Dissipative Hydrodynamics

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Quantum Fields at Finite Temperature "From Tera to Nano Kelvin" Discussion II with Professor Jean-Paul Blaizot

November 25th 2009, Komaba, The University of Tokyo

AM and T. Hirano, Phys. Rev. C **80**, 054906 (2009)
AM and T. Hirano, in preparation

Outline

1. Introduction

   Relativistic hydrodynamics and heavy ion collisions

2. Distortion of Distribution

   How to express δf by dissipative currents

3. Effects on Observables

   Numerical results of δf on observables

4. Summary and Outlook

   How to obtain dissipative currents

1. Introduction

Next: 2. Distortion of Distribution

Introduction

- Relativistic hydrodynamics

Macroscopic description of a very hot, strongly-coupled system

⇨ Various applications are expected
(heavy ion collisions, early universe, cold atomic gas, etc…)

- What hydrodynamics does:

Inputs	Outputs
Initial condition Equation of state Transport coefficients	Energy-momentum tensor $T^{\mu\nu}(x)$ Conserved charge current $N_B^\mu(x)$ Temperature $T(x)$ Chemical potential $\mu_B(x)$ Flow $u^\mu(x)$

Microscopic physics is integrated in. Macroscopic dynamics comes out.

Introduction

- Tensor decompositions – Projection to/against $u^\mu(x)$

$$T^{\mu\nu} = eu^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + 2W^{(\mu}u^{\nu)} + \pi^{\mu\nu}$$
$$N_B^\mu = n_B u^\mu + V^\mu$$

e : energy density
P_0: hydrostatic pressure
n_B: charge density

Π : bulk pressure
$\pi^{\mu\nu}$: shear stress tensor
W^μ: energy current
V^μ: charge current

Dissipative currents (= 0 in ideal hydro)

- Multi-component system

(multi component theory) $\neq \sum$(single component theory)

because of (i) difference of particle masses, (ii) pair creation/annihilation

Not considered seriously, but *important* for applications

⇨ I discuss a multi-component theory in this talk

Relativistic Heavy Ion Collisions

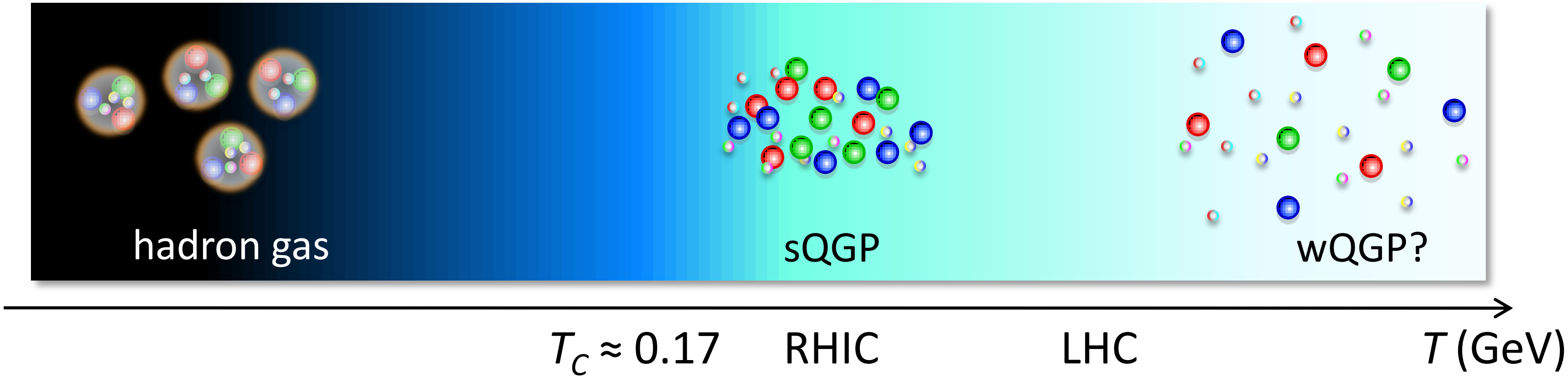
- RHIC experiment (2000-)

The quark-gluon plasma (QGP) created at heavy ion collisions $\sqrt{s_{NN}} = 200\text{GeV}$

⇨ It obeys relativistic ideal hydrodynamic models well.

- Strongly-coupled QGP (sQGP)

The success of ideal hydrodynamics suggests sQGP at RHIC



Relativistic Heavy Ion Collisions

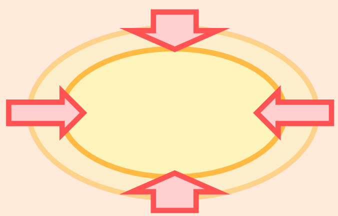
- LHC experiment (2010?-)

  Coupling constant "runs" to become smaller as energy gets higher

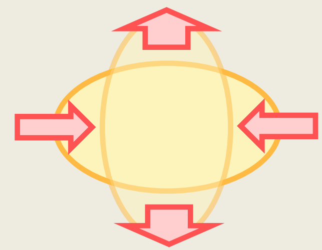
  ⇨ Dissipative hydrodynamic models will become more important

- Viscosity in QGP

Kharzeev & Tuchin ('08) …



bulk viscosity =
response to volume change



shear viscosity =
response to deformation

Bulk viscosity is usually neglected, *BUT* is not so small near T_c

⇨ I put emphasis on bulk viscous effects in this talk

Relativistic Heavy Ion Collisions

- How does viscosity affects observables?

One needs a convertor of flow field into particles at freezeout

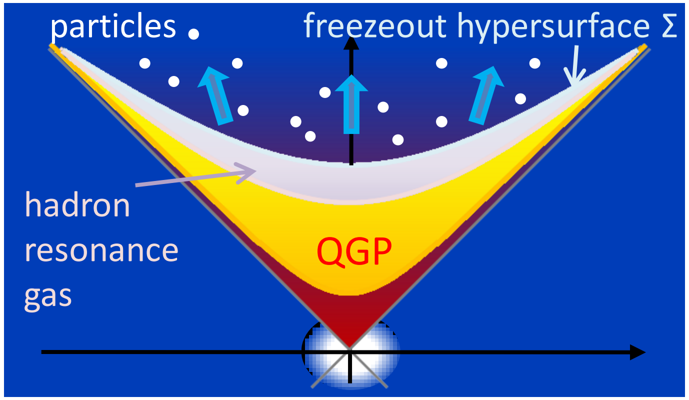
hydro result — observables

Cooper-Frye formula

$$\frac{d^2 N_i}{d^2 p_T dy} = \frac{g_i}{(2\pi)^3} \int_\Sigma p_i^\mu d\sigma_\mu (f_0^i + \delta f^i)$$

variation of the flow/hypersurface

modification of the distribution



We need to estimate both δf^i and δu^μ in a multi-component system

2. Distortion of Distribution

Previous: 1. Introduction

Next: 3. Effects on Observables

Macroscopic to Microscopic

- Express δf^i in terms of dissipative currents

Israel & Stewart ('76)

Macroscopic quantities

$\Pi, W^\mu, V^\mu, \pi^{\mu\nu}$

Dissipative currents
(given from hydro)

Microscopic quantities

δf^i

Distortion of distribution
(unknown)

14 "bridges" from Relativistic Kinetic Theory

$$\Pi = -\frac{1}{3}\Delta_{\mu\nu}\sum_i \int \frac{g_i d^3p}{(2\pi)^3 E_i} p_i^\mu p_i^\nu \delta f^i \qquad \pi^{\mu\nu} = \sum_i \int \frac{g_i d^3p}{(2\pi)^3 E_i} p_i^{\langle\mu} p_i^{\nu\rangle} \delta f^i$$

$$W^\mu = \Delta^\mu{}_\nu u_\rho \sum_i \int \frac{g_i d^3p}{(2\pi)^3 E_i} p_i^\nu p_i^\rho \delta f^i \qquad 0 = u_\mu \sum_i \int \frac{b_i g_i d^3p}{(2\pi)^3 E_i} p_i^\mu \delta f^i$$

$$V^\mu = \Delta^\mu{}_\nu \sum_i \int \frac{b_i g_i d^3p}{(2\pi)^3 E_i} p_i^\nu \delta f^i \qquad 0 = u_\mu u_\nu \sum_i \int \frac{g_i d^3p}{(2\pi)^3 E_i} p_i^\mu p_i^\nu \delta f^i$$

δf^i in Multi-Component System

- Grad's 14-moment method ⇨ 14 unknowns ε^μ, $\varepsilon^{\mu\nu}$

$$\delta f^i = -f_0^i(1 \pm f_0^i)[p_i^\mu \varepsilon_\mu + p_i^\mu p_i^\nu \varepsilon_{\mu\nu}]$$

No scalar, but **non-zero trace tensor**

$\partial_\mu s^\mu = \varepsilon_{\mu\nu} \partial_\alpha I^{\mu\nu\alpha} \geq 0$: 2nd law of thermodynamics

\+ $\partial_\alpha I^{\mu\alpha}_\mu \neq 0$ in multi-comp. system ⇨ $\varepsilon^\mu_\mu \neq 0$

- The distortion is uniquely obtained:

New tensor structure for multi-component system

$$\varepsilon_\mu = D_0 \Pi u_\mu + D_1 W_\mu + \tilde{D}_1 V_\mu$$

$$\varepsilon_{\mu\nu} = (B_0 \Delta_{\mu\nu} + \tilde{B}_0 u_\mu u_\nu) \Pi + 2B_1 u_{(\mu} W_{\nu)} + 2\tilde{B}_1 u_{(\mu} V_{\nu)} + B_2 \pi_{\mu\nu}$$

where D_i and B_i are calculated in kinetic theory.

3. Effects on Observables

Previous: 2. Distortion of Distribution

Next: 4. Summary and Outlook

Model Inputs

- Estimation of particle spectra (with bulk viscosity in δf):

$$\frac{d^2N_i}{d^2p_T dy} = \frac{g_i}{(2\pi)^3}\int_\Sigma p^\mu d\sigma_\mu [f_0^i - f_0^i(1\pm f_0^i)(D_0 \Pi u_\mu p^\mu + (B_0\Delta_{\mu\nu} + \tilde{B}_0 u_\mu u_\nu)\Pi p^\mu p^\nu)]$$

Flow u^μ, freezeout hypersurface $d\sigma_\mu$:
(3+1)-D ideal hydrodynamic model
Hirano et al.('06)

Bulk pressure: $\Pi = -\zeta \nabla_\mu u^\mu$
Navier-Stokes limit

Equation of State: 16-component hadron resonance gas
(hadrons up to $\Delta(1232)$, under $\mu \to 0$)

Transport coefficients:

$$\zeta = \alpha\left(\frac{1}{3} - c_s^2\right)^2 \eta,\ \eta = \frac{1}{4\pi}s$$

where $c_s \equiv \sqrt{\frac{\partial p}{\partial e}}$: sound velocity

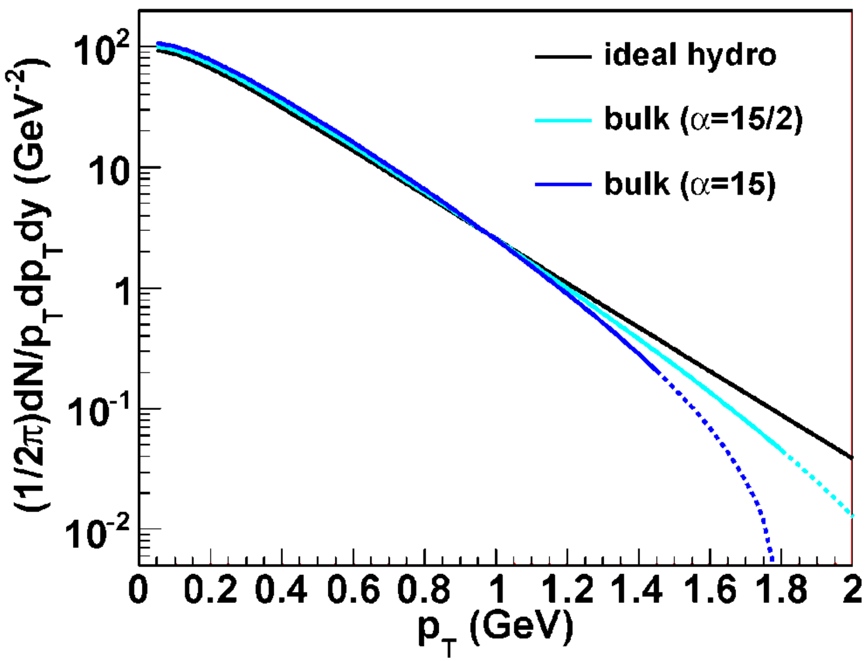
s: entropy density

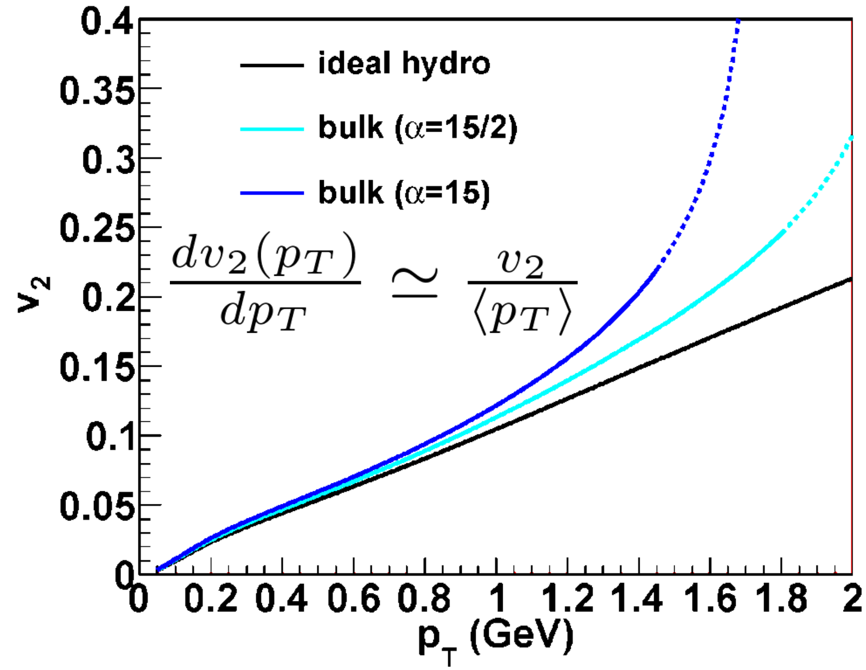
Weinberg ('71) *Kovtun et al.('05)*

Freezeout temperature: T_f = 0.16(GeV)

Bulk Viscosity and Particle Spectra

- *Au+Au*, $\sqrt{s_{NN}} = 200(\mathrm{GeV})$, *b* = 7.2(fm), p_T-spectra and $v_2(p_T)$ of π^-





p_T-spectra ⇨ suppressed

$v_2\ (p_T)$ ⇨ enhanced

Even “small” bulk viscosity may have significant effects on particle spectra

4. Summary and Outlook

Previous: 3. Effects on Observables

Next: Appendix

Summary and Outlook

- Determination of δf^i in a multi-component system
  - Viscous correction $\varepsilon_{\mu\nu}$ has non-zero trace.
- Visible effects of δf_{bulk} on particle spectra
  - p_T-spectra is *suppressed*; $v_2(p_T)$ is *enhanced*

- Bulk viscosity can be important in extracting information (e.g. transport coefficients) from experimental data.
- Full *Viscous* hydrodynamic models need to be developed to see more realistic behavior of the particle spectra.

Estimation of Dissipative Currents

- 2nd order Israel-Stewart theory

AM and T. Hirano, in preparation

⇨ Naïve generalization to a multi-component system does *NOT* work

- Constitutive equations in a multi-component system:

__*Bulk pressure*__

$$
\begin{aligned}
\Pi = & -\zeta \nabla_\mu u^\mu && \text{Navier-Stokes term} \\
& - \tau_\Pi D\Pi + \chi^a_{\Pi W} W_\mu D u^\mu + \chi^a_{\Pi V} V_\mu D u^\mu && \\
& + \chi^b_{\Pi W} \nabla^\mu W_\mu + \chi^b_{\Pi V} \nabla^\mu V_\mu && \text{Israel-Stewart 2nd order terms} \\
& + \chi^a_{\Pi\Pi} \Pi \nabla_\mu u^\mu + \chi^b_{\Pi\Pi} \Pi D \frac{\mu_B}{T} + \chi^c_{\Pi\Pi} \Pi D \frac{1}{T} && \\
& + \chi^c_{\Pi W} W_\mu \nabla^\mu \frac{\mu_B}{T} + \chi^c_{\Pi V} V_\mu \nabla^\mu \frac{\mu_B}{T} && \text{Post Israel-Stewart 2nd order terms} \\
& + \chi^d_{\Pi W} W_\mu \nabla^\mu \frac{1}{T} + \chi^d_{\Pi V} V_\mu \nabla^\mu \frac{1}{T} + \chi_{\Pi\pi} \pi_{\mu\nu} \nabla^{\langle\mu} u^{\nu\rangle} &&
\end{aligned}
$$

Shear tensor $\pi^{\mu\nu}$ in conformal limit reduces to AdS/CFT result (*Baier et al. '08*)

Thank You

- The numerical code will become available at

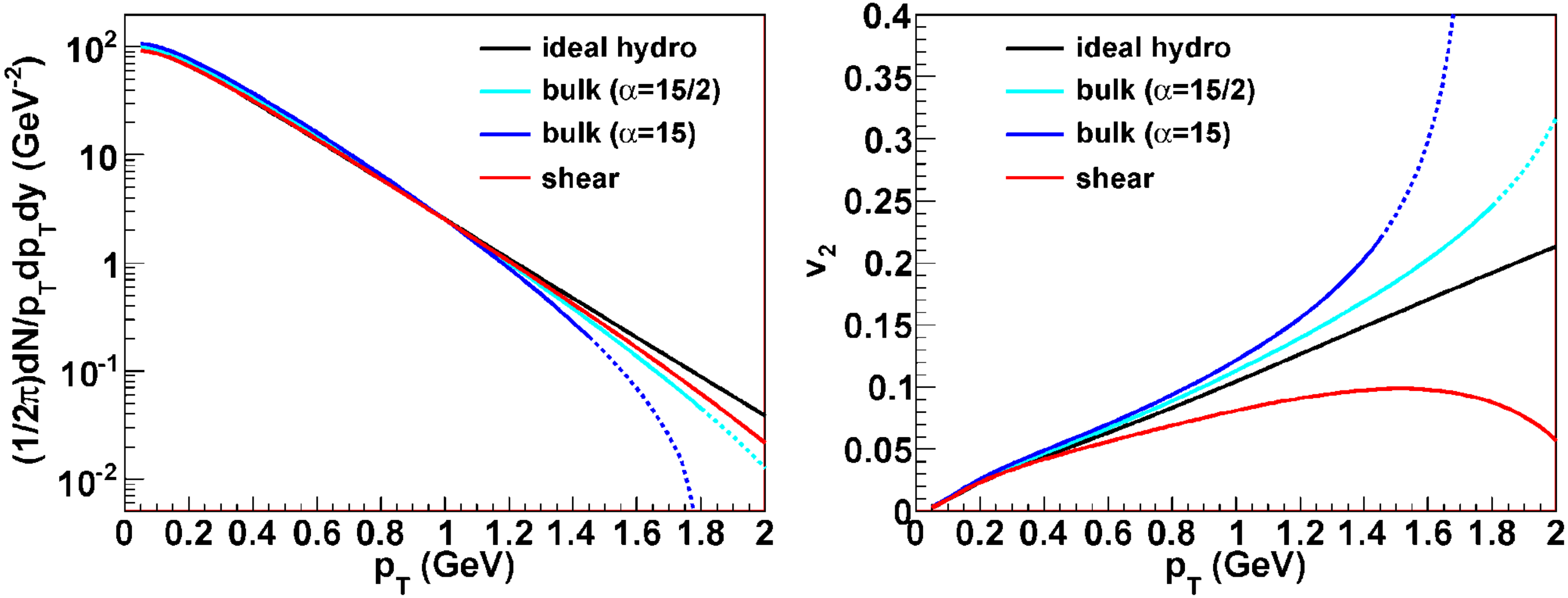
  http://tkynt2.phys.s.u-tokyo.ac.jp/~monnai/distributions.html

Appendix

Previous: 4. Summary and Outlook

Shear Viscosity and Particle Spectra

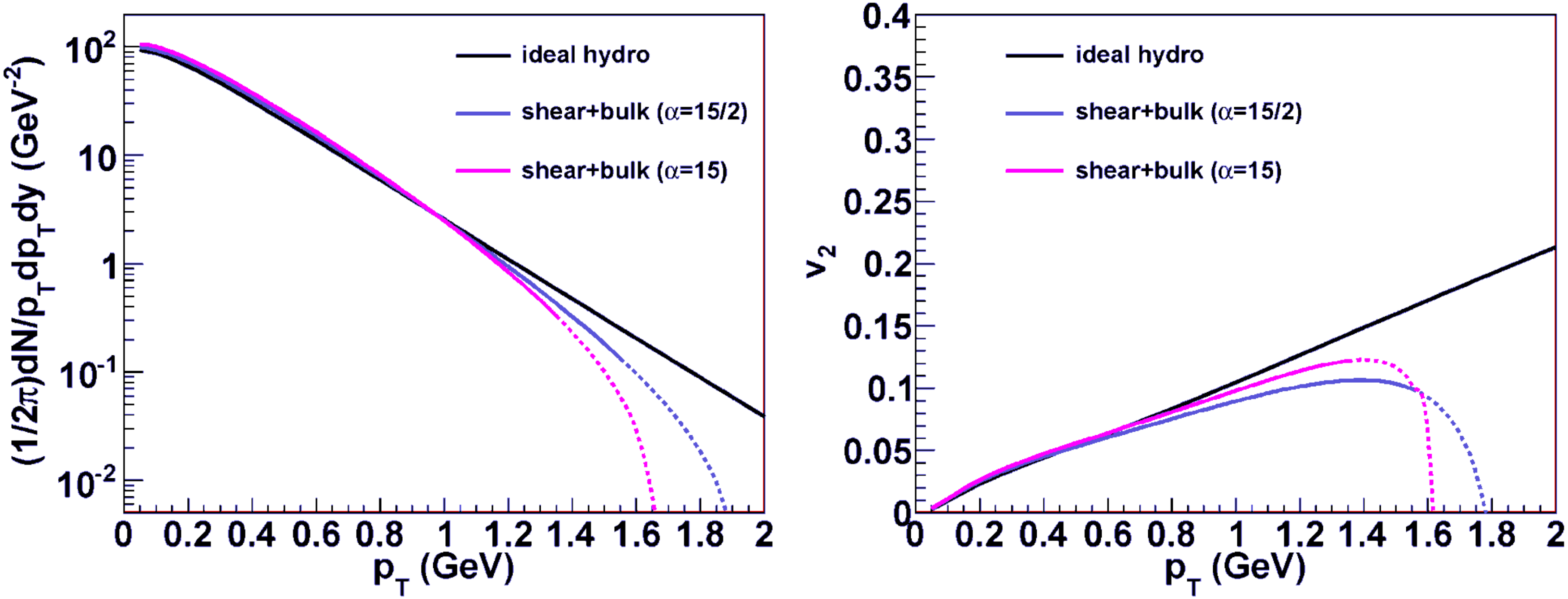
- p_T-spectra and $v_2(p_T)$ of π^- with shear viscous correction



Non-triviality of shear viscosity; both p_T-spectra and $v_2(p_T)$ suppressed

Shear & Bulk Viscosity on Spectra

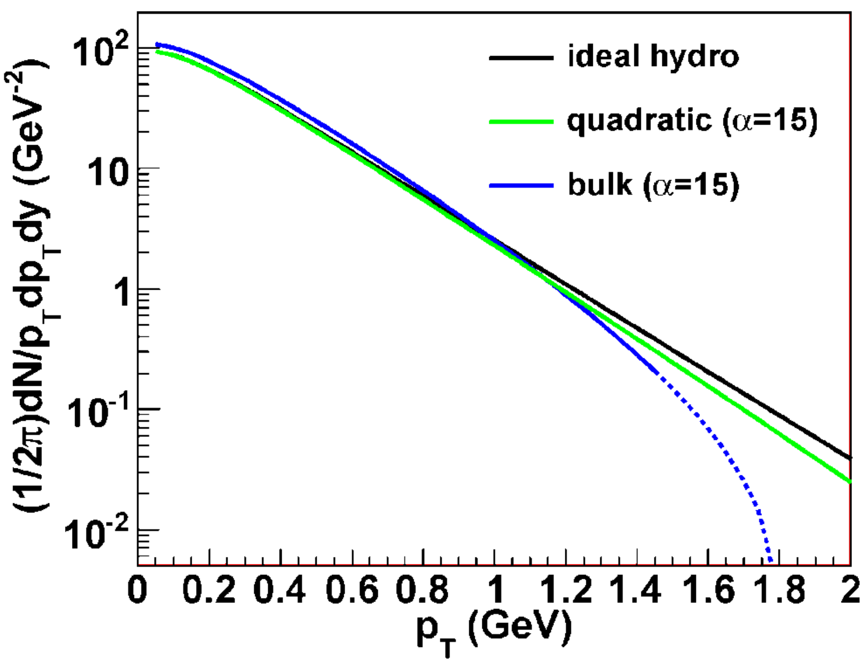
- p_T -spectra and $v_2(p_T)$ of π^- with corrections from shear and bulk viscosity

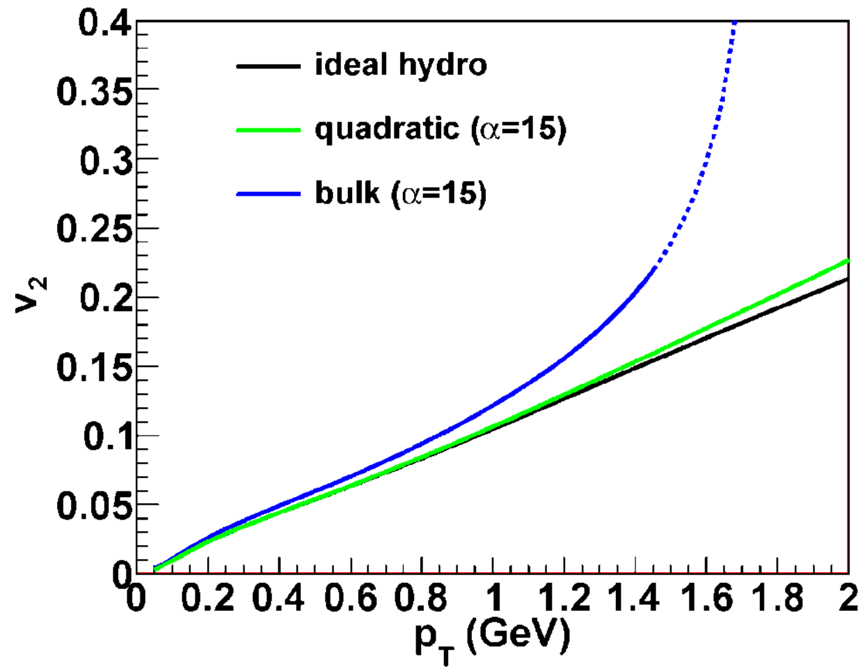


Overall viscous correction suppresses $v_2(p_T)$; consistent with experiments

Quadratic Ansatz

- p_T-spectra and $v_2(p_T)$ of π^- when $\varepsilon_{\mu\nu} = C_1\pi_{\mu\nu} + C_2\Delta_{\mu\nu}\Pi$





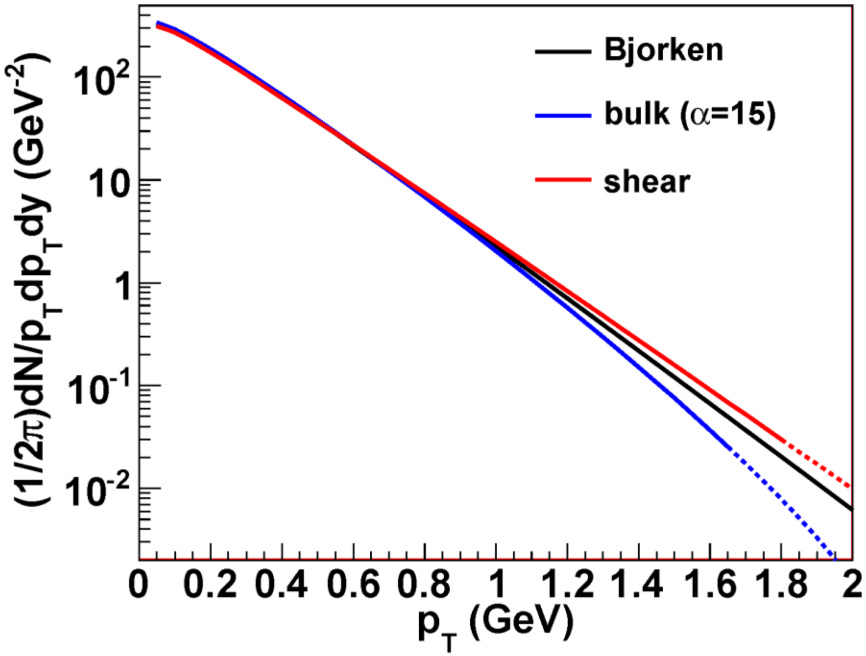
Effects of the bulk viscosity is underestimated in the quadratic ansatz.

Bjorken Model

- p_T-spectra and $v_2(p_T)$ of π^- in Bjorken model with cylindrical geometry: $R_0 = 10.0\text{fm}, \tau = 7.5\text{fm}$

$$u^\tau = 1,\ u^r = u^\phi = u^\eta = 0$$

$$d\sigma_\tau = \tau d\eta r dr d\phi,\ d\sigma_r = d\sigma_\phi = d\sigma_\eta = 0$$



Bulk viscosity suppresses p_T-spectra
Shear viscosity enhances p_T-spectra

Blast wave model

- p_T-spectra and $v_2(p_T)$ of π^-

$$u^r = u_0 \frac{r}{R_0}[1 + u_2 \cos(2\phi)]\Theta(R_0 - r)$$
$$u^\tau = \sqrt{1 + (u^r)^2}$$
$$u^\phi = u^{\eta_s} = 0$$

$$R_0 = 7.5 \text{ fm}, \tau = 5.25 \text{ fm}$$
$$u_0 = 0.55, u_2 = 0.2$$

Shear viscosity *enhances* p_T-spectra and suppresses $v_2(p_T)$.