Review of Chiral Random Matrix(ChRM) Models
- ChRM Models with Determinant Interaction
- Numerical Results
- Conclusions & Outlook

2. Chiral Random Matrix Models

The Banks-Casher relation

Bnaks & Casher (1980)

The Banks-Casher relation : small eigenmodes vs. chiral condensate

$$\langle \bar{\psi}\psi \rangle = -\pi \frac{\rho(0)}{V}$$

$\rho(\lambda)$: Dirac eigenvalue distribution

$$|\langle \bar{\psi}\psi \rangle_{\text{QCD}}| = \frac{1}{N_f}\frac{\partial}{\partial m}\log Z(m) = \frac{1}{N_f}\frac{\partial}{\partial m}\log \langle \det(D+m)\rangle_{\text{YM}}$$

$$= \frac{1}{Z}\left\langle \text{tr}\frac{1}{D+m}\det(D+m)\right\rangle_{\text{YM}}$$

$$= \left\langle \text{tr}\frac{1}{D+m}\right\rangle_{\text{QCD}}$$

Eigenvalues: $\pm i\lambda$

$$\rho(\lambda) = \langle \text{tr}\delta(D - i\lambda)\rangle_{\text{QCD}}$$

$$\left\langle \text{tr}\frac{1}{D+m}\right\rangle_{\text{QCD}} = \left\langle \sum_{i\lambda_j>0} \frac{1}{i\lambda_j + m} + \frac{1}{-i\lambda_j + m}\right\rangle_{\text{QCD}}$$

$$= \left\langle \sum_{i\lambda_j>0} \frac{2m}{\lambda_j^2 + m^2}\right\rangle_{\text{QCD}} \to \int d\lambda \frac{2m}{\lambda^2+m^2}\rho(\lambda) \to \int d\lambda \pi\delta(\lambda)\rho(\lambda)$$

Chiral Random Matrix Models

- Model definition (Vacuum)

Shuryak & Verbaarschot (1993)

  - Partition function

$$Z^{\mathrm{RM}} = \int dW \prod_{f=1}^{N_f} \det(D + m_f) \underline{e^{-N\Sigma^2 \mathrm{tr} W^\dagger W}}$$

Gaussian

•Dirac operator: truncated in zero(soft)-mode space $\gamma_5 = \mathrm{diag}(+1, -1)$

$$D = \begin{pmatrix} 0 & iW \\ iW^\dagger & 0 \end{pmatrix}$$

$\{D, \gamma_5\} = 0$: Chiral Symmetry

$W : N \times N$: Complex random matrix

- Gauge fields → random elements of D
- Thermodynamic limit: # of (quasi-)zero modes $N \to \infty \, (N \propto V)$

Effective Potential Ω

Gaussian integral over W ➔ $$\int d\bar{\psi} d\psi e^{-\frac{1}{N\Sigma^2}\psi_L^{\dagger f}\psi_R^g\psi_R^{\dagger g}\psi_L^f - m\bar{\psi}\psi}$$

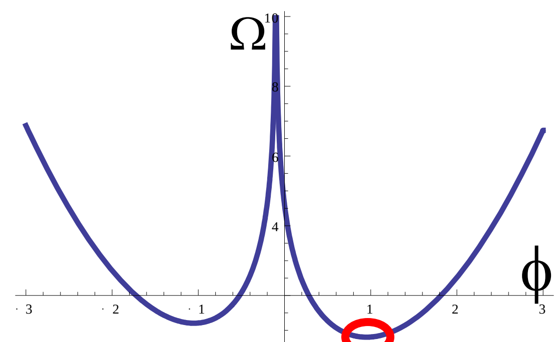
Hubbard-Stratonovitch transformation (bosonization) ➔

$$Z^{\mathrm{RM}} = \int dS e^{-N\Sigma^2 \mathrm{tr} S^\dagger S} \det{}^N[(S+m)(S^\dagger + m)] \equiv \int dS e^{-2N\Omega(S)}$$

$$\langle \bar{\psi}^f \psi^g \rangle \sim S = \mathrm{diag}(\phi,\ \ldots,\ \phi)$$

→ The effective potential

$$\Omega(\phi, m) = \frac{N_f}{2}\left[\Sigma^2\phi^2 - \log(\phi+m)^2\right]$$



Broken phase

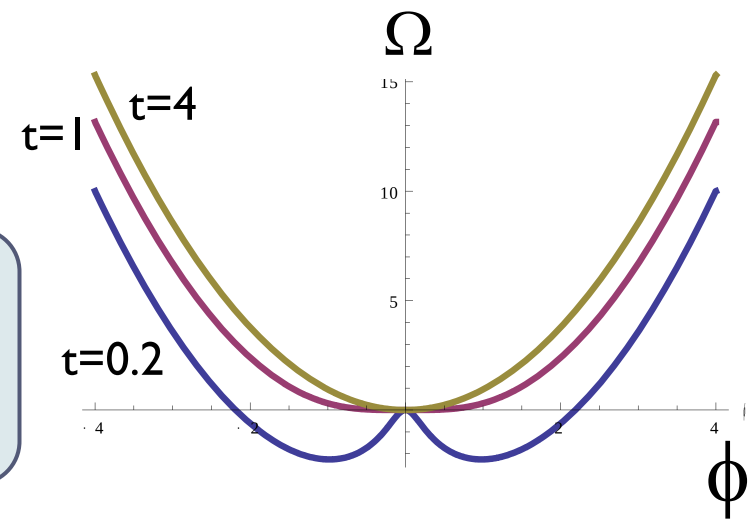
➔ Chirally broken ground state 

Finite Temperature ChRM Model

- Non-random external field t

Jackson & Verbaarschot(1996)

$$D = \begin{pmatrix} 0 & iW + it \\ iW^\dagger + it & 0 \end{pmatrix}$$



effective potential

$$\Omega = \frac{N_f}{2}\left[\Sigma^2\phi^2 - \log\{(\phi+m)^2 + t^2\}\right]$$

- Chiral symmetry is restored at finite T
- 2nd-order for any number of Nf
  - Inadequate as an effective model for QCD

Determinant interaction should be incorporated

3. ChRM models with determinant interaction

Janik, Nowak & Zahed (1997)

Extension of Zero-mode Space

Categorize Dirac zero modes(indices) into two types

- $N+$, $N-$: Topological (quasi-) zero modes = instanton origin (localized)
- 2N : near zero modes ➔ matter effects

$$D = \begin{pmatrix} 0 & iW + i\mathcal{T} \\ iW^\dagger + i\mathcal{T}^T & 0 \end{pmatrix}$$

$$W \in \mathbf{C}^{(N+N_+)\times(N+N_-)}$$

$$iW + i\mathcal{T} = \begin{pmatrix} \overbrace{iA + it\mathbf{1}_{N\times N}}^{N} & \overbrace{iX}^{N_+} \\ iY & iB \end{pmatrix} \begin{matrix} \}\, N \\ \}\, N_- \end{matrix}$$

$$Z_{N_+,\,N_-} = \int dS e^{-N\Sigma^2 \mathrm{tr} S^\dagger S} \det{}^N[(S+m)(S^\dagger + m^\dagger) + t^2] \times \det{}^{N_+}(S+m) \det{}^{N_-}(S^\dagger + m^\dagger)$$

- N+=N-=0 ➔ reduced to the conventional model

Sum over Instanton Distribution

QCD vacuum ➔ composed by multi-instanton configs.

$$Z^{\rm RM}(\theta) = \sum_{N_+,\, N_-} P(N_+)P(N_-)e^{i\theta\nu} Z_{N_+,N_-} \equiv \int dS e^{-2N\Omega}$$

Example: Poisson dist. 🡐 free instantons

't Hooft (1986)

$$P_{\rm Po}(N_\pm) = \frac{(gN)^{N_\pm}}{N_\pm !}$$

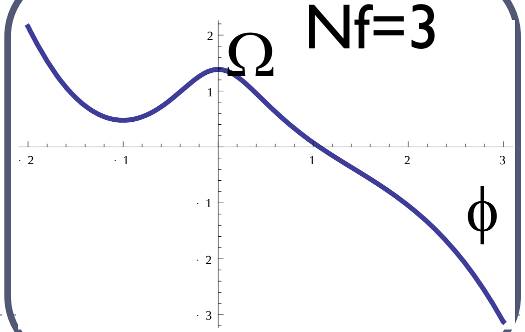
$$\Omega_{\rm Po} = \frac{1}{2}\Sigma^2 {\rm tr} S^\dagger S - \frac{1}{2}\log\det\left[(S+m)(S^\dagger + m^\dagger) + t^2\right]$$
$$- \frac{g}{2}\left[e^{i\theta}\det(S+m) + e^{-i\theta}\det(S^\dagger + m^\dagger)\right]$$

't Hooft int.

$$S = {\rm diag}(\phi,\ \ldots,\ \phi)$$

Unfortunately…

Unbound potential



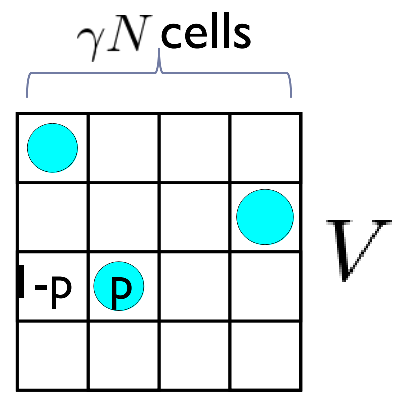
•ϕ^3 terms dominate (Nf=3)

Binomial Distribution for N+, N-

TS, H. Fujii & M. Ohtani (2009)

$\phi \to \infty$ corresponds $\langle N_+ \rangle$, $\langle N_- \rangle \to \infty$

➔ Regularized distribution $N_+,\ N_- \le \gamma N$



$$P(N_\pm) = \binom{\gamma N}{N_\pm} p^{N_\pm}(1-p)^{\gamma N - N_\pm}$$

p: single instanton existence probability

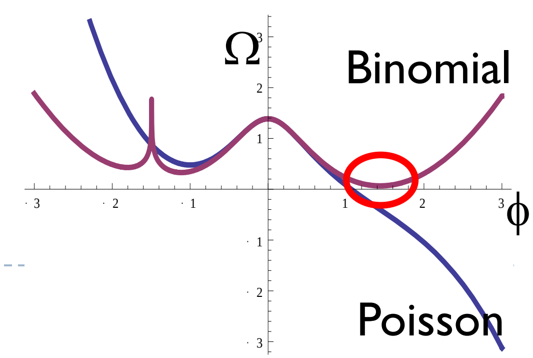
With binomial summation formula,

$$\alpha = p/(1-p)$$

$$\Omega = \frac{1}{2}\Sigma^2 \mathrm{tr} S^\dagger S - \frac{1}{2}\log\det\left[(S+m)(S^\dagger+m^\dagger)+t^2\right] - \frac{\gamma}{2}\left[\log(\alpha e^{i\theta}\det(S+m)+1)+\log(\alpha e^{-i\theta}\det(S^\dagger+m^\dagger)+1)\right]$$

'‘t Hooft int. appears under the log.

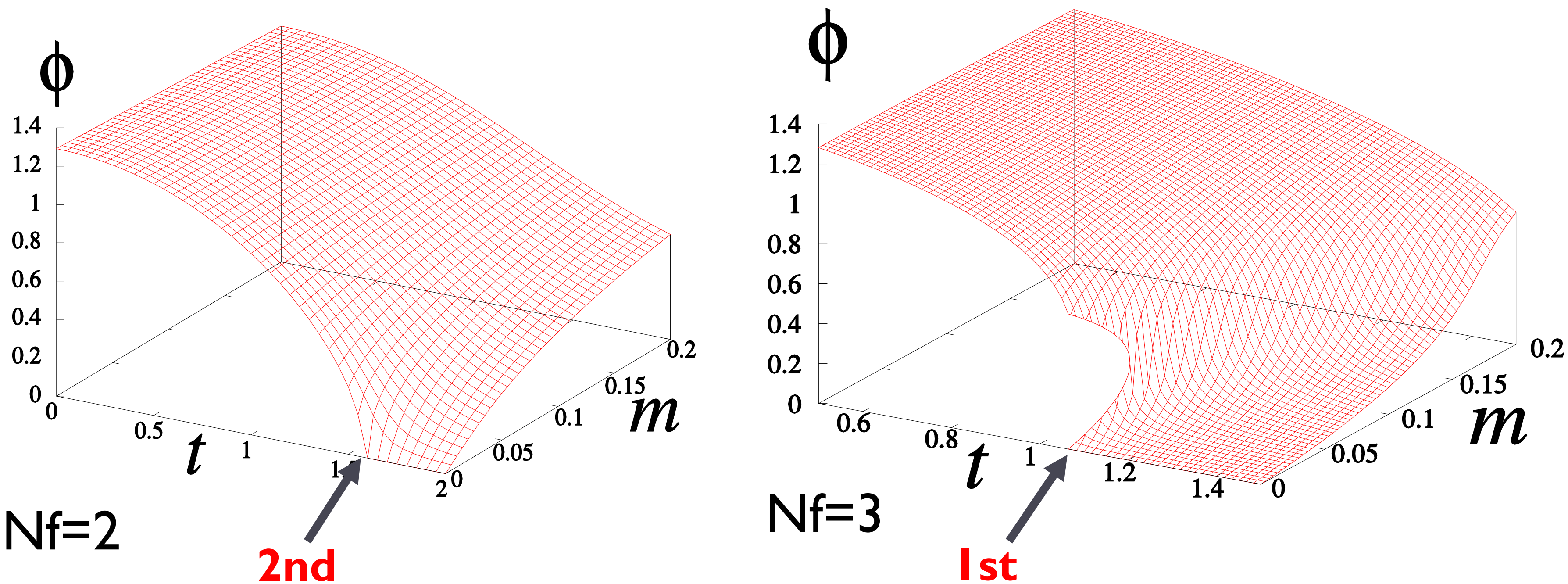
➔ Stable ground state



4. Numerical results

Results of equal-mass case: Phase Transition

$m = \mathrm{diag}(m, \ldots, m)$ $\quad S = \mathrm{diag}(\phi, \ldots, \phi)$ $\qquad \Sigma=1, \alpha=0.3, \gamma=2$

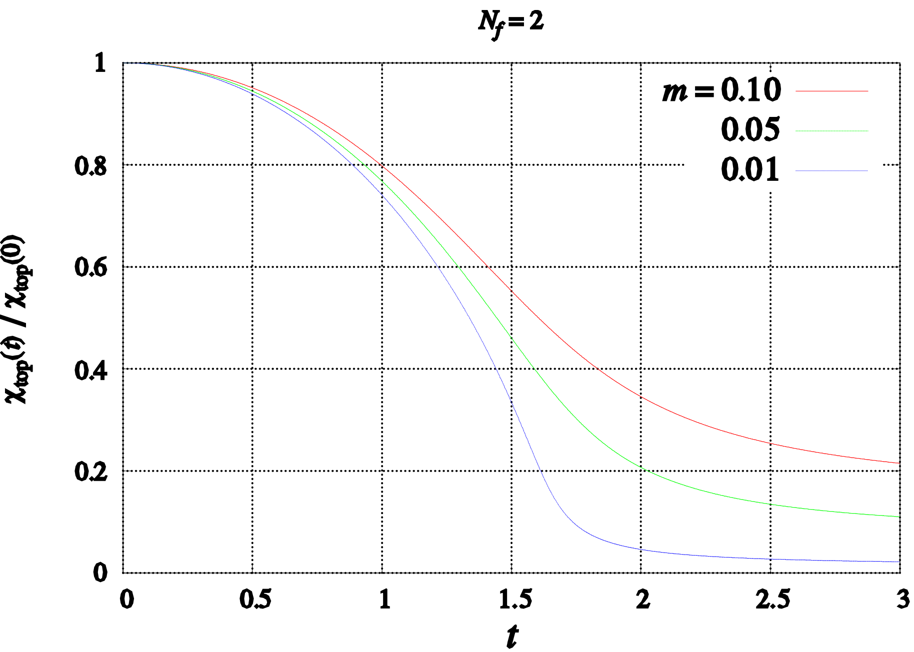


•2nd-order for Nf=2 & 1st-order for Nf=3 in the chiral limit

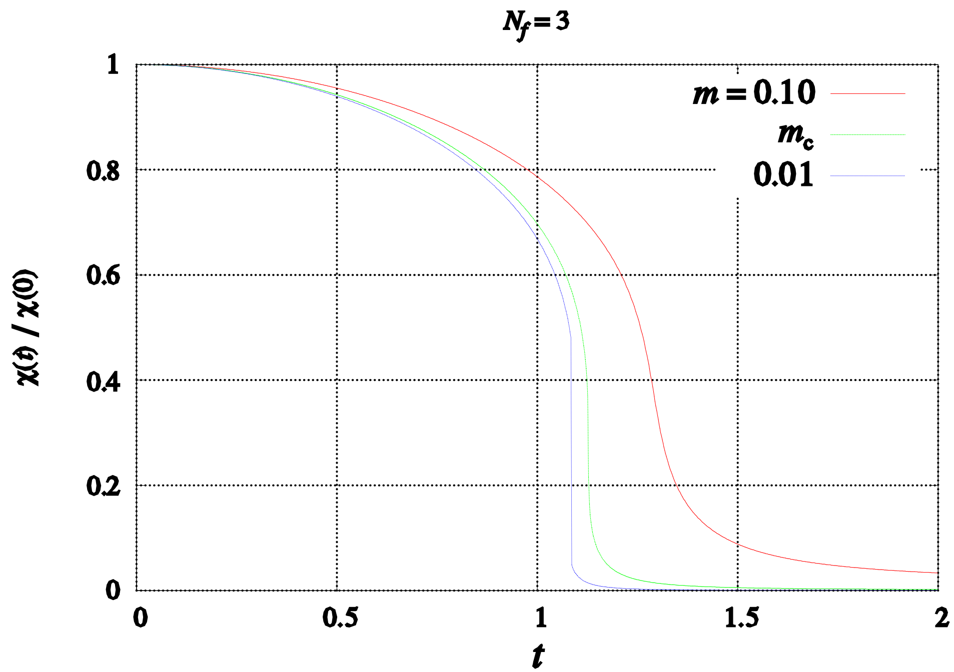
For the first time !

Topological Susceptibility

$$\chi_{\rm top} = -\frac{1}{2N}\frac{\partial^2}{\partial\theta^2}\log Z_\theta\Big|_{\theta=0}$$



Nf=2



Nf=3

- No unphysical suppression
- correct θ dependence: $Z(m,\theta) = Z(me^{i\theta/N_f}, 0)$

➔Axial Ward identity :

$$\chi_{\rm top} = -\frac{m^2}{N_f}\chi_{\rm ps} - \frac{m}{N_f}\langle\bar\psi\psi\rangle$$

2+1 flavor case at Finite T & μ

$$D = \begin{pmatrix} 0 & iW + iC \\ iW^\dagger + iC^T & 0 \end{pmatrix}$$

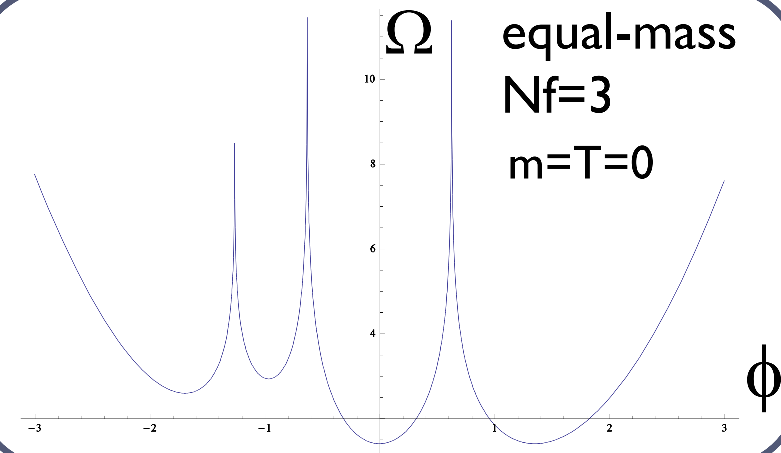
$$W \in \mathbf{C}^{(N+N_+)\times(N+N_-)}$$

near-zero mode

$$iC = \begin{pmatrix} (\mu + iT)\times \mathbf{1}_{N/2} & 0 & 0 \\ 0 & (\mu - iT)\times \mathbf{1}_{N/2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- μ→-μ symmetry

Halasz et. al. (1998)



$$\Omega = \frac{1}{2}\Sigma^2 \,\mathrm{Tr} S^\dagger S - \frac{1}{4}\log\left[\det\left((S+m)(S^\dagger+m^\dagger)-(\mu+iT)^2\right)\det\left((S+m)(S^\dagger+m^\dagger)-(\mu-iT)^2\right)\right]$$
$$-\frac{\gamma}{2}\log\left[\left(\alpha\det(S+m)+1\right)\left(\alpha\det(S^\dagger+m^\dagger)+1\right)\right]$$

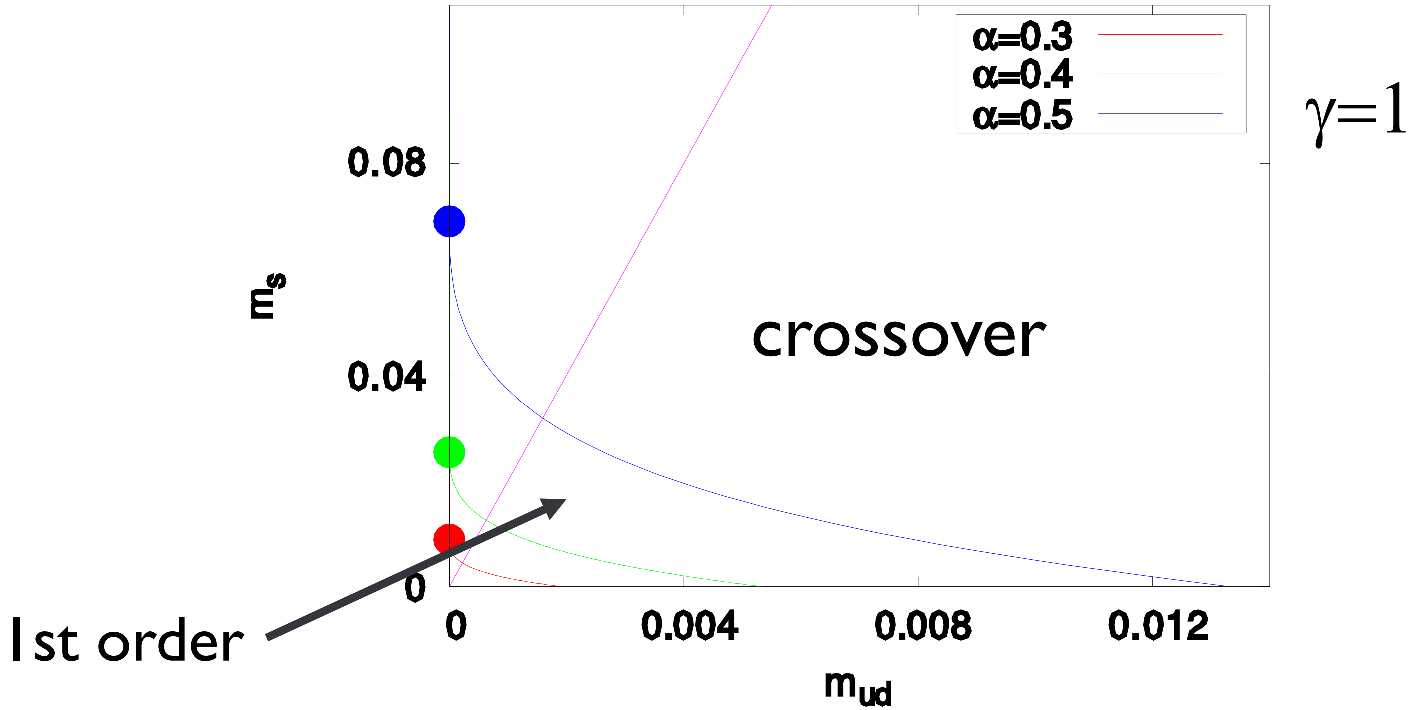
$$m = \mathrm{diag}(m_{ud},\ m_{ud},\ m_s)$$
$$S = \mathrm{diag}(\phi_{ud},\ \phi_{ud},\ \phi_s)$$

- Σ can be absorbed: Σ=1
- α & γ : “anomaly effects”

The 2+1 flavor case: μ=0 Plane

$$m = \mathrm{diag}(m_{ud}, m_{ud}, m_s)$$ → mass-dependent phase transition

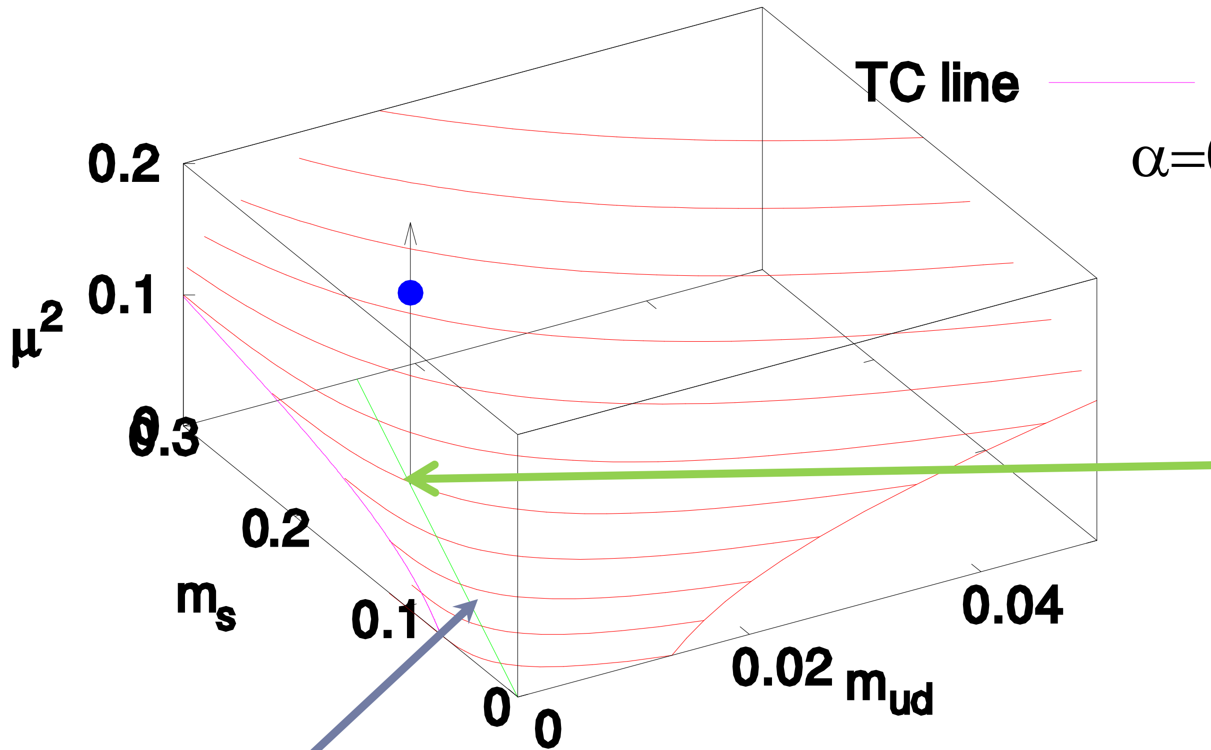
$m_s/m_{ud} = 20$



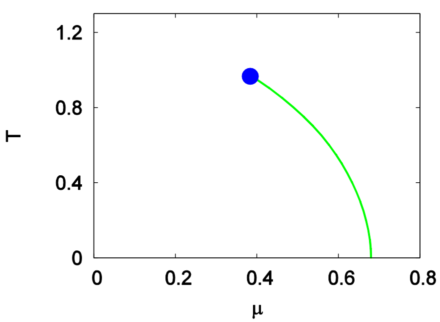
- Critical line on mud-ms plane
- TCP on ms axis

Critical Surface

- Positive curvature for all m which is the same as usual with effective models



$$m_s/m_{ud} = 20$$



Conclusions & Further Studies

- We have constructed the ChRM model with U(1) breaking determinant term
  - Stable ground state solution ← binomial distribution
  - 1st order phase transition for Nf=3 at finite T
- We apply the model to the 2+1 flavor case at finite T & μ
  - Critical surface: Positive curvature → Critical point exists

- Outlook
  - Isospin & strangeness chemical potential
  - Two color model: diquark condensate
  - Color superconductivity cf. Vanderheyden,& Jackson (2000)
  - …etc

Thank you