Proper Heavy $Q\bar{Q}$ Potential
from Lattice QCD

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See also: A.R., T. Hatsuda, S. Sasaki arXiv:0910.2321

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Heavy Quarkonia @ finite T

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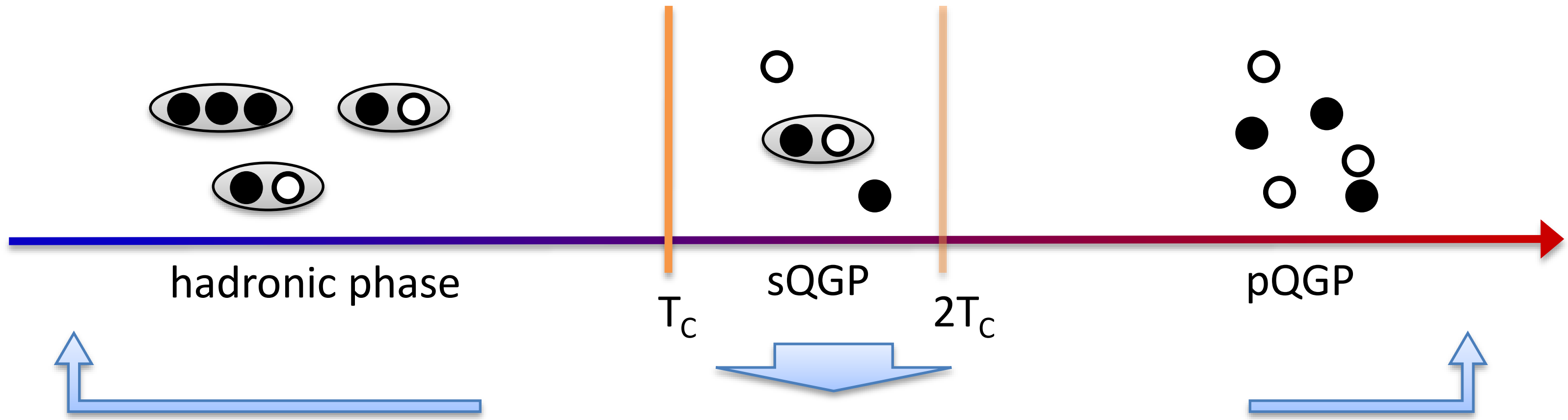
- $m \gg T$: non-relativistic description using a static potential



T=0 case Wilson loop

$$\psi(t) = e^{imt}\, e^{ig\int_0^t A^0(t)dt}\psi(0)$$

$$V(R) \overset{\tau\to\infty}{=} -\frac{1}{\tau}\log\Big[WL(\tau,R)\Big]$$

Potential Models

$$V(R) \sim \Delta F_{Q\bar{Q}}(R) \in \mathbb{R}$$

$$= -\frac{1}{\beta}\log\Big[\langle L(R)L^\dagger(0)\rangle\Big]$$

Hard Thermal Loop

$$V(R) = V_{DS}(R) + iV_{LD}(R)$$

$Q\bar{Q}$ is transient state

We propose a GAUGE-INVARIANT and NON-PERTURBATIVE definition of the proper potential based on the spectral function

Outline

- Motivation: Heavy Quark Potential
- Proper Potential from the Wilson Loop
  - Idea and formulation
  - First numerical results for $Re[V_0(R,T)]$
- Conclusion and Outlook

Formulation

- Starting point is the QQbar correlator and its spectral function:
  - QQbar mesic operator: $M(x,y) = \bar{\psi}(x)\,\Gamma\,U(x,y)\psi(y)$
  - QQbar forward correlator: $D^{>}(t,R) = \langle M_R(t) M_R^{\dagger}(0)\rangle$
  - Spectral function at finite T:

$$\rho(\omega,R) = \frac{1}{Z}\sum_{n,n'} |\langle n|M(0)|n'\rangle|^2 \left(e^{-\beta E_n} - e^{-\beta E_{n'}}\right)\delta(E_{n'} - E_n - \omega)$$

Antisymmetry

- In the spectral function we find three mutually exclusive cases:
  - Case I
  - Case II
  - Case III

Case I	Case II	Case III
Pure **medium** in $\lvert n\rangle$	Pure **medium** in $\lvert n'\rangle$	Q **or** Qbar +medium in $\lvert n'\rangle$
QQbar + medium in $\lvert n'\rangle$	**QQbar + medium** in $\lvert n\rangle$	Qbar **or** Q +medium in $\lvert n\rangle$

redundant

No interaction: irrelevant

Formulation II

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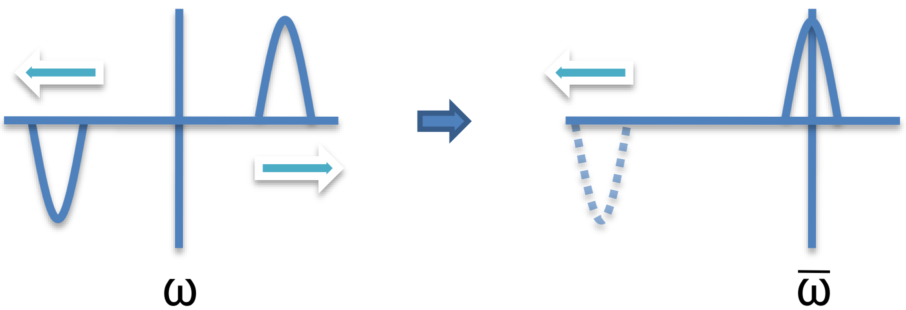
- Preparations for a consistent m → ∞ limit: frequency shift
  - Non-interacting case, two delta peaks
  - Retain positive ω peak: case I

$$\bar{\omega} = \omega - 2m$$



- Physics of the interaction lies in the relative position of the peak to ω̄=0

$$\rho^I(\bar{\omega}, R) = \frac{1}{Z_0} \sum_{n,n'} |\langle n|M(0)|Q\bar{Q} \in n'\rangle|^2 \delta(\bar{\omega} - (\epsilon_{n'}(R) - \epsilon_n))e^{-\beta E_n}$$

  - Finite Temperature effects from sum over n: $\epsilon_{n'}(R) = E_{n'}(R) - 2m$ is T independent

- Heavy mass limit: retarded and forward correlator are equal

$$D^I_>(t) = \int_{-\infty}^{\infty} e^{-i\omega t} D^I_>(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} \rho^I(\omega) = e^{-2imt} \int_{-\infty}^{\infty} e^{-i\bar{\omega} t} \rho^I(\bar{\omega})$$

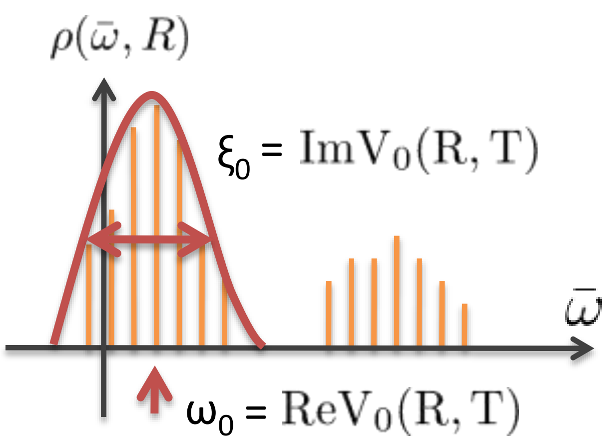
The Schrödinger Equation

$$i\partial_t D^I_>(t) = e^{-2imt}\int_{-\infty}^{\infty} d\bar{\omega}\Big(2m+\bar{\omega}\Big)e^{-i\bar{\omega}t}\rho^I(\bar{\omega})$$
$$= 2mD^I_>(t) + e^{-2imt}\int_{-\infty}^{\infty} d\bar{\omega}\Big(\bar{\omega}\Big)e^{-i\bar{\omega}t}\rho^I(\bar{\omega})$$

- Non interacting case:

$$\rho^I(\bar{\omega}) \propto \delta(\bar{\omega})$$
$$i\partial_t D^I_>(t) = 2mD^I_>(t)$$

- In a finite volume all energies are discrete but their envelope can exhibit broad peaks



- „Ground state potential": lowest lying peak structure
- Validity of Schrödinger description can be checked:

$$\int_{-\infty}^{\infty} d\bar{\omega}\Big(\bar{\omega}\Big)e^{-i\bar{\omega}t}\rho^I(\bar{\omega},R) \stackrel{!}{=} V(R)\int_{-\infty}^{\infty} d\bar{\omega}e^{-i\bar{\omega}t}\rho^I(\bar{\omega},R)$$

- In the high T region e.g. a Breit Wigner shape

$$i\partial_t D^I_>(t) = \Big[2m + \mathrm{ReV}_0(R,T) - i\mathrm{ImV}_0(R,T)\Big]D^I_>(t)$$

- Note: There is **no Schrödinger** equation for the **full** $D^>(t)$

$$D_>(t) = e^{-i\omega_0 t} - e^{i\omega_0 t} \quad \Longrightarrow \quad i\partial_t D_>(t) = \omega_0\Big[e^{-i\omega_0 t} + e^{i\omega_0 t}\Big]$$

Connection to Lattice QCD

- Analytic continuation gives:

$$D^I_>(\tau,R) = e^{-2m\tau}\int_{-\infty}^{\infty} d\bar{\omega}\, e^{-\bar{\omega}\tau}\rho^I(\bar{\omega},R)$$

  - Cannot yet take the heavy mass limit
  - What form does $D^I_>$ have?

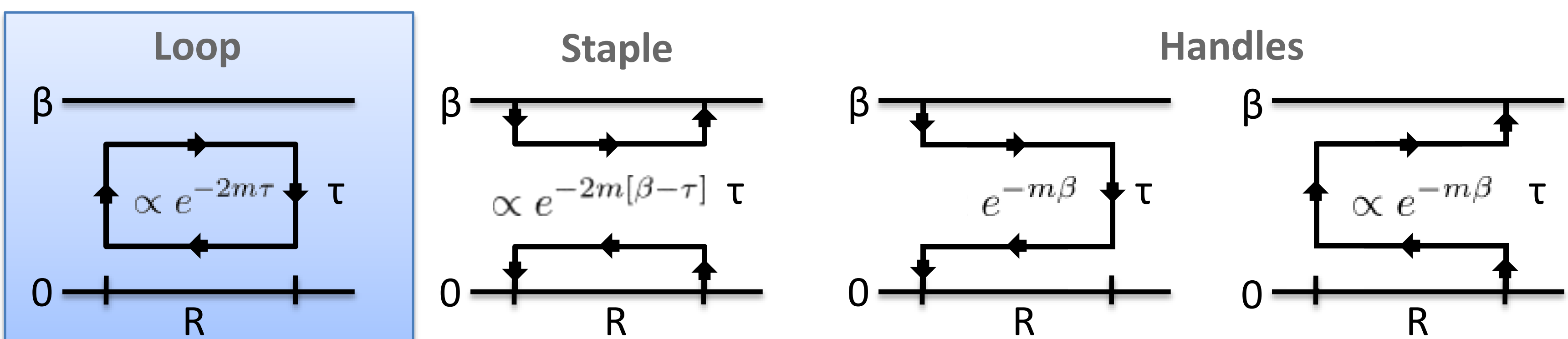
- Finding an expression for $D_>(\tau)$:

$$\langle M_R(\tau)M_R^\dagger(0)\rangle = -\sum_n \langle n|e^{-\beta H}\psi(x,0)\bar{\psi}(x,\tau)\Gamma U_\tau(\vec{x},\vec{y})\psi(y,\tau)\bar{\psi}(y,0)U_0(\vec{y},\vec{x})\bar{\Gamma}|n\rangle$$

$$S_E(z,z') = \langle T_\tau\, \psi(z,\tau)\bar{\psi}(z',\tau')\rangle \quad \xrightarrow{\text{static limit}} \quad (-i\gamma_4 D_\tau + m)S_E(\tau,\tau') = \delta(\tau-\tau')$$

  - Additionally: boundary conditions in τ direction

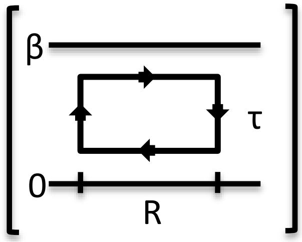
- In the **heavy mass limit**: only **Wilson lines** remain for $D^>(\tau)$



Exploring the Proper Potential

- Combining both spectral function and the imaginary time correlator:

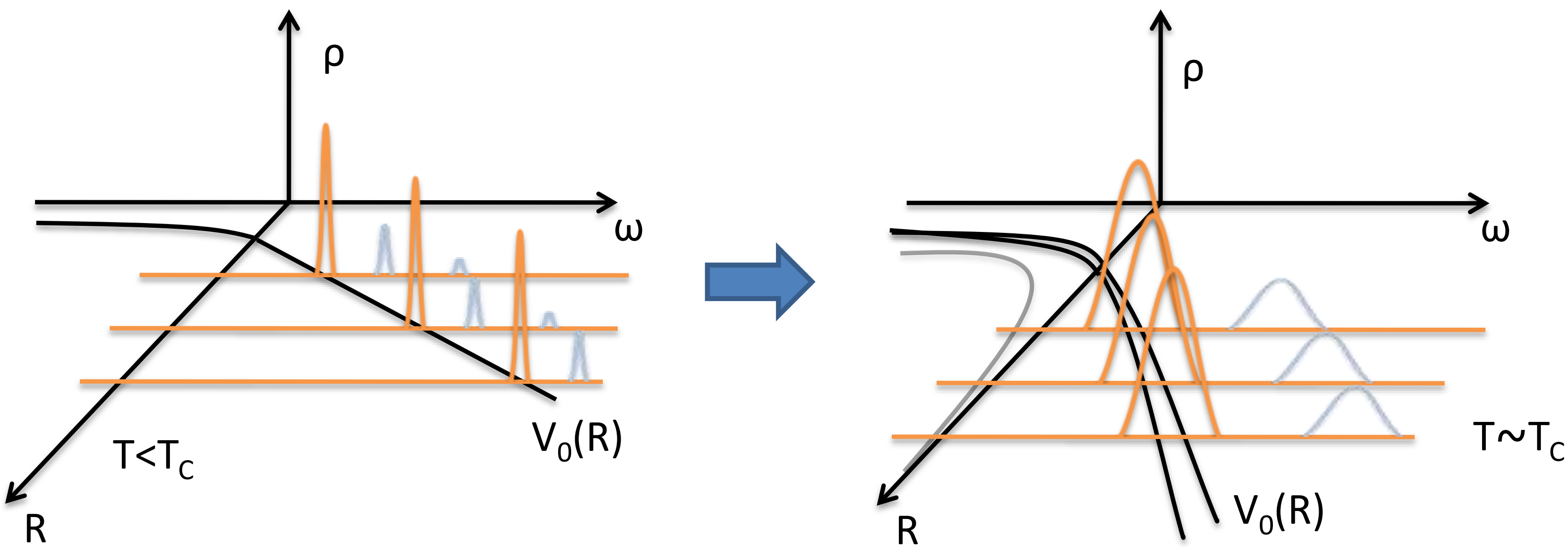
$$\frac{D^{\rm loop}_{>}(\tau,R)}{e^{-2m\tau}} = \mathrm{Tr}\Big[$$

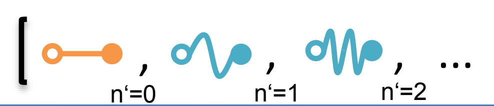


$$\Big] = \int_{-\infty}^{\infty} d\bar{\omega}\, e^{-\tau\bar{\omega}} \rho^{\rm loop}(\bar{\omega},R)$$

Lattice QCD — **MEM / Exponential Fitting**

- **Reconstruct** spectral function for **different R**: map the shape of the potential:





Simulation Details:

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- Anisotropic Lattices
  - Since τ direction is compact: need to increase # of points
  - Renormalized anisotropy needs to be determined: compare spatial and temporal Wilson Loop ratios

Pure SU(3): Heatbath algorithm, naive Wilson action

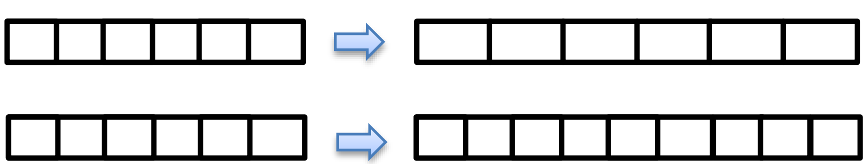
NX=20 NT=32 $T=2.33T_C$
NT=96 $T=0.78T_C$

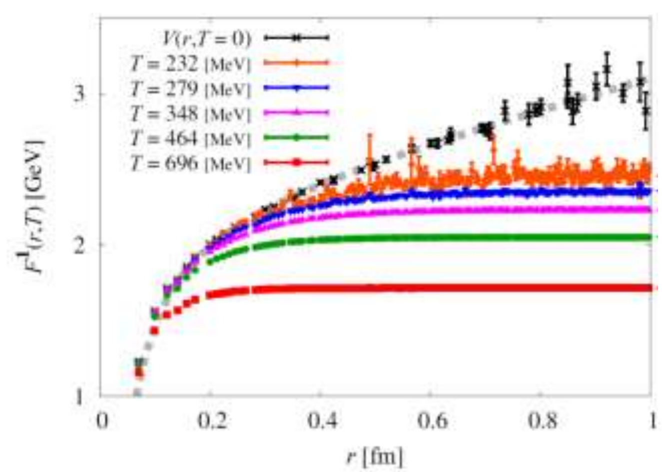
β=7.0 $\xi_0=3.5$ $a_\tau= ¼a_\sigma =0.01$fm

- How to determine the physical scale
  - Sommer Scale, relies on the Free-Energies "Potential": $$r^2 \frac{\partial V(r)}{\partial r}\Big|_{r=r_0} = 1.65 \qquad r_0 = 0.5fm$$
  - Using rho-mesons operator: compare mass to the chiral limit value Asakawa, Hatsuda, PRL92, 2004
- Fixed scale vs. Fixed size Lattice QCD
  - Change NT instead of a_τ : **$T = 1/(NT \times a_\tau)$**
  - Renormalization parameter same for all T

Δβ

ΔN_τ





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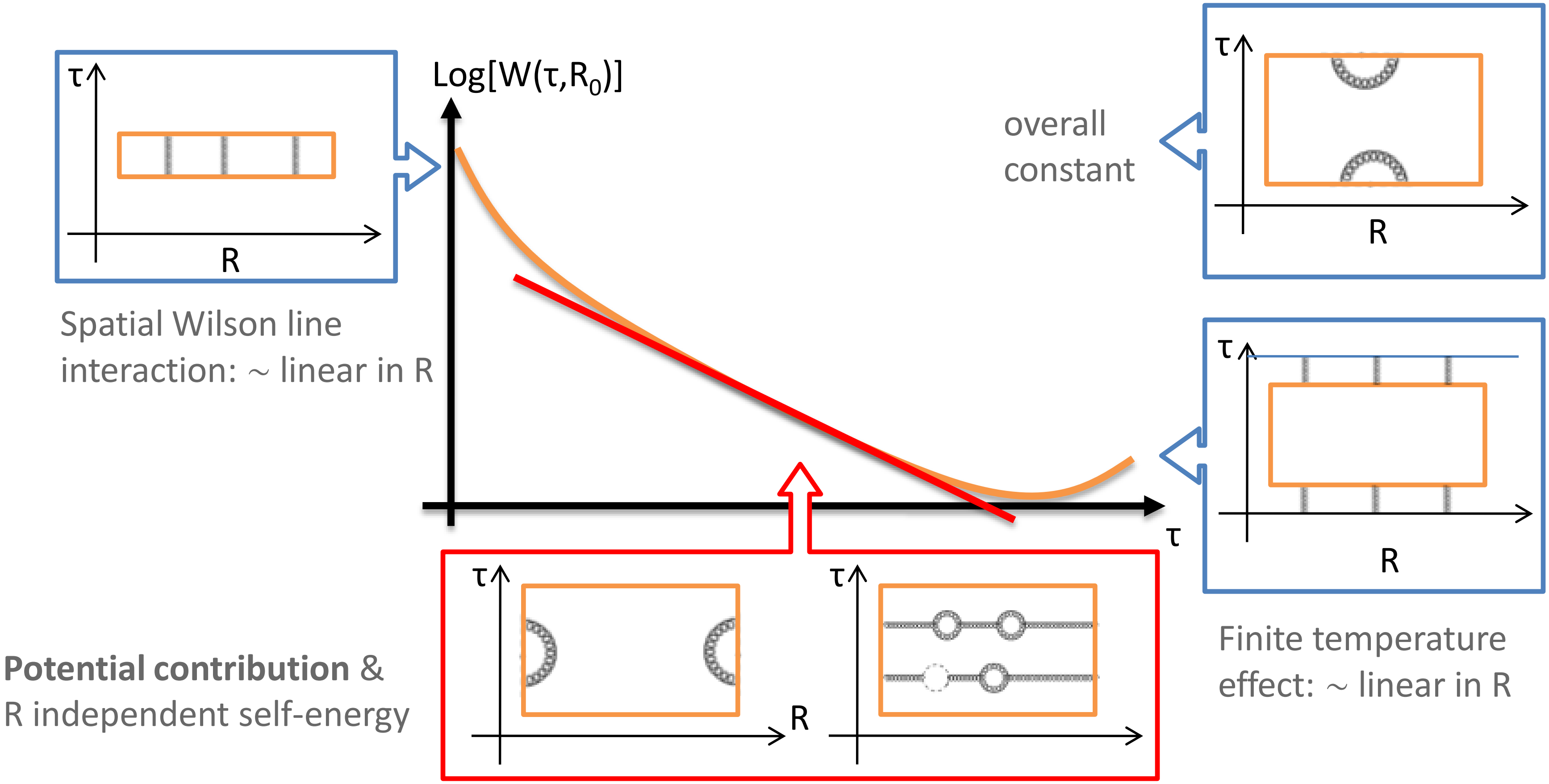
small R physics is T independent

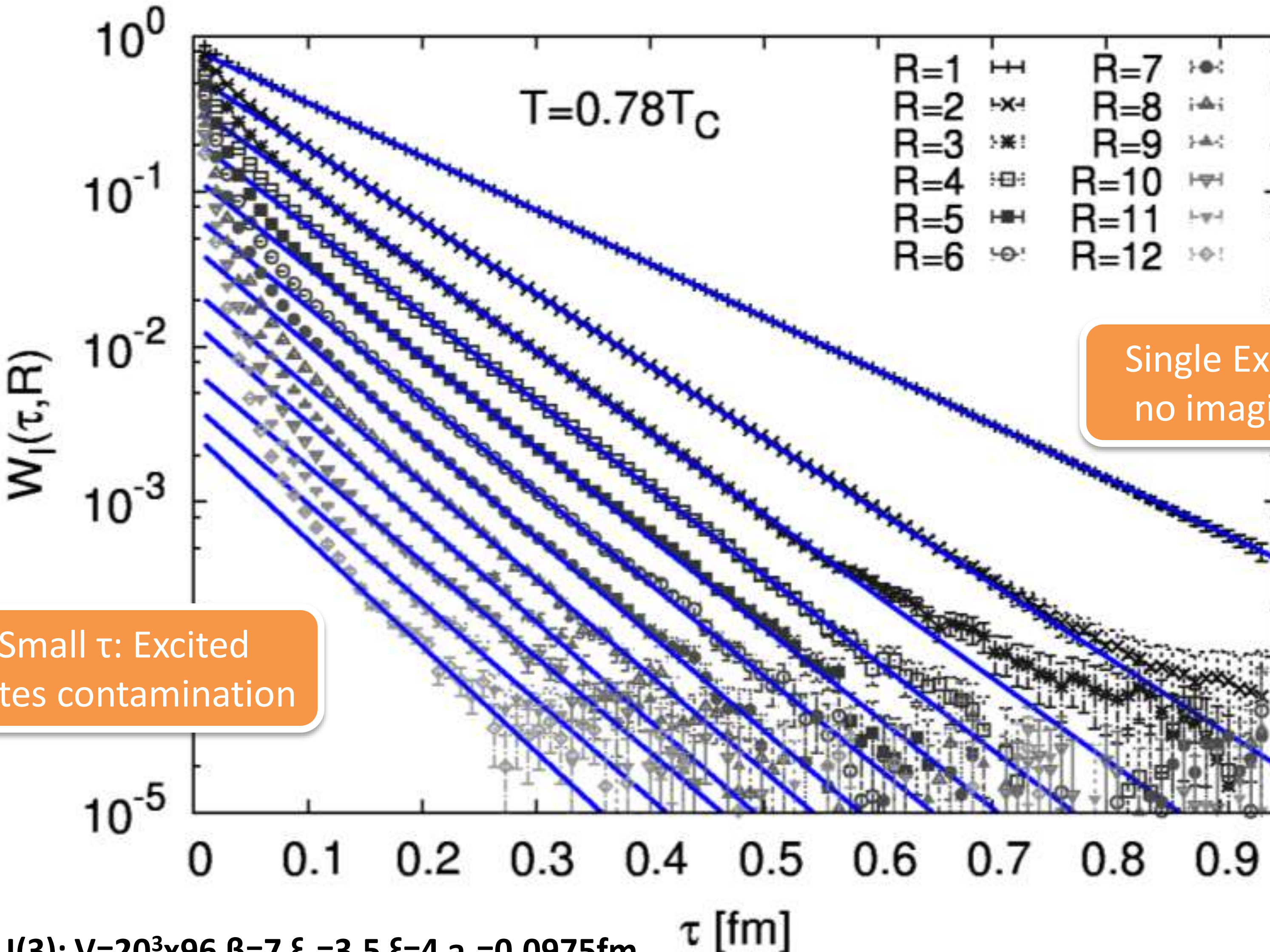
Extracting Re[V(R)]

- Exponential fitting analysis:
  - Identify the range that corresponds to the $\bar{Q}Q$ interaction and use a delta peak ansatz

$$\rho(\bar{\omega}, R_0) = C_0 \delta(\bar{\omega} - \bar{\omega}_0)$$ ⇒ X^2 fitting



Simulation results at $T=0.78T_C$



Single Exponential, no imaginary part

Small τ: Excited states contamination

Pure SU(3): $V=20^3 x96$ $\beta=7$ $\xi_0=3.5$ $\xi=4$ $a_\tau=0.0975fm$

Simulation results at T=0.78T_C

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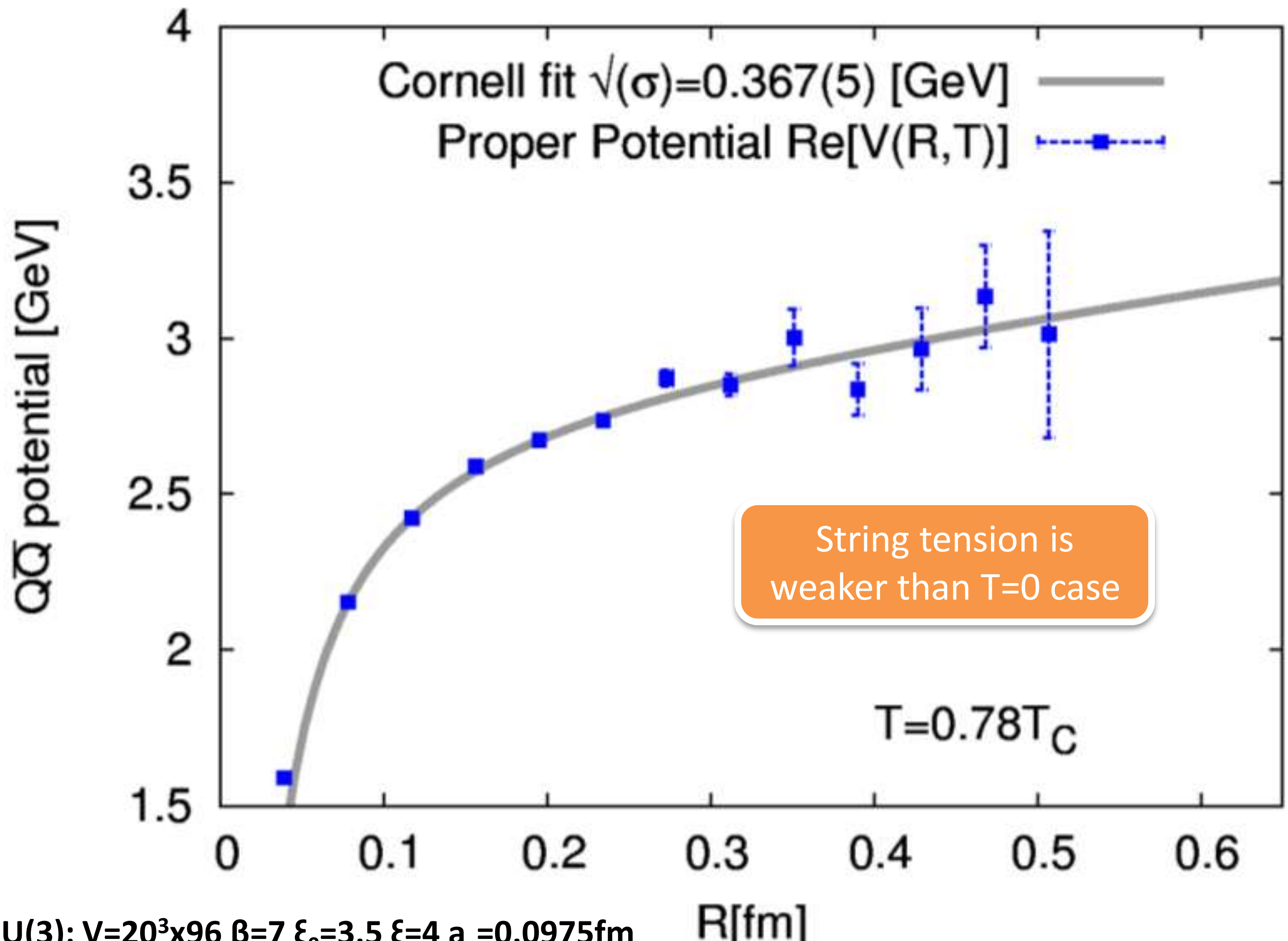
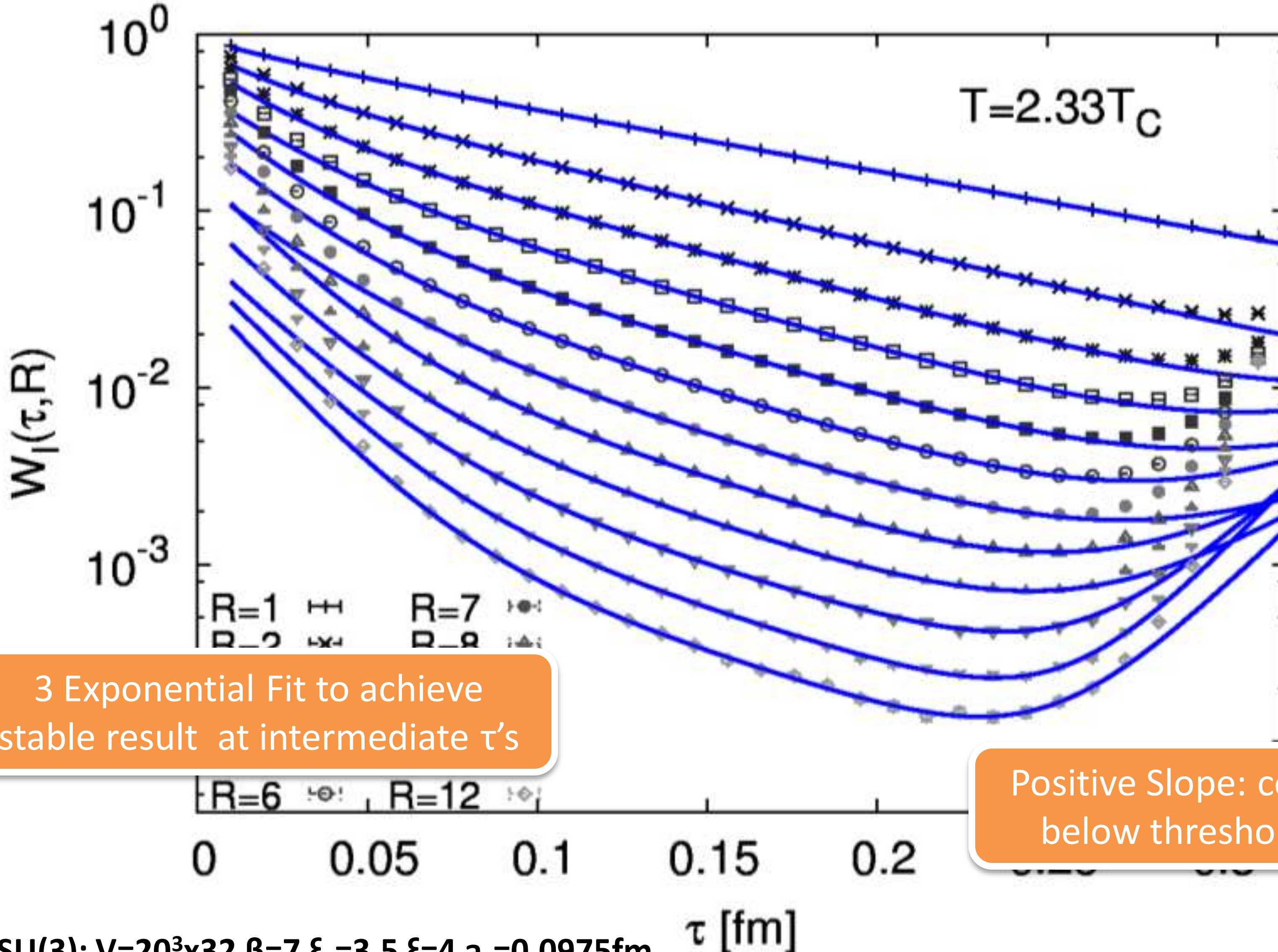


Pure SU(3): V=20^3x96 β=7 ξ_0=3.5 ξ=4 a_τ=0.0975fm

Simulation results at $T=2.33T_C$



3 Exponential Fit to achieve stable result at intermediate τ's

Positive Slope: contribution below threshold in $\rho(\bar{\omega})$

Pure SU(3): $V=20^3$x32 $\beta=7$ $\xi_0=3.5$ $\xi=4$ $a_\tau=0.0975$fm

Simulation results at T=2.33T_C

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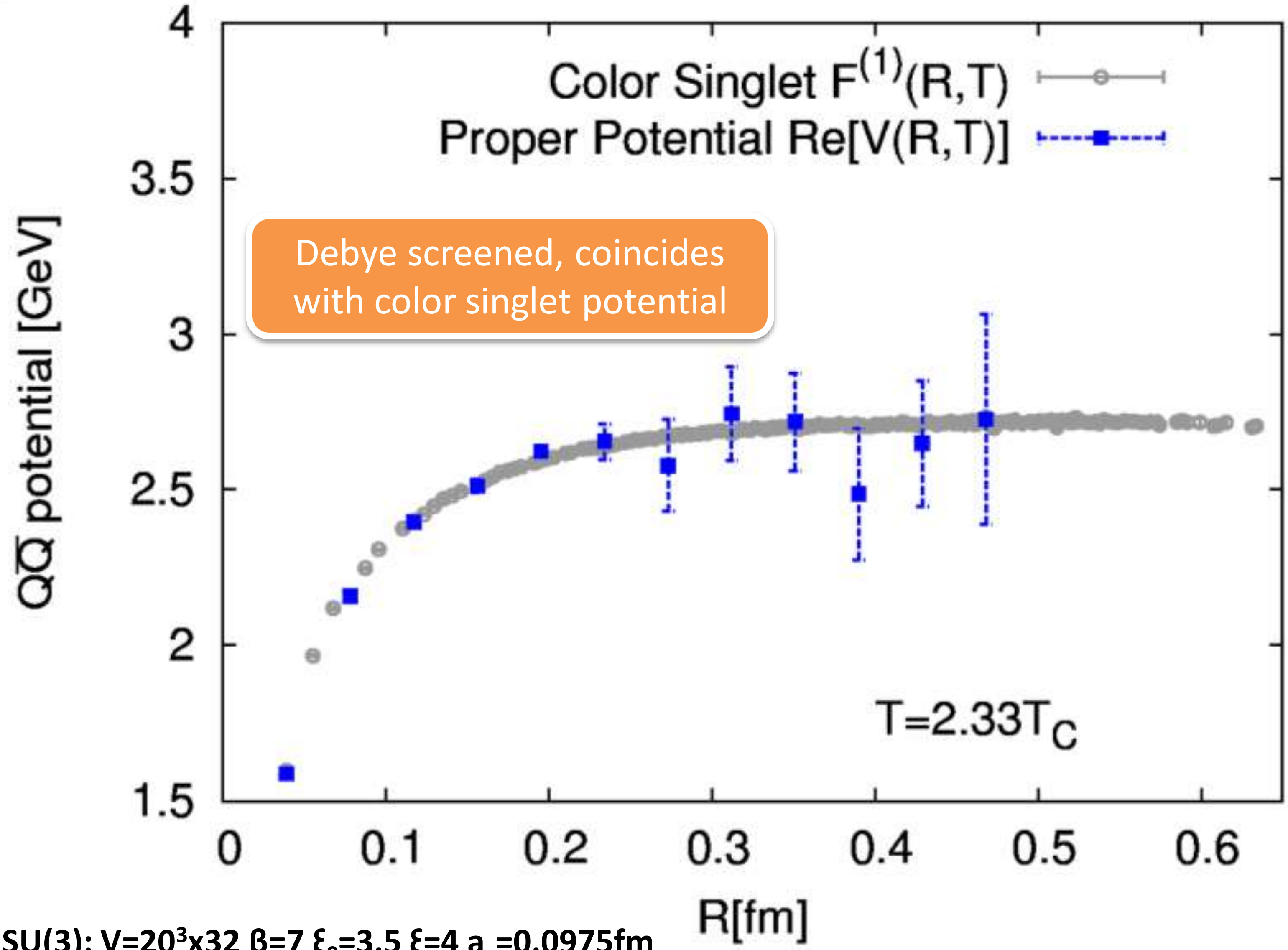

Pure SU(3): V=20^3x32 β=7 ξ_0=3.5 ξ=4 a_τ=0.0975fm

Conclusion & Outlook

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- First-principles definition of the heavy quark potential:
  - Wilson Loop and its spectral function connected to V(R,T) **non-perturbatively**
  - Ground state peak envelope of the spectral function leads to a **Schrödinger Equation**
  - Peak structure of the spectral function provides **real part** (position) and **imaginary part** (width) of the potential V(R,T)
  - Current results with $\mathbf{m_Q=\infty}$ and **quenched QCD:**

    Re[V(R,T)] below $\mathbf{T_C}$: Confining potential with weakened string tension ($\sqrt{\sigma}$ < 430 MeV)

    Re[V(R,T)] above $\mathbf{T_C}$: Debye screened , coincides with color singlet potential from free energies

- Work in progress and future directions:
  - MEM Investigation above T_C with focus on a possible imaginary part -> Melting
  - Use of full QCD data to include the influence of light quarks in the medium (WHOT)
  - Extend the definition to include finite mass corrections

The End

Thank you for your attention