A study of charmonium dissociation temperatures using a finite volume technique in the lattice QCD

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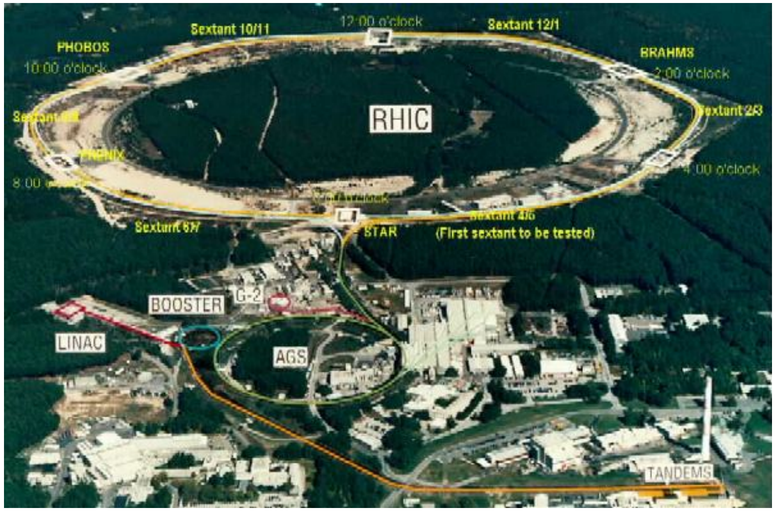
Contents

- Introduction
  - J/ψ suppression scenarios
  - Spectral functions in Lattice QCD
  - Comments on MEM
- Charmonium in a finite box
  - Spectral function in a finite box
  - Finite volume technique
  - Variational analysis
  - Test with free quarks
- Numerical results
- Summary & outlook

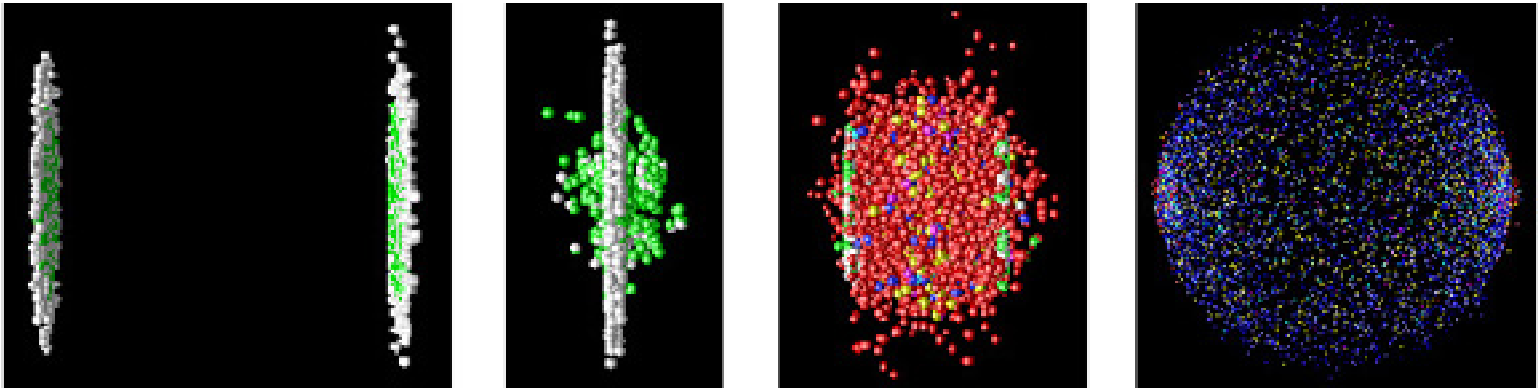
Preliminary results !!

total 30 pages

Experiments



- SPS : CERN (– 2005)
  Super Proton Synchrotron
- RHIC: BNL (2000 –)
  Relativistic Heavy Ion Collider
- LHC : CERN (2009 –)
  Large Hadron Collider



from the Phenix group web-site

J/ψ suppression in Exp.

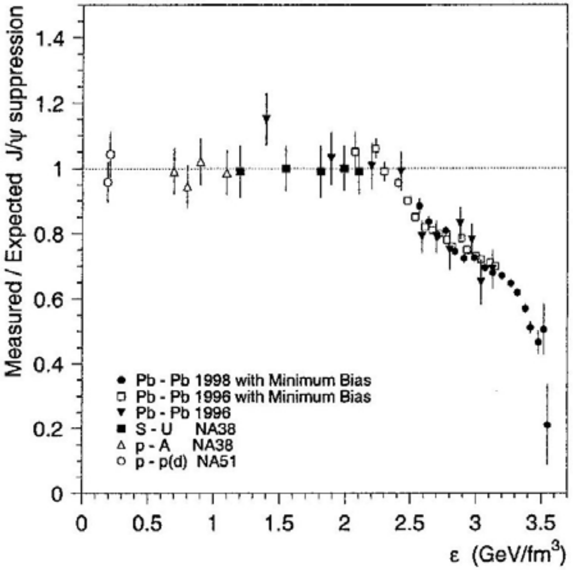


Fig. 7. Measured J/ψ production yields, normalised to the yields expected assuming that the only source of suppression is the ordinary absorption by the nuclear medium. The data is shown as a function of the energy density reached in the several collision systems.

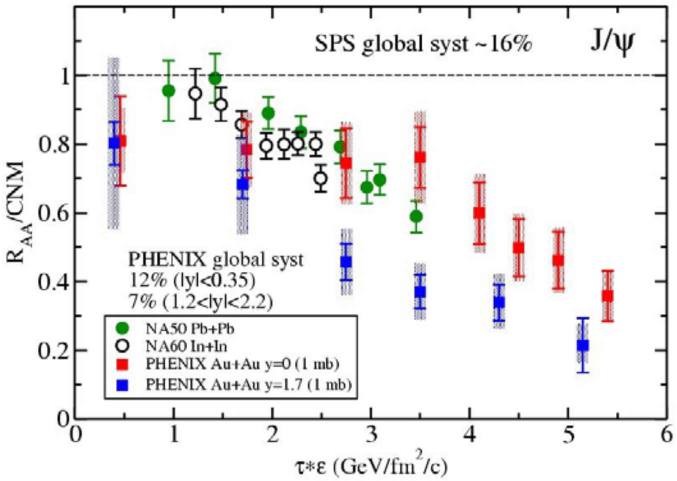


Figure 10: Survival fraction (R_{AA}/CNM) vs energy density comparison of PHENIX Au+Au suppression to that from NA38/50 at CERN.

Phy

NA5

ation

Energy density $0.5\text{–}1.5\mathrm{GeV/fm^3} = 1.0T_c$

$10\ \mathrm{GeV/fm^3} = 1.5T_c$

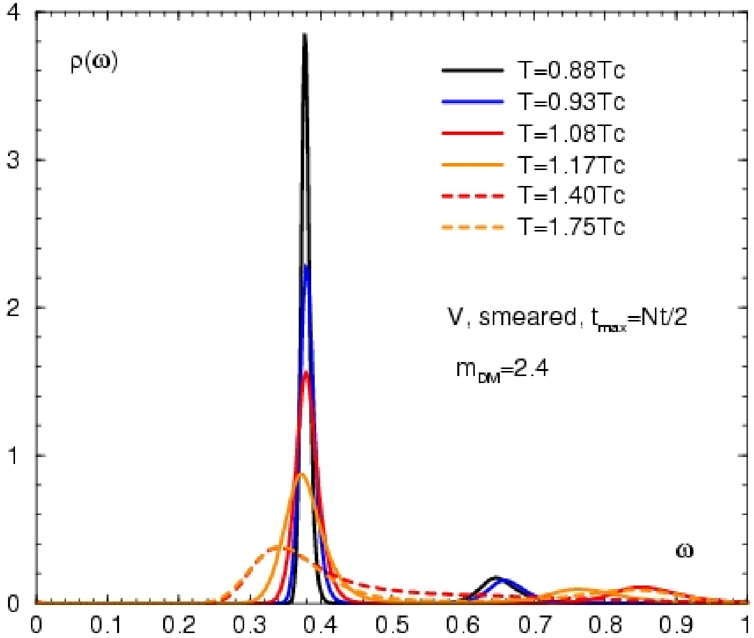
$30\ \mathrm{GeV/fm^3} = 2.0T_c$

Charmonium spectral function

$$
\begin{aligned}
C(t) &= \langle O(t)O^\dagger(0)\rangle \\
&= \int d\omega \rho(\omega) \frac{e^{-\omega t}+e^{-\omega(N_t-t)}}{1-e^{-\omega N_t}}
\end{aligned}
$$

Maximum entropy method

C(t) → $\rho(\omega)$



Quenched QCD

- *T. Umeda et al., EPJC39S1, 9, (2005).*
- *S. Datta et al., PRD69, 094507, (2004).*
- *T. Hatsuda & M. Asakawa, PRL92, 012001, (2004).*
- *A. Jakovac et al., PRD75, 014506 (2007).*

Full QCD

- *G. Aarts et al., arXiv:0705.2198.*

Charmonium spectral function

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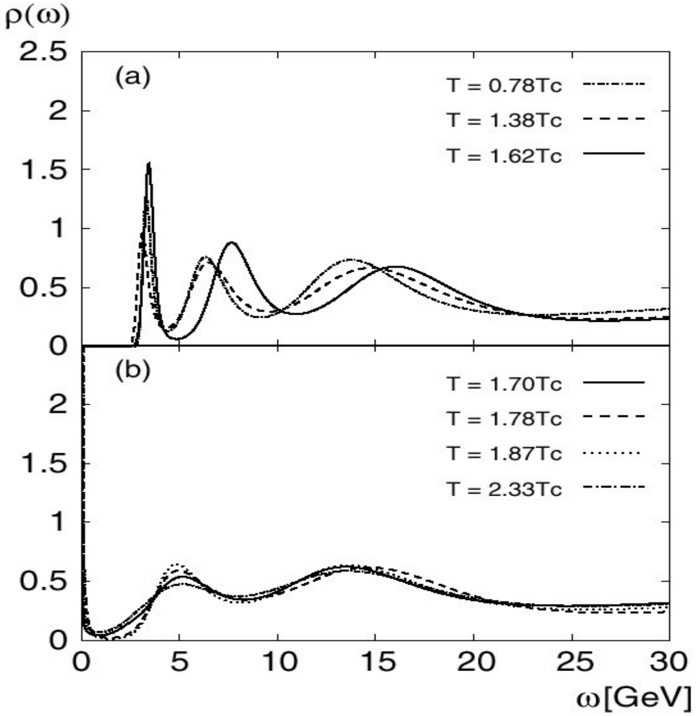

Maximum entropy method
C(t) → ρ(ω)

Quenched QCD

- T. Umeda et al., EPJC39S1, 9, (2005).
- S. Datta et al., PRD69, 094507, (2004).
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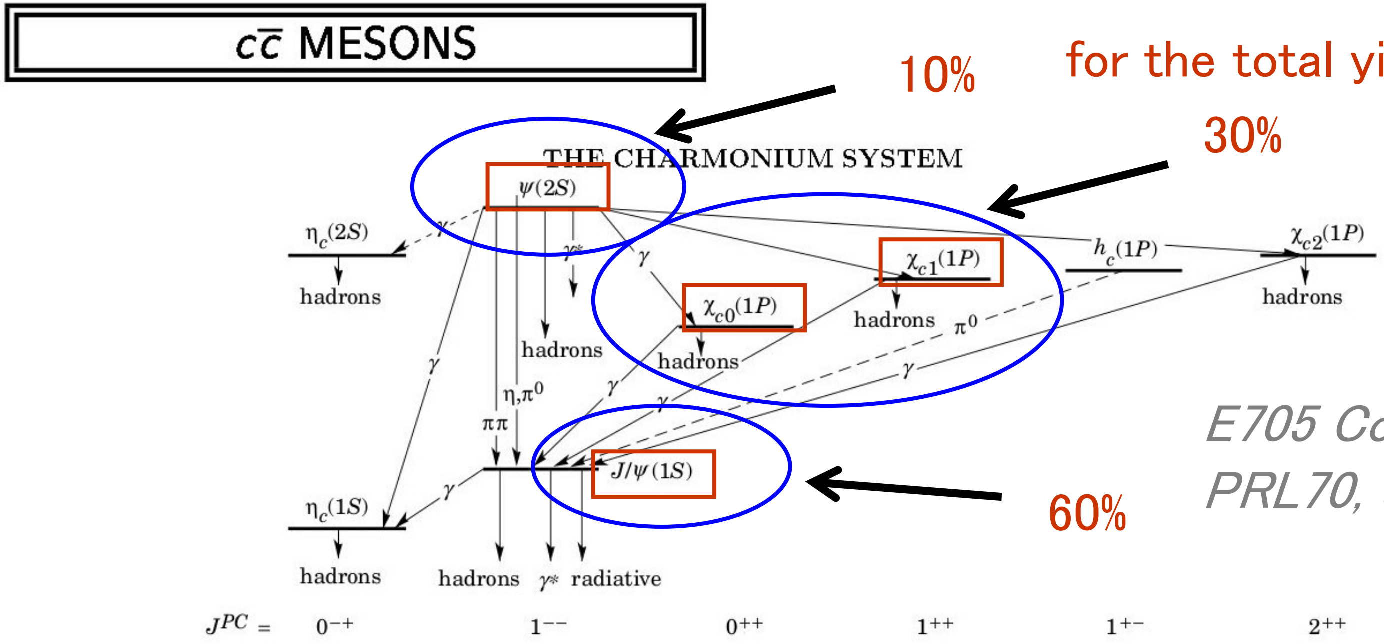
Full QCD

- G. Aarts et al., arXiv:0705.2198.

All studies indicate survival of J/ψ state above T_c ($1.5T_c$?)

Sequential J/ψ suppression

Particle Data Group (2006)



10%

for the total yield of J/ψ

30%

60%

E705 Collaboration, PRL70, 383, (1993).

Ps V S Av

Dissociation temperatures of J/ψ and ψ' & χ_c are important for QGP phenomenology.

χ_c states dissolve ?

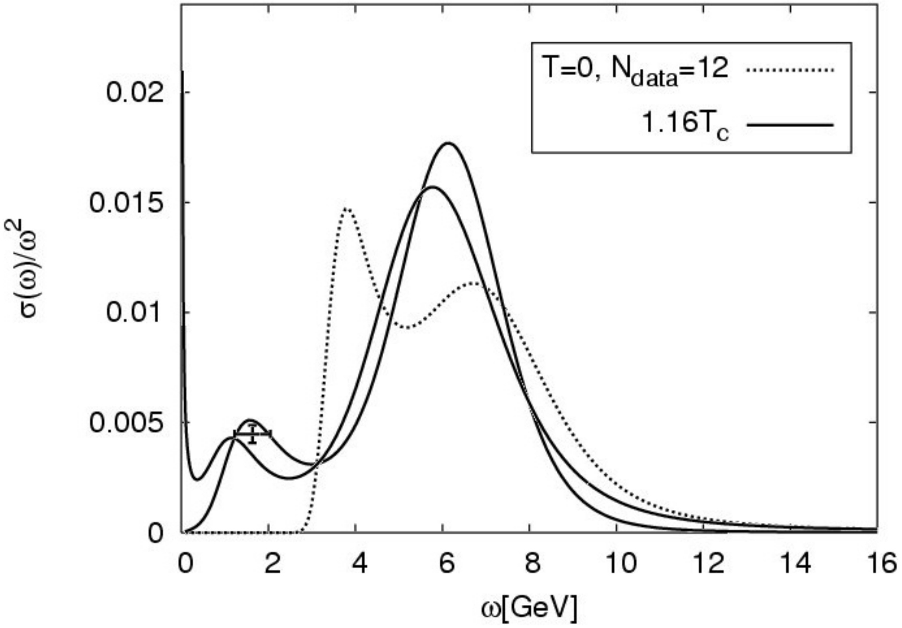


FIG. 19: The scalar spectral function for $\beta = 6.1$ at $T = 1.16T_c$ and at zero temperature reconstructed using $N_{data} = 12$. At finite temperature two default models $m(\omega) = 0.01$ and $m(\omega) = 0.038\omega^2$ have been used.

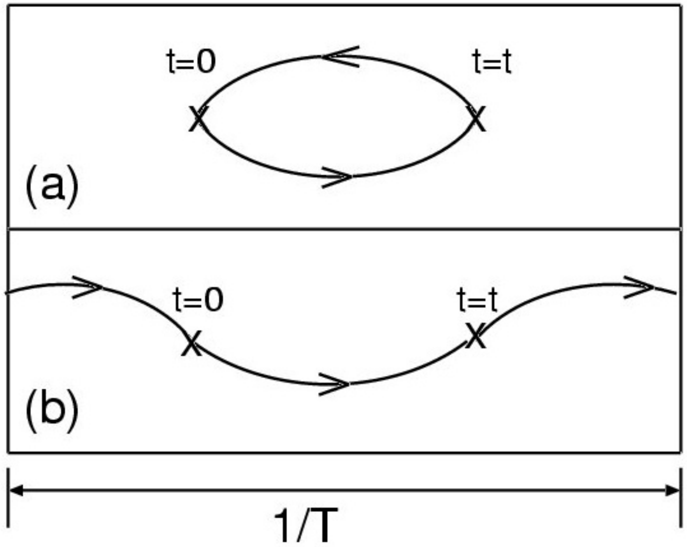
A.Jakovac et al., PRD75,014506)2007.

charmonium spectral func.
for P−wave states
→ χ_c states

S. Datta et al.,
PRD69, 094507 (2004).
A.Jakovac et al.,
PRD75, 014506 (2007).
G.Aarts et al.,
arXiv:0705.2198

- the results of SPFs for P−states are not so reliable.
  e.g. large default model dep.
- the drastic change just above Tc is reliable results.

Constant mode



$\exp(-m_q t) \times \exp(-m_q t)$
$= \exp(-2m_q t)$

m_q is quark mass
or single quark energy

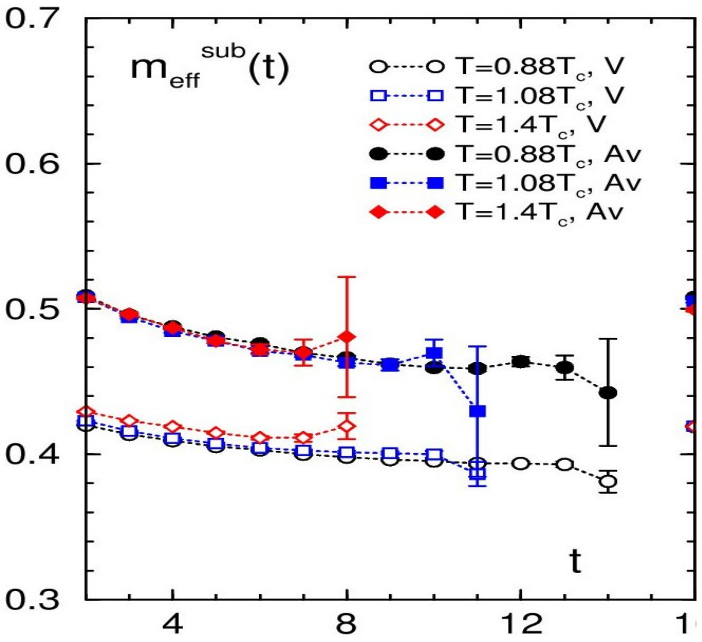
$\exp(-m_q t) \times \exp(-m_q(L_t - t))$
$= \exp(-m_q L_t)$

L_t = temporal extent

Imaginary time formalism
$L_t = 1/\text{Temp.}$
gauge field : periodic b.c.
quark field : anti-periodic b.c.

- confined phase （$T<T_c$）
  $m_q = \infty$
  constant mode disappears
- deconfined phase （$T>T_c$）
  m_q = finite
  constant mode appears

χ_c states dissolve ?



Midpoint subtracted correlator

$$\bar{C}(t) = C(t) - C(N_t/2)$$

$$\frac{C(t)}{C(t+1)} = \frac{\cosh[m_{eff}(t)(N_t/2-t)]}{\cosh[m_{eff}(t)(N_t/2-t-1)]}$$

$$\frac{\bar{C}(t)}{\bar{C}(t+1)} = \frac{\sinh^2\left[\frac{1}{2}m^{sub}_{eff}(t)(N_t/2-t)\right]}{\sinh^2\left[\frac{1}{2}m^{sub}_{eff}(t)(N_t/2-t-1)\right]}$$

Small changes even in P-states (apart from the const. mode)

Comments on MEM

- MEM is a kind of constrained χ^2 fitting

  # of fit parameters ≤ # of data points

- Prior knowledge of SPFs is needed

  as a default model function, m(w)

  The default model function

  plays crucial roles in MEM

MEM maximizes $Q = \alpha S - L$

L : Likelihood function (χ^2 term)

α : parameter (to be integrated out)

$$S = \int d\omega \left[\rho(\omega) - m(\omega) - \rho(\omega) \ln \frac{\rho(\omega)}{m(\omega)} \right]$$

default model func.

Fit form in MEM

Singular Value Decomposition (SVD)

$$\begin{aligned} K(\tau,\omega) &= e^{-\omega\tau} + e^{-\omega(T-\tau)} \\ &= V(\tau,\tau')w(\tau',\tau'')U(\omega,\tau'')^t \end{aligned}$$

diagonal matrix

fit form for spectral function : $\rho(\omega)$

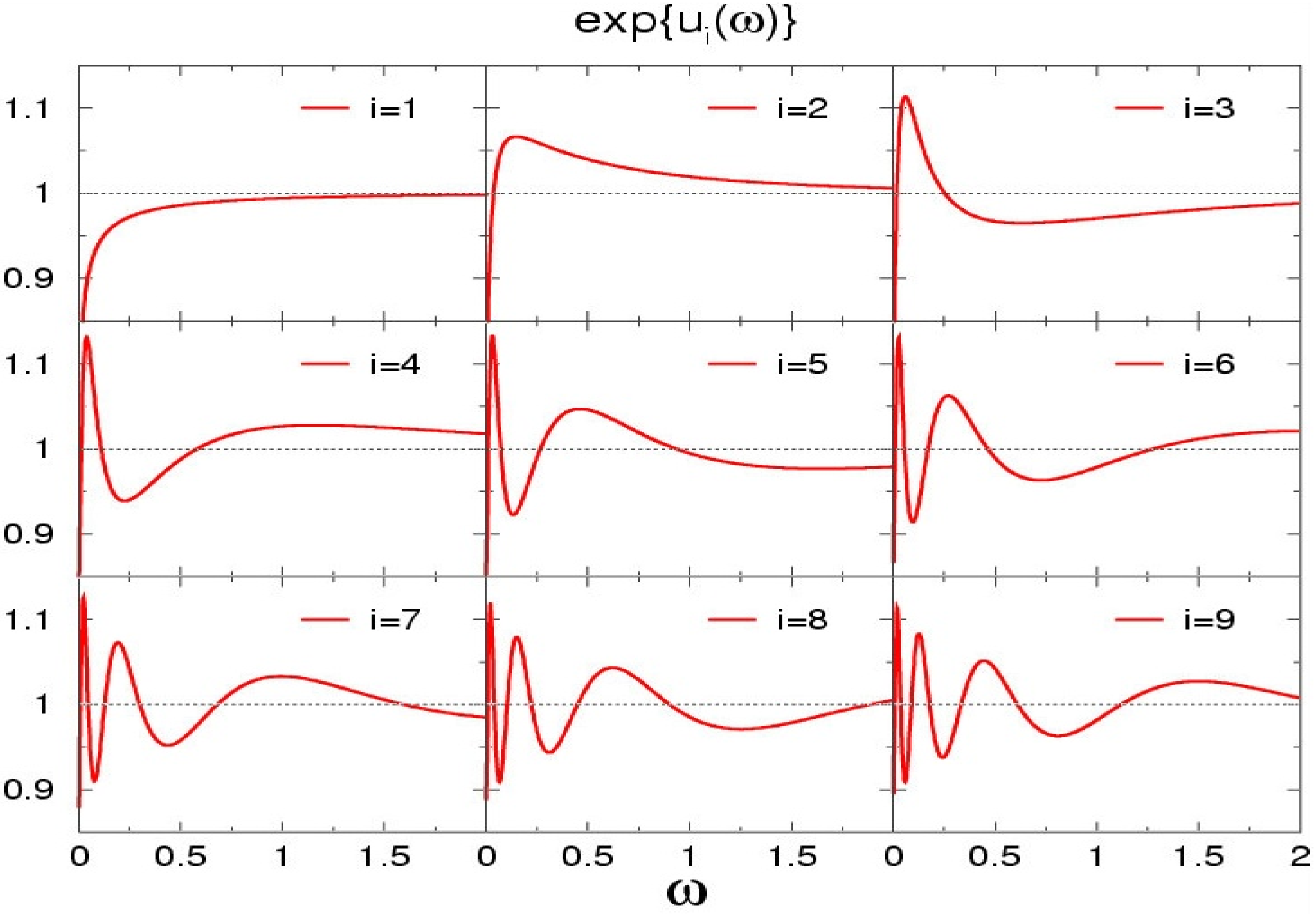
$$\rho(\omega) = m_0\omega^2 \prod_{i=1}^{N} \exp\{b_i u_i(\omega)\}$$

$u_i(\omega)(= U(\omega,\tau_i))$: basis in singular space

b_i : fit parameters

Fit form in MEM

$$\rho(\omega) = m_0 \omega^2 \prod_{i=1}^{N} \exp\{b_i u_i(\omega)\}$$



Goal of our study

Dissociation temperatures for J/ψ, χ_{c}, ψ(2S),…
from Lattice QCD calculations

Comments on our Lattice QCD calculations

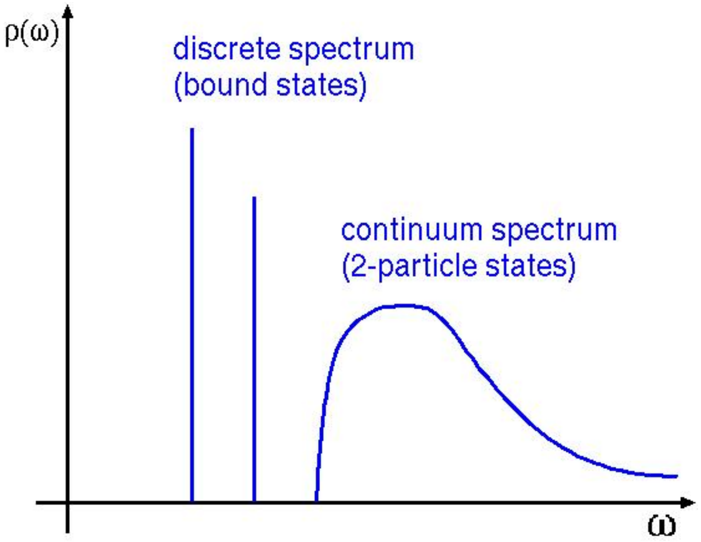
- ☐ Thermal equilibrium system
- ☐ Charmonium with zero total momentum
- ☐ Quenched approximation
  - Tc∽260MeV (∽190MeV in full QCD)
  - 1st order transition (cross over? in full QCD)
  - Static free energy vs T/Tc
  - ∽10% deviation in hadron spectra
- ☐ Homogeneous in finite volume

Spectral functions in finite volume

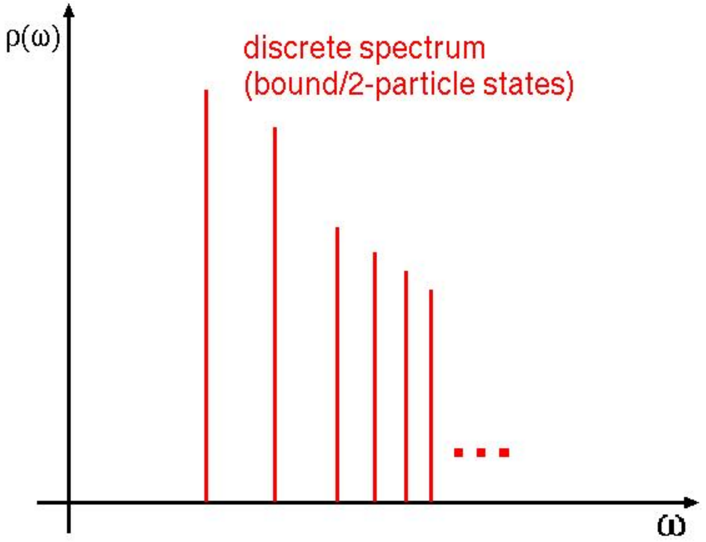
In a finite box of $(La)^3$, momenta are discretized

$$p/a = 2n\pi/L \quad (n = 0, 1, 2, \cdots)$$ for the periodic BC

infinite volume case



finite volume case



bound state or scattering state ?

(1) volume dependence of the discrete spectra

bound states → small volume dependence

scattering state → volume dependence from discretized momenta

(2) wave function

bound states → localized shape

scattering states → broad shape

(3) boundary condition dependence

bound states → insensitive to B.C.

scattering states → sensitive to B.C.

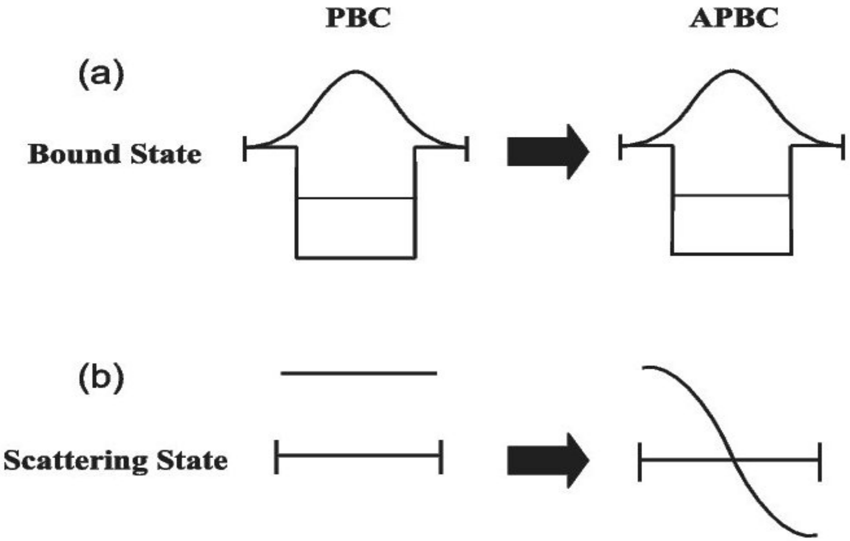
finite volume techniques

Spectrum on different (spatial) boundary conditions

Periodic B.C. → $p/a = 2n\pi/L$

Anti-periodic B.C. → $p/a = (2n+1)\pi/L$

$(n = 0, 1, 2, \cdots)$



$E_{bound}(PBC) \simeq E_{bound}(APBC)$

$E_{scatt}(PBC) < E_{scatt}(APBC)$

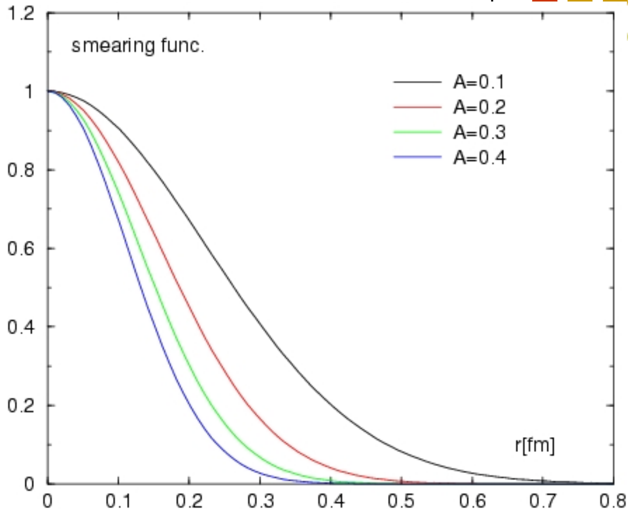
H. Iida et al., PRD74, 074502 (2006).

Variational analysis

We prepare N operators for charmonia

$$O_i^{\Gamma}(t) = \sum_{\vec{x}\vec{y}} \phi_i(\vec{y})\bar{q}(\vec{x}-\vec{y},t)\Gamma q(\vec{x},t)$$

$$\phi_i(\vec{y}) \propto \exp\left(-A_i|x|^2\right)$$



and calculate the correlator matrix

$$\begin{aligned} C_{ij}(t) &= \langle O_i(t)O_j^{\dagger}(0)\rangle \\ &= \sum_n V_{in}\lambda_n(t)V_{nj}^{\dagger} \end{aligned}$$

$$\lambda_n(t) \propto \cosh\left(E_n(t-\frac{N_t}{2})\right)$$

$$\boxed{C(t)C(t_0)^{-1} = V\lambda(t-t_0)V^{-1}}$$

Numerical results

- Test with free quarks
  - Wilson quarks in 20^3 x 32 lattice
  - bare quark mass is tuned as

$$2 * am_q \simeq m_{charmonium}/8GeV$$

- Quenched QCD results

Effective mass (local mass)

Definition of effective mass

$$\begin{aligned} C(t) &= A_0 e^{-m_0 t} + A_1 e^{-m_1 t} + \cdots \\ &\equiv A e^{-m_{eff}(t) t} \end{aligned}$$

$$\frac{C(t)}{C(t+1)} = e^{-m_{eff}(t)}$$

$$\boxed{m_{eff}(t) \to m_0 \quad \text{when} \quad (m_1 - m_0)t \gg 1}$$

In the (anti) periodic b.c.

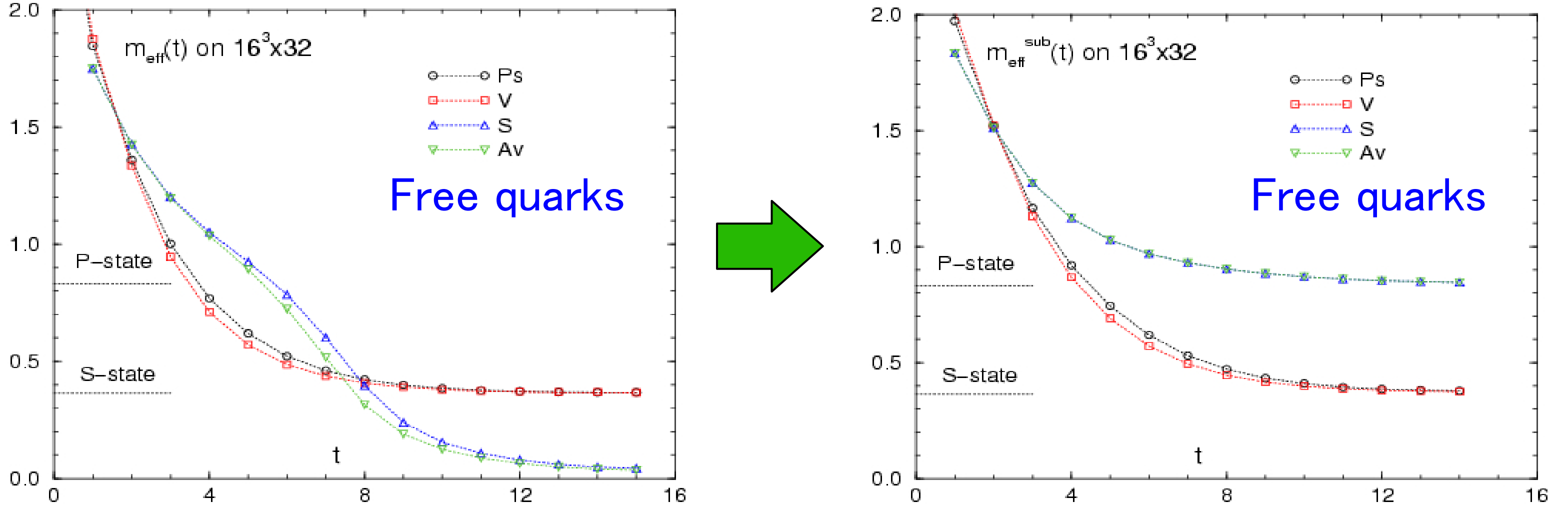
$$\frac{C(t)}{C(t+1)} = \frac{\cosh[m_{eff}(t)(N_t/2 - t)]}{\cosh[m_{eff}(t)(N_t/2 - t - 1)]}$$

Midpoint subtraction

An analysis to avoid the constant mode

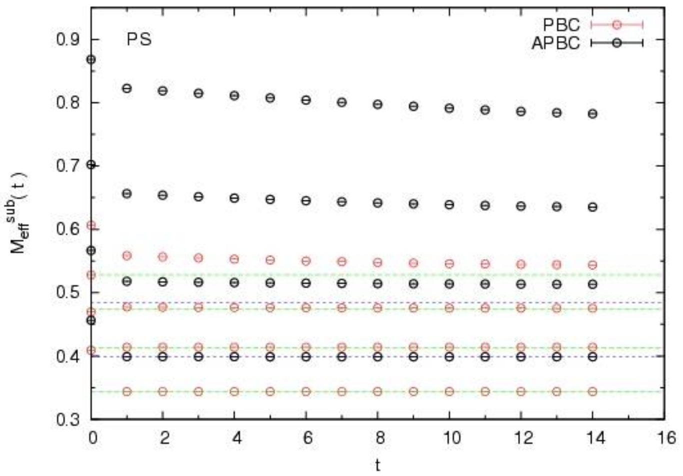
Midpoint subtracted correlator

$$\bar{C}(t) = C(t) - C(N_t/2) \quad \Rightarrow \quad \frac{\bar{C}(t)}{\bar{C}(t+1)} = \frac{\sinh^2\left[\frac{1}{2}m^{sub}_{eff}(t)(N_t/2-t)\right]}{\sinh^2\left[\frac{1}{2}m^{sub}_{eff}(t)(N_t/2-t-1)\right]}$$

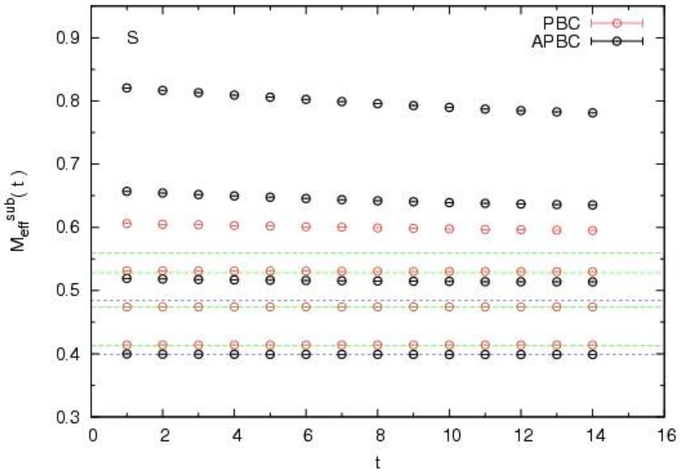


Test with free quarks

S−wave states



P−wave states



$$E(PBC) = 2\sqrt{m_q^2 + \left(\frac{2\vec{n}\pi}{L}\right)^2}$$

$$E(APBC) = 2\sqrt{m_q^2 + \left(\frac{(2\vec{n}+1)\pi}{L}\right)^2}$$

S−wave : PBC APBC

$$\vec{p}(1S) = \frac{\pi}{L}(0,0,0), \boxed{1} \quad \frac{\pi}{L}(1,1,1) \boxed{2}$$

$$\vec{p}(2S) = \frac{\pi}{L}(2,0,0), \boxed{3} \quad \frac{\pi}{L}(3,1,1) \boxed{4}$$

P−wave : PBC APBC

$$\vec{p}(1P) = \frac{\pi}{L}(2,0,0), \boxed{2} \quad \frac{\pi}{L}(-1,1,1) \boxed{1}$$

$$\vec{p}(2P) = \frac{\pi}{L}(2,2,0), \boxed{3} \quad \frac{\pi}{L}(3,1,1) \boxed{4}$$

Lattice QCD results

Lattice setup

- Quenched approximation (no dynamical quark effect)
- Anisotropic lattices

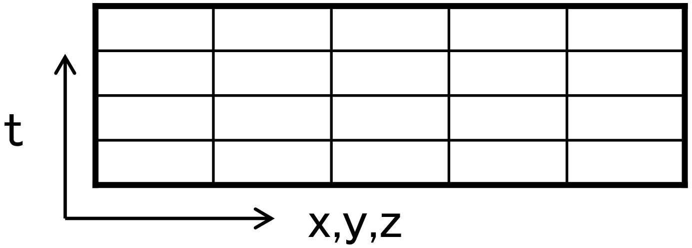
  lattice size : 20^3 x N_t

  lattice spacing : $1/a_s = 2.03(1)$ GeV,

  anisotropy : $a_s/a_t = 4$

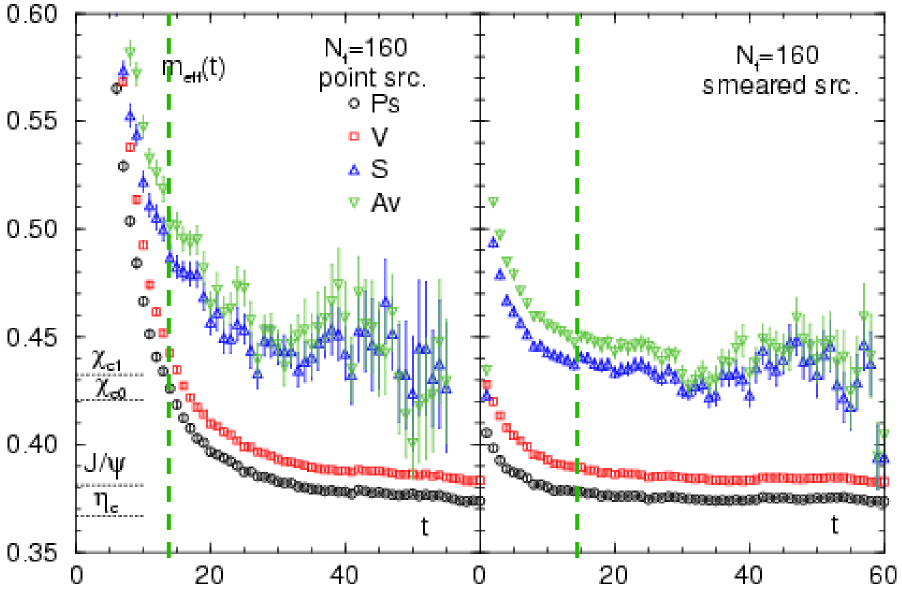
- Quark mass

  charm quark (tuned with J/ψ mass)



N_T	160	32	26	20	12
T/T_c	~ 0	0.88	1.08	1.40	2.33
# of conf.	60	100	100	100	100

At zero temperature



(our lattice results)

M_{PS} = 3033(19) MeV

M_V = 3107(19) MeV

(exp. results from PDG06)

$M_{\eta c}$ = 2980 MeV

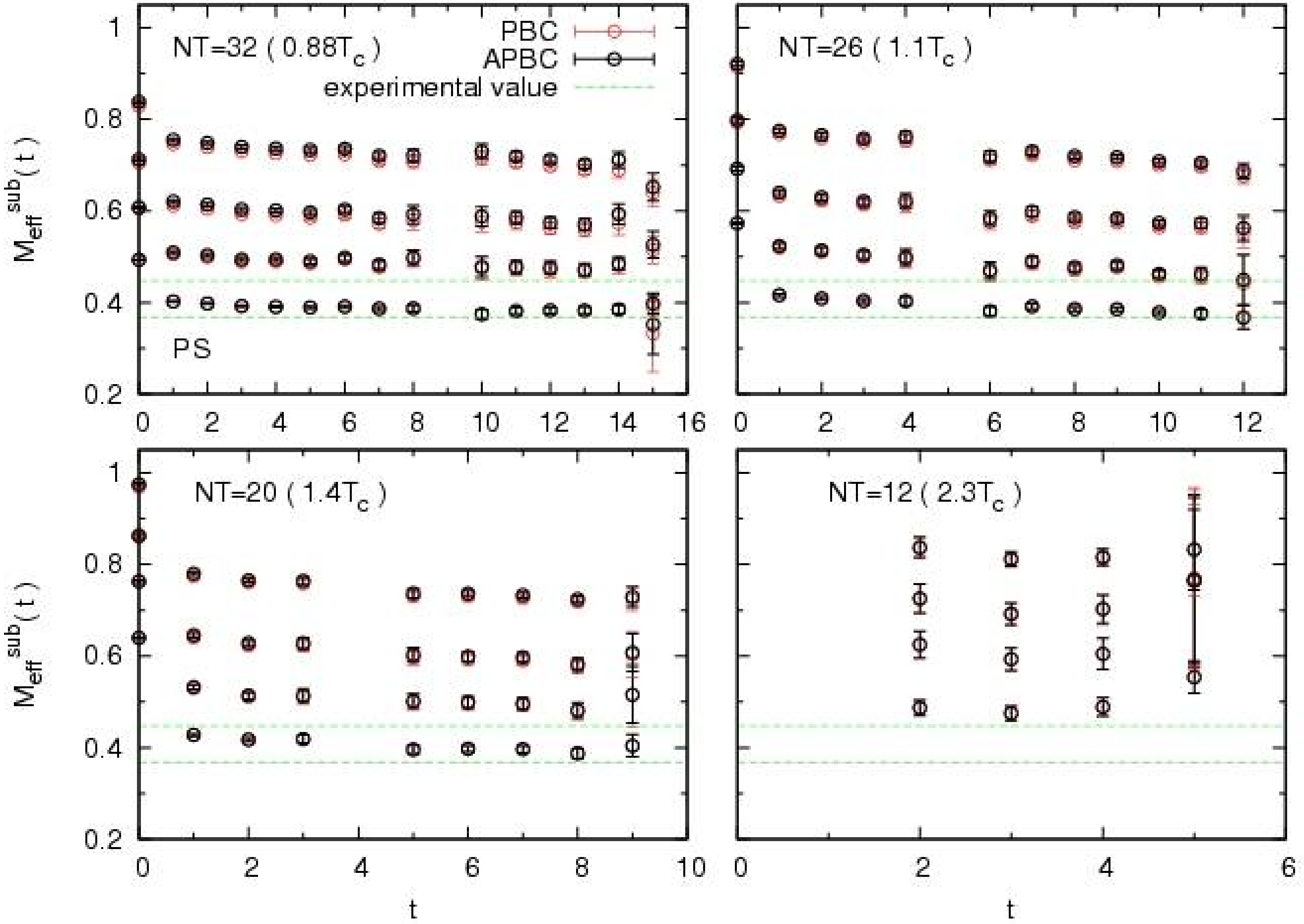
$M_{J/\psi}$ = 3097 MeV

$M_{\chi c0}$ = 3415 MeV

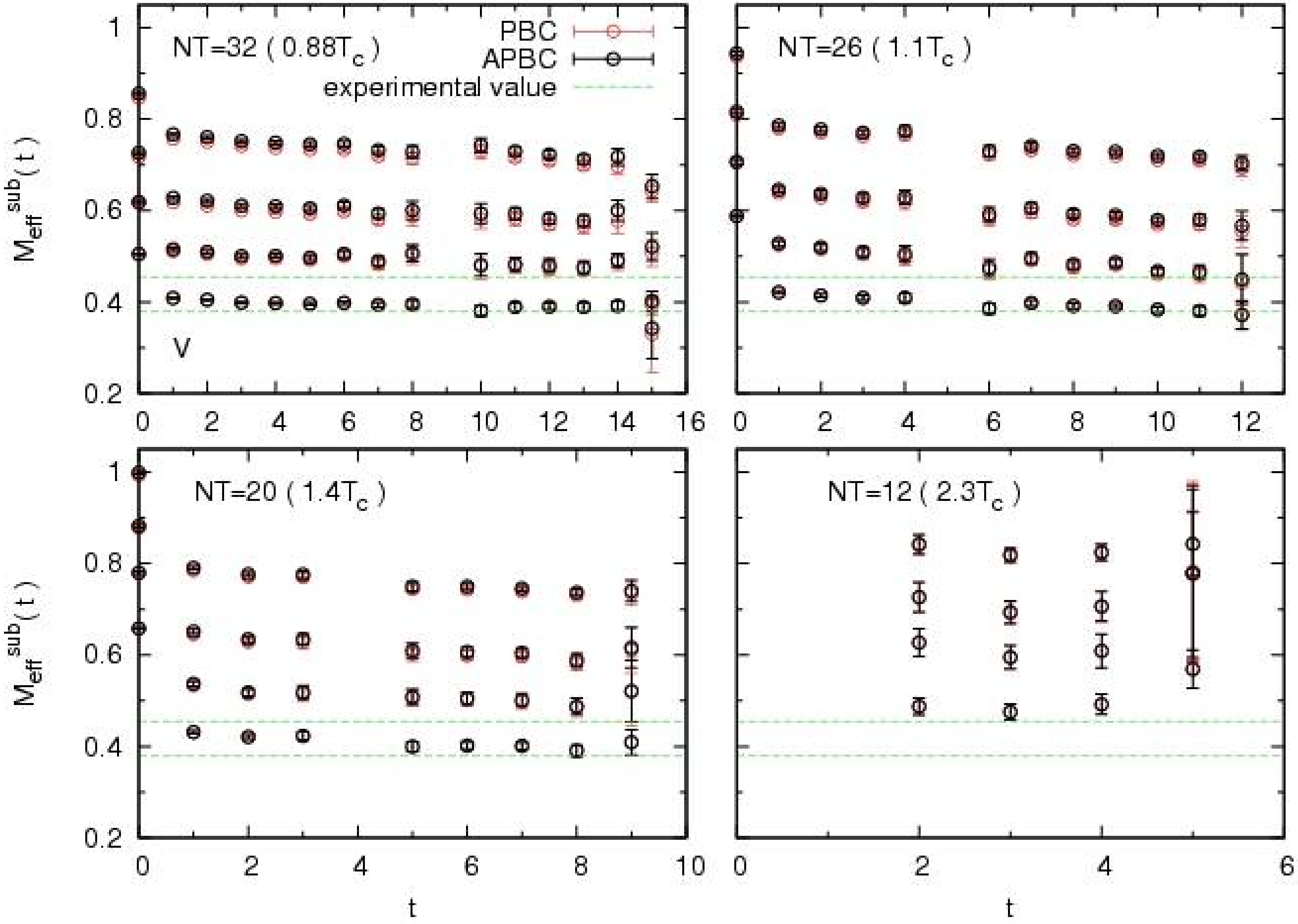
$M_{\chi c1}$ = 3511 MeV

- In our lattice $N_t \simeq 28$ at T_c
  $t = 1 - 14$ is available near T_c
- Spatially extended (smeared) op. is discussed later

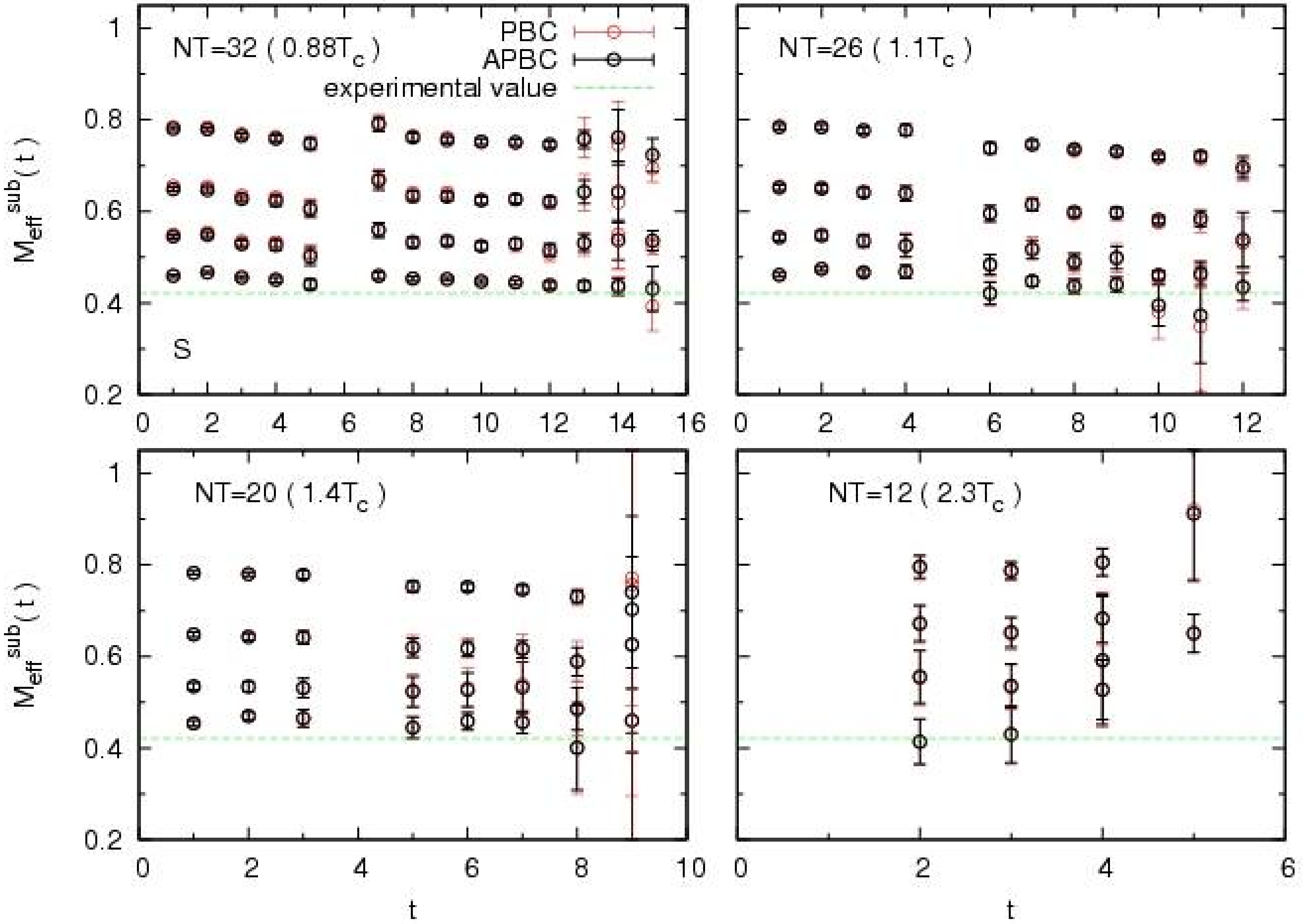
Results for PS channel



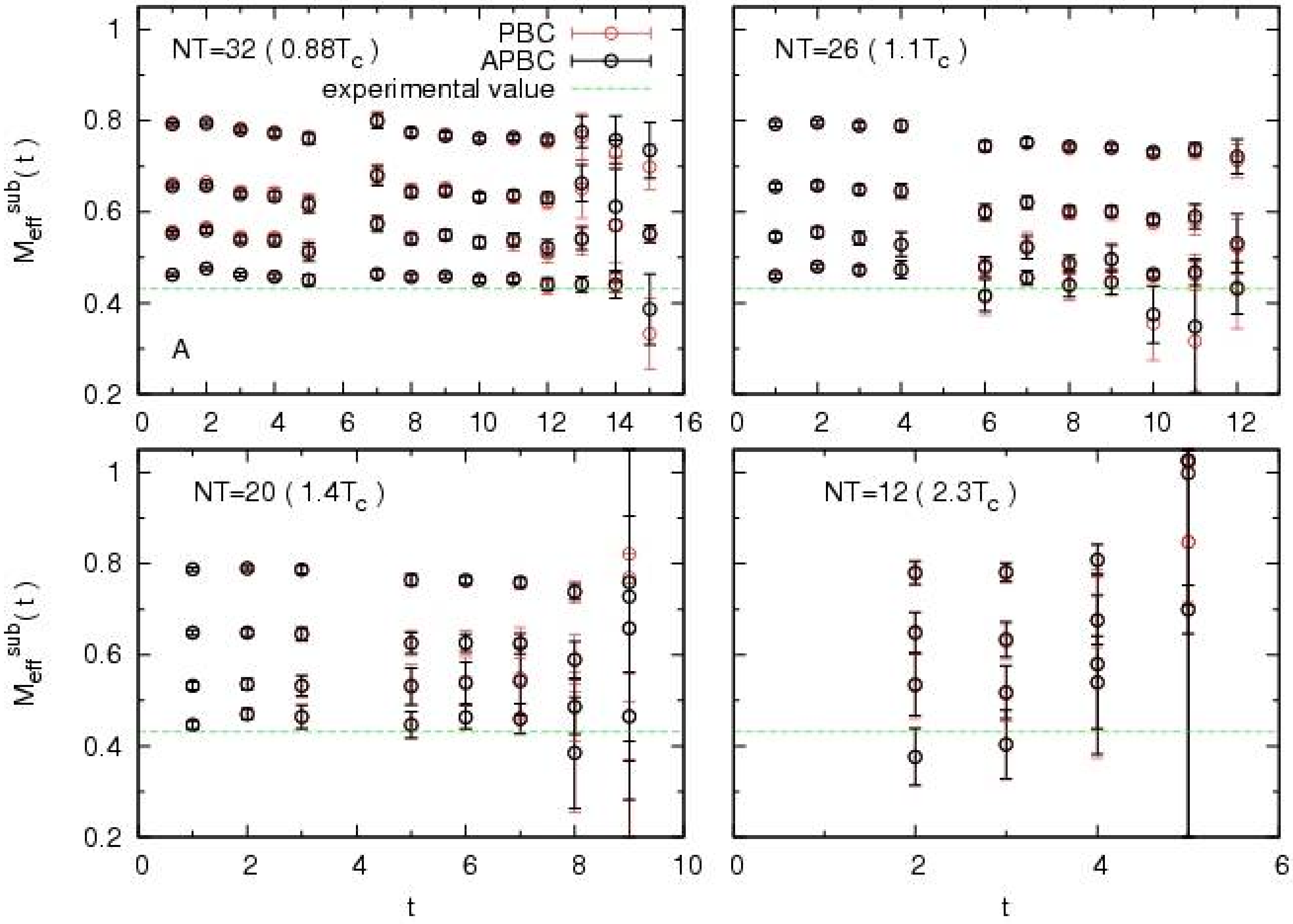
Results for V channel



Results for S channel



Results for Av channel



Summary

We investigated Tdis of charmonia from Lattice QCD using the finite volume technique

- boundary condition dependence
- variational analysis

■ Our approach can control the system with free quarks

■ In quenched QCD case,
we found no boundary condition dependence
and no mass shift (except for 2.3Tc)

☐ Our results may suggest
both S & P states survive upto 2Tc (?)

Outlook

- more efficient smearing func.
- more configurations & temperatures

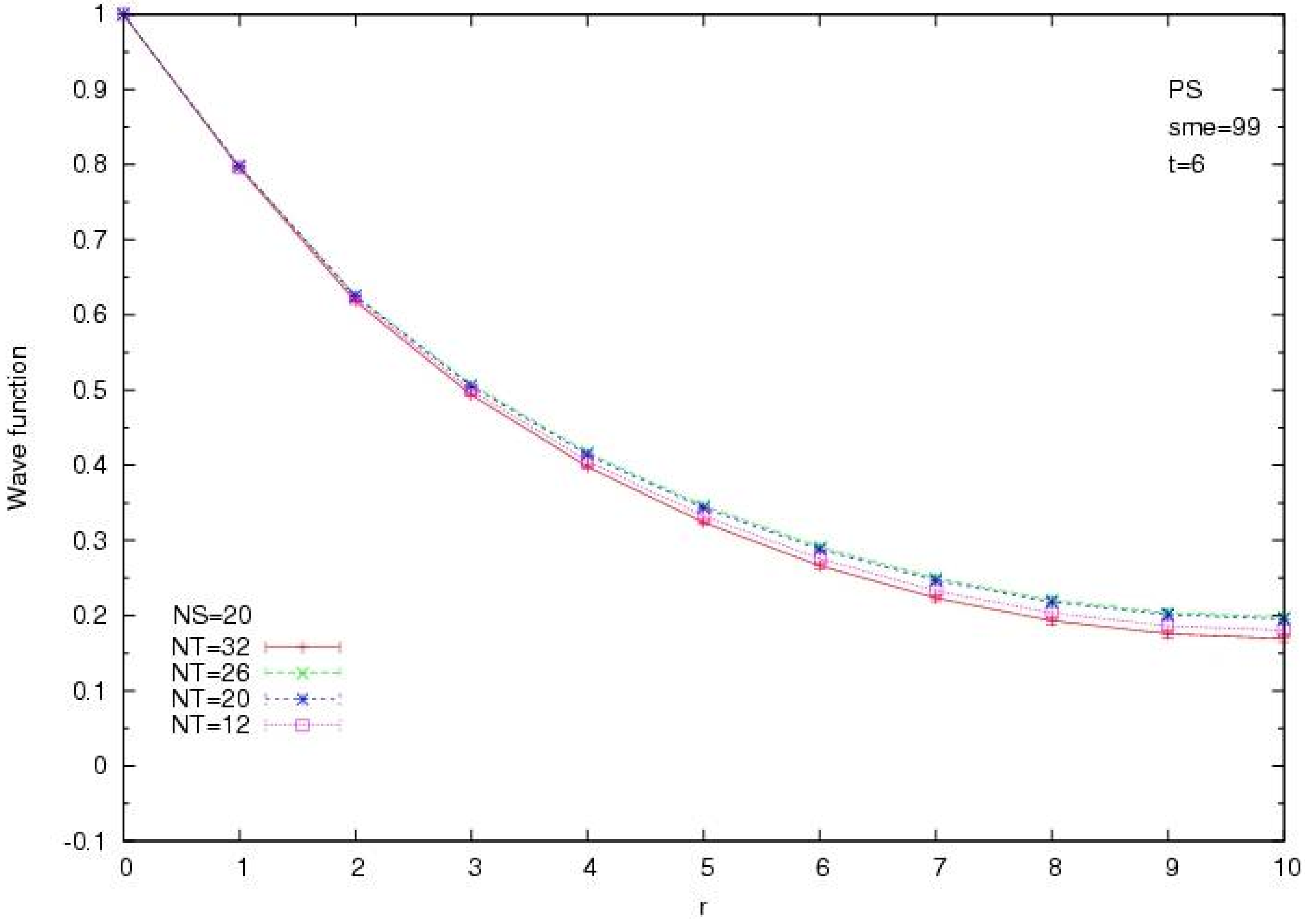
  up to 1000confs. at each temp.

  Nt=32,26,20,(16),12,(8)
- volume dependence

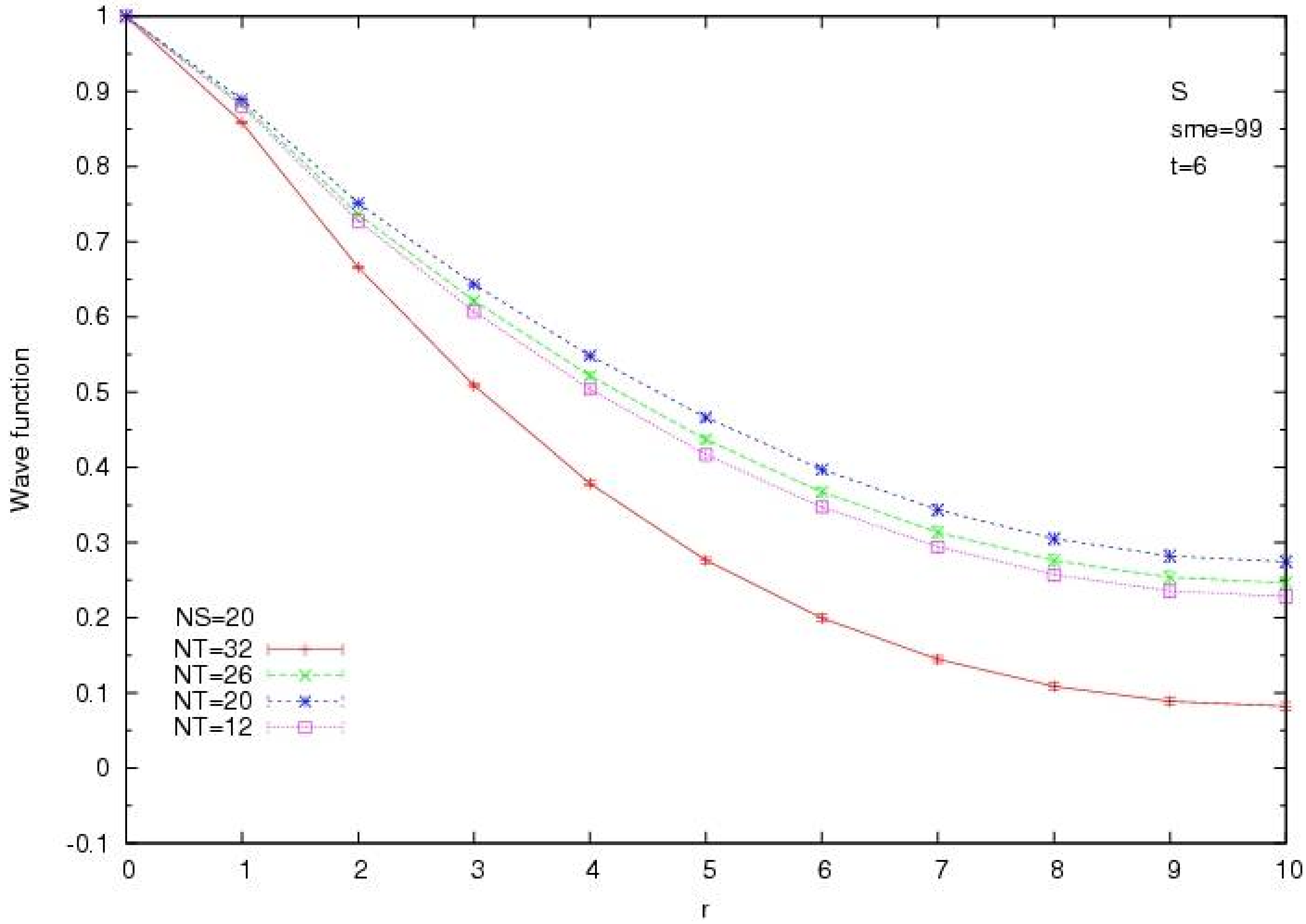
  L=20, (16)

☐ wave function (Bethe–Salpeter amplitude)

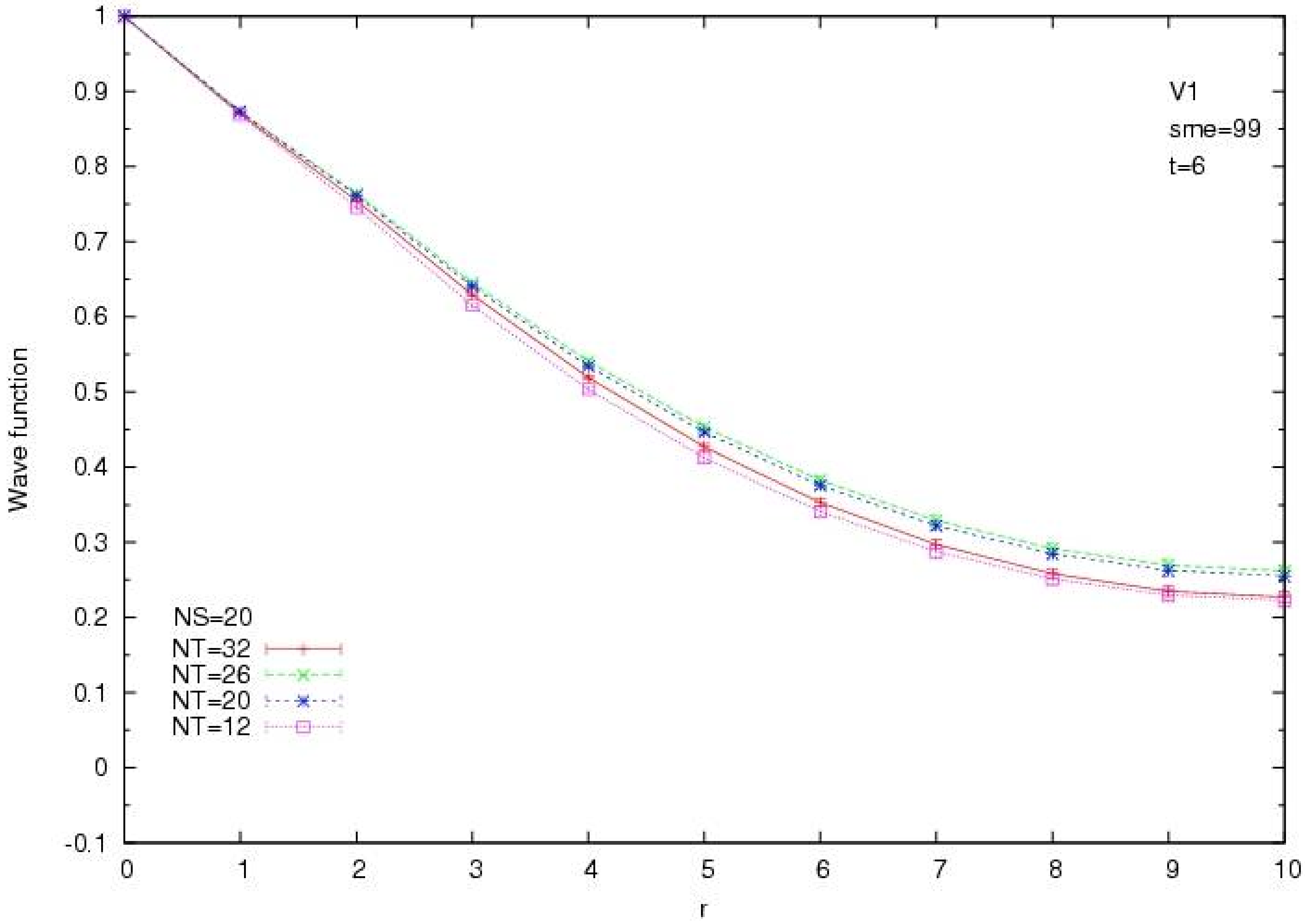
Wave functions at T>0



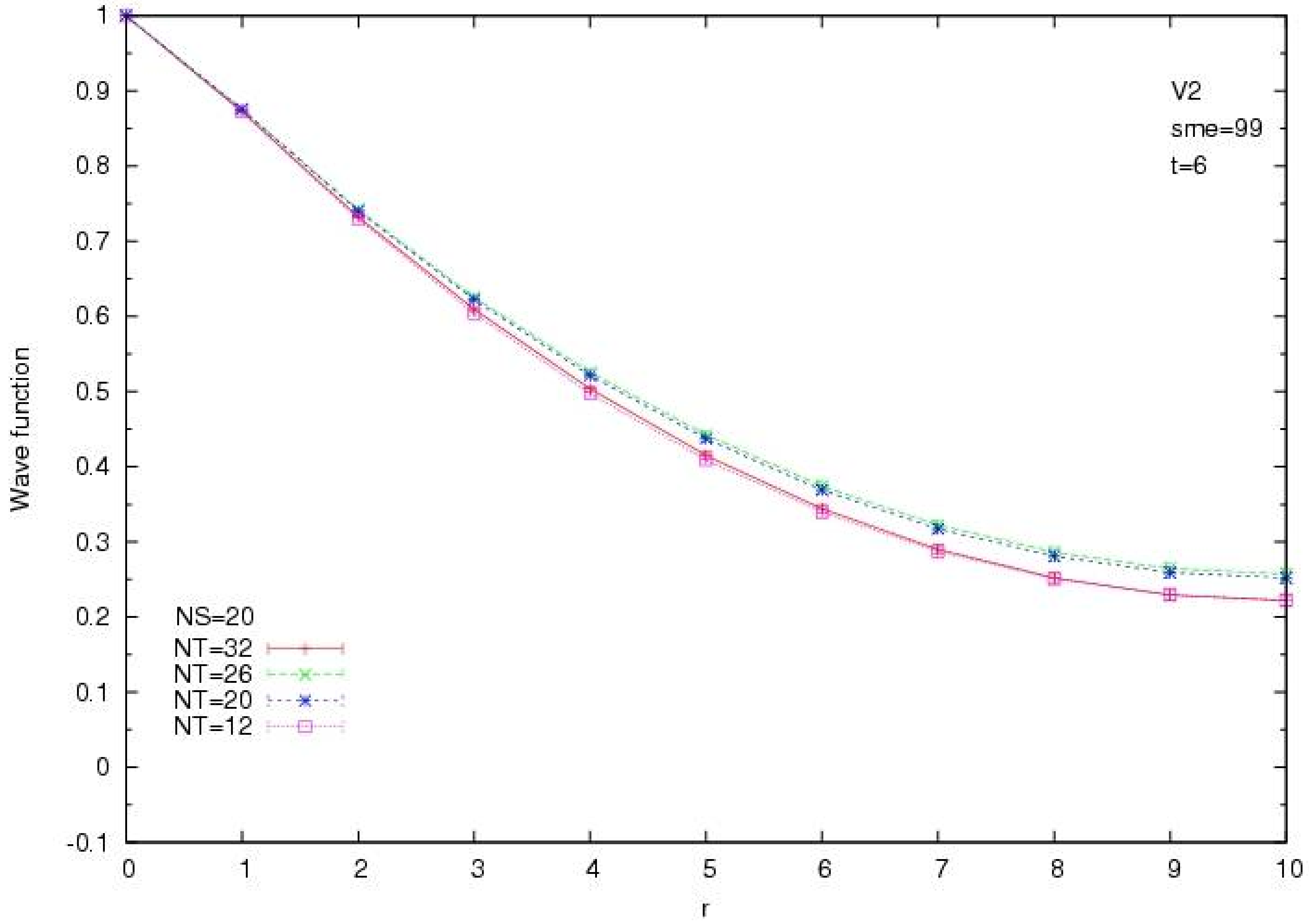
Wave functions at T>0



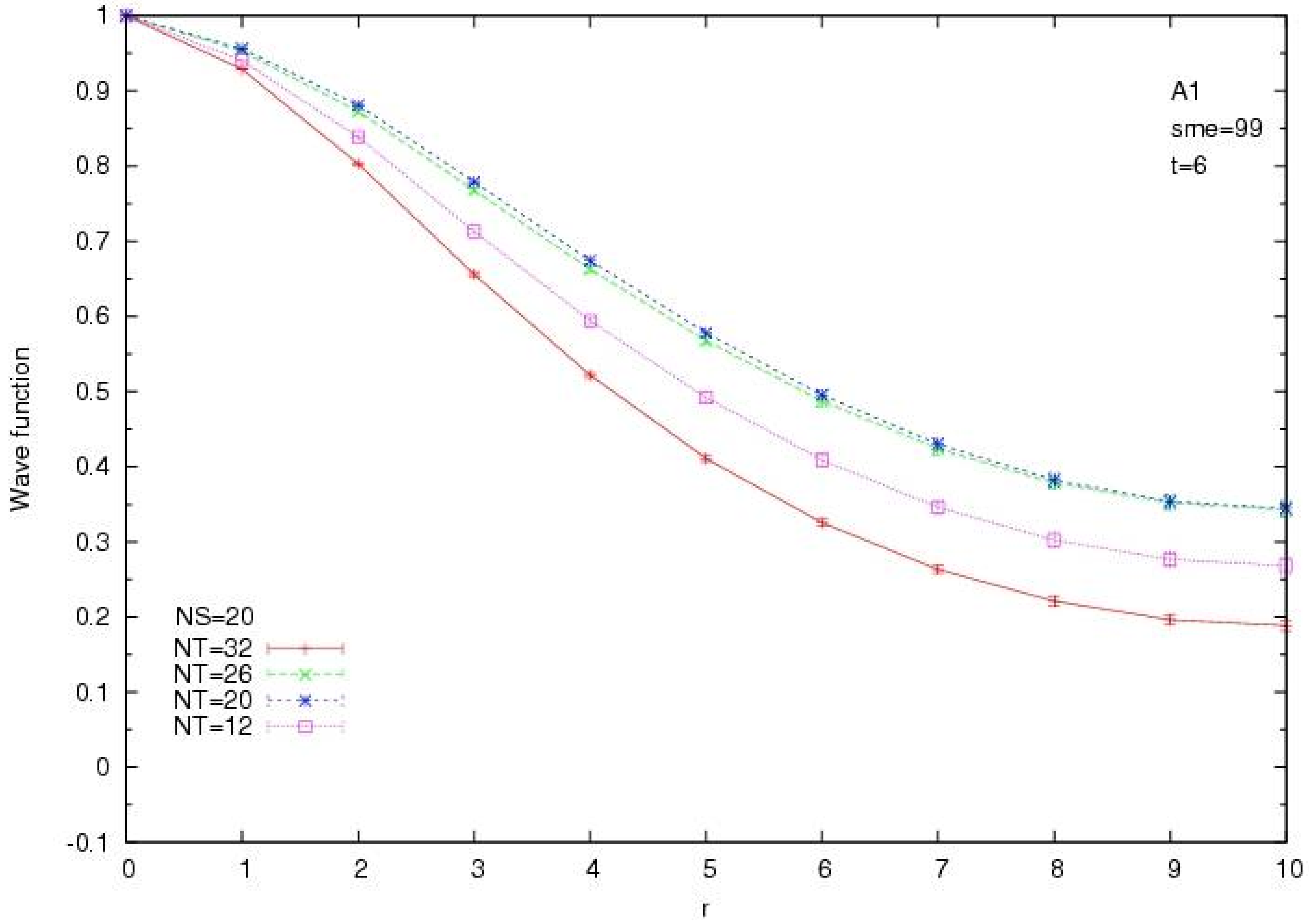
Wave functions at T>0



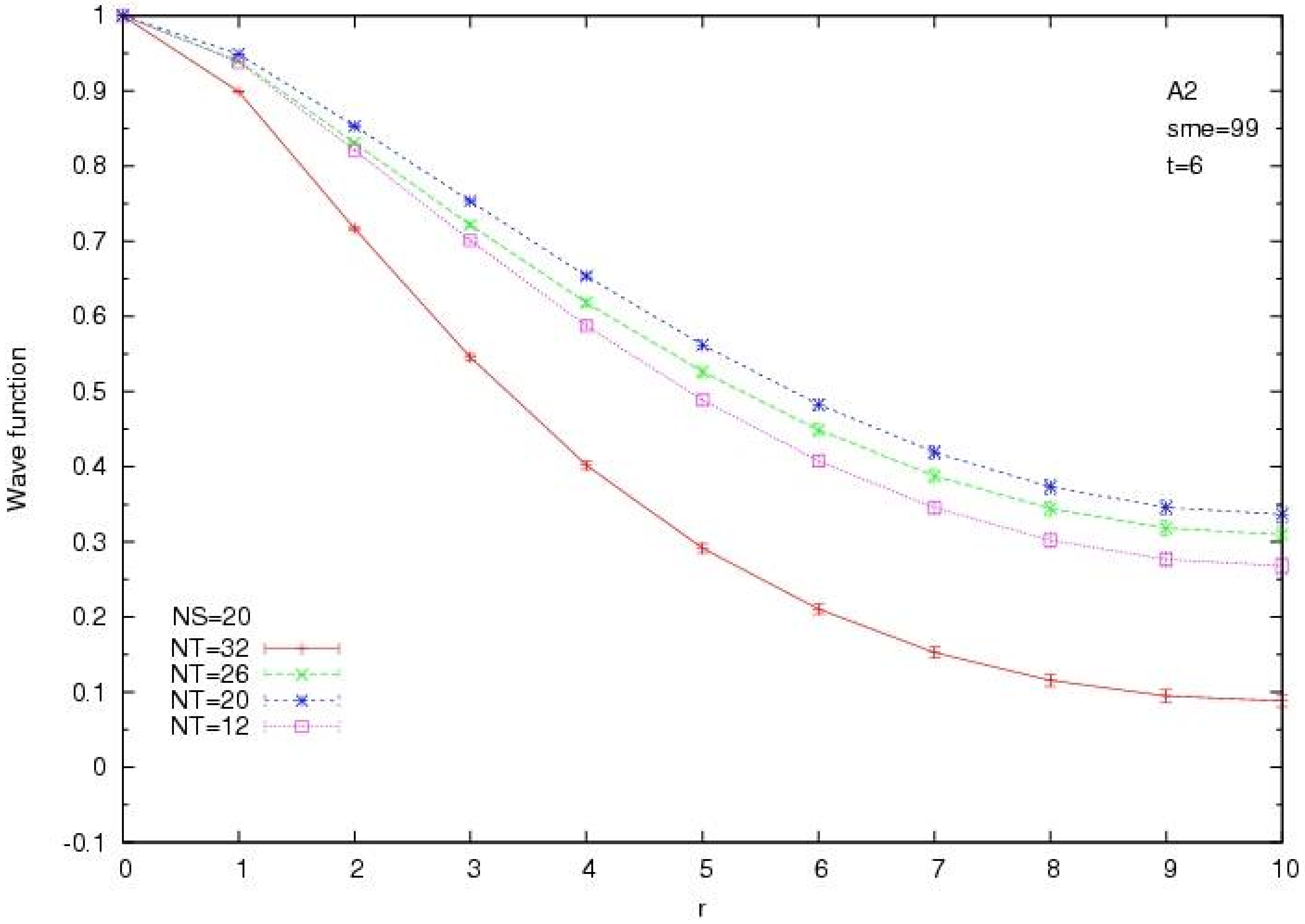
Wave functions at T>0



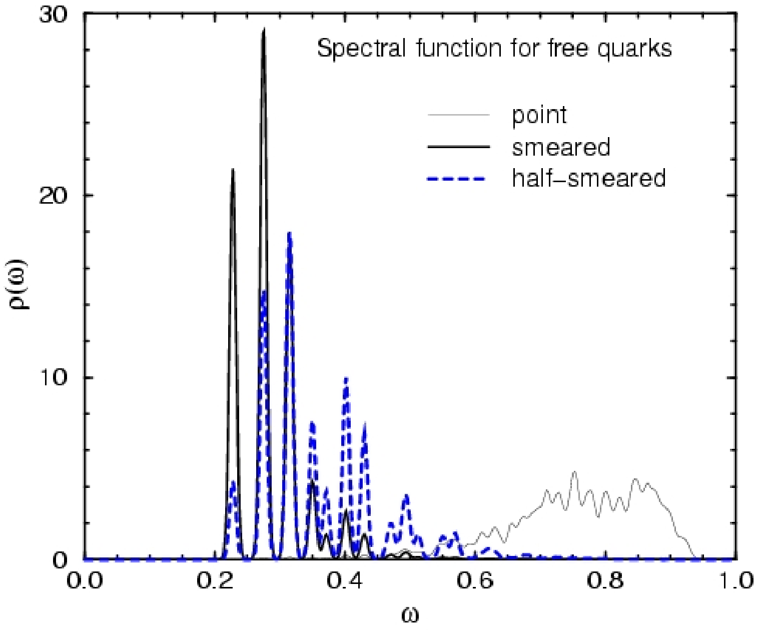
Wave functions at T>0



Wave functions at T>0



Test with free quarks



Charmonium in Lattice QCD

Lattice QCD enables us to perform

nonperturbative calculations of QCD

$$\langle X \rangle = \frac{1}{Z_{QCD}} \int Dq(x) D\bar{q}(x) DA_\mu(x) X(q, \bar{q}, A_\mu) e^{-S_{QCD}}$$

Path integral by MC integration

QCD action on a lattice

Wilson quark,
Staggered (KS) quark,
Domain Wall quark, etc

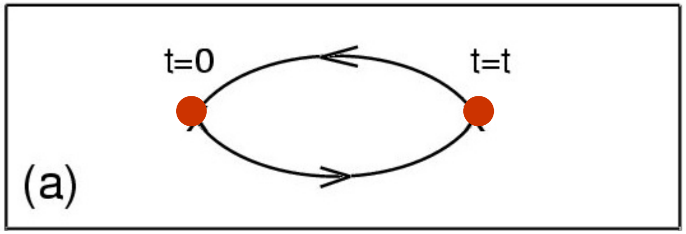
det(D)=1 is the quenched approx.

Charmonium correlation function

$$\langle X \rangle = \frac{1}{Z_{QCD}} \int Dq(x) D\bar{q}(x) DA_\mu(x) X(q,\bar{q},A_\mu) e^{-S_{QCD}}$$

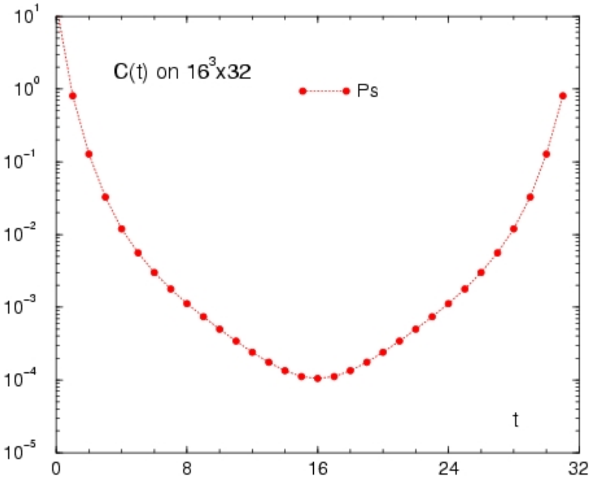
Correlation function

$$C(t) = \langle O(t) O^\dagger(0) \rangle$$



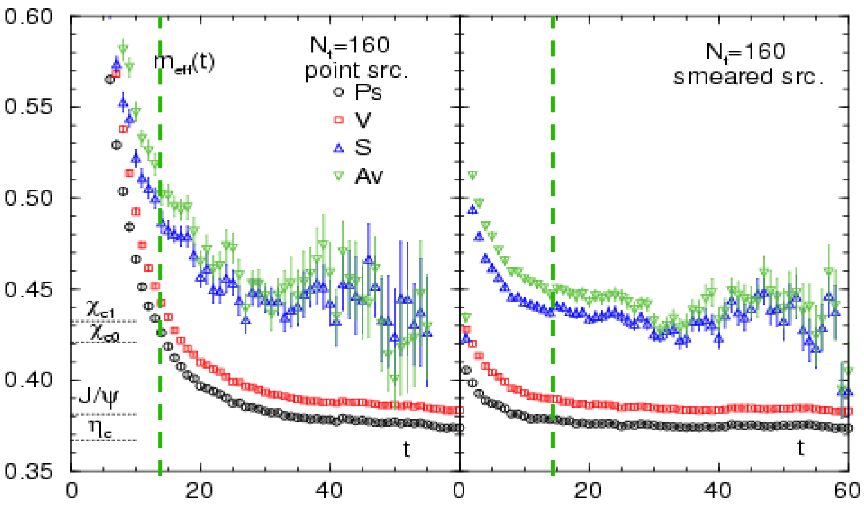
Charmonium operators

Pseudescalar(Ps)	$J^{PC} = 0-+$	η_{c}, ...
Vector (V)	$J^{PC} = 1--$	J/ψ, $\psi(2S)$, ...
Scalar (S)	$J^{PC} = 0++$	χ_{c0}, ...
Axialvector (Av)	$J^{PC} = 1++$	χ_{c1}, ...



LQCD provides C(t) of charmonium at T>0

Results with extended op.

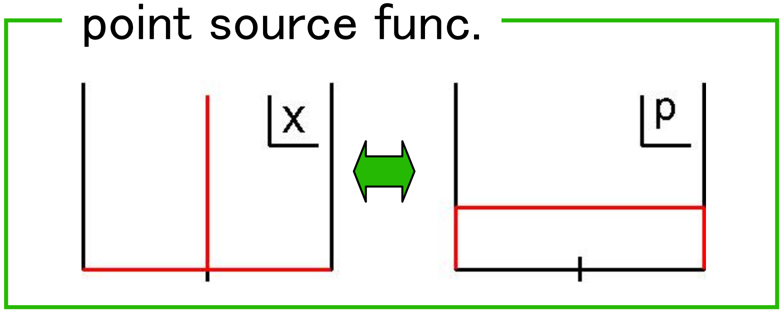


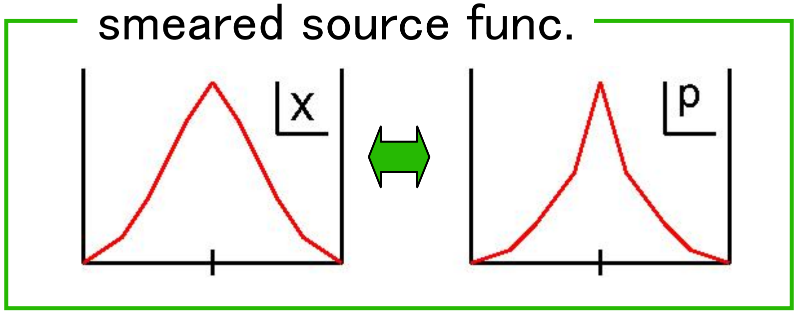
Spatially extended operators:

$$O_\Gamma(\vec{x},t) = \sum_{\vec{y}} \phi(\vec{y})\bar{q}(\vec{x}-\vec{y},t)\Gamma q(\vec{x},t)$$

with a smearing func. ϕ(x)

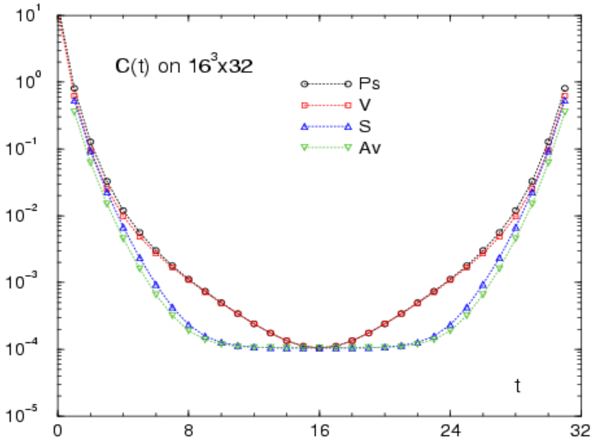
in Coulomb gauge

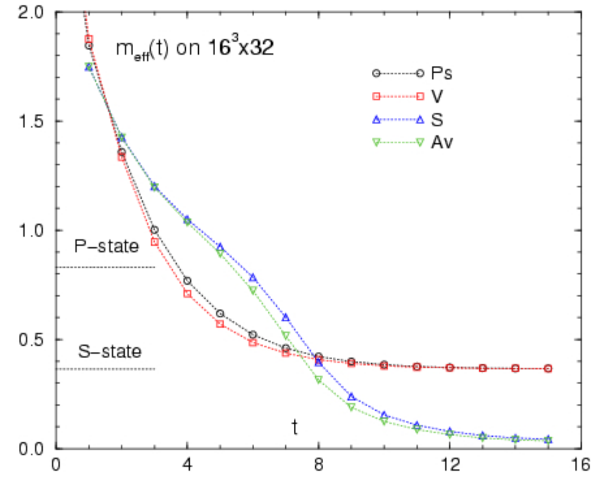

point source func.


smeared source func.

The extended op. yields large overlap with lowest states

Free quark calculations





Continuum form of the correlators

calculated by S. Sasaki

$$C(t) = \sum_{\vec{p}} \frac{4}{\cosh\left(E_p N_t/2\right)} \times$$

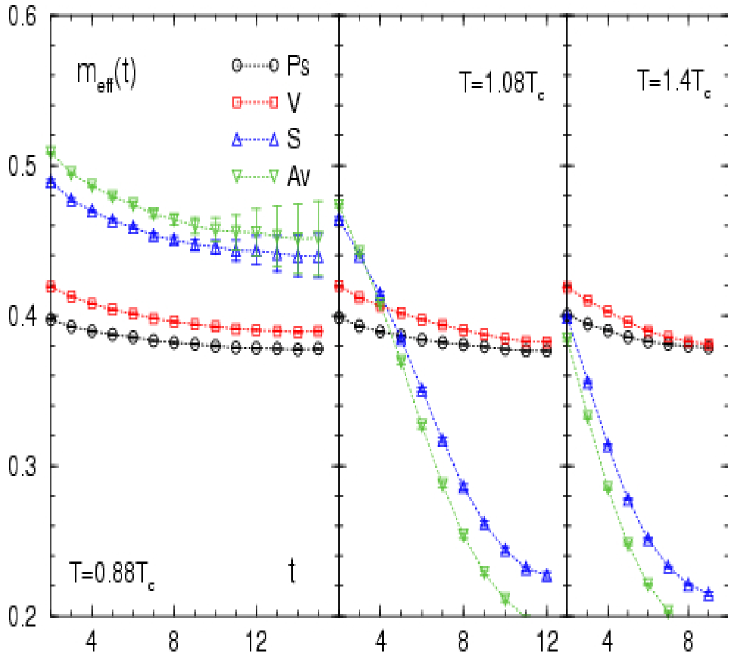
$$\begin{cases} \left(E_p^2 \cosh\left[2E_p(t-N_t/2)\right]\right) & \text{for } \Gamma = \gamma_5 \\ \left((E_p^2 - p_i^2)\cosh\left[2E_p(t-N_t/2)\right] + p_i^2\right) & \text{for } \Gamma = \gamma_i \\ -\left(p^2 \cosh\left[2E_p(t-N_t/2)\right] + (E_p^2 - p^2)\right) & \text{for } \Gamma = 1 \\ -\left((p^2 - p_i^2)\cosh\left[2E_p(t-N_t/2)\right] + (E_p^2 - p^2 + p_i^2)\right) & \\ & \text{for } \Gamma = \gamma_i\gamma_5 \end{cases}$$

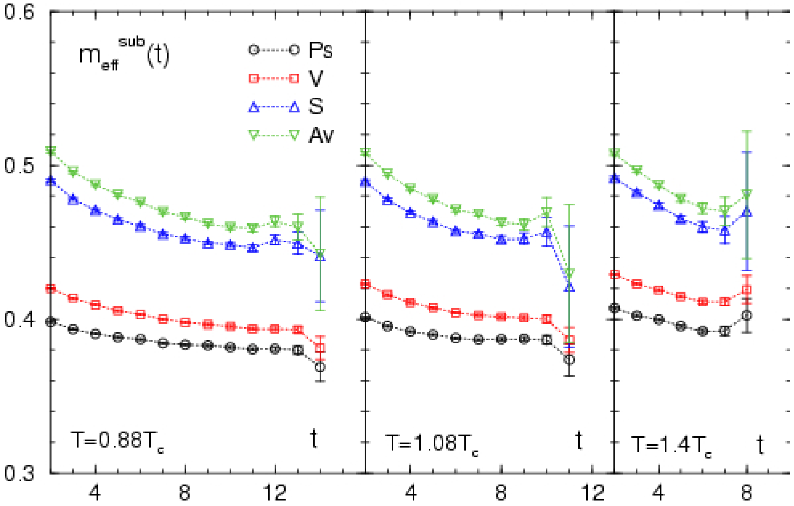
where

E_p : single quark energy with relative mom. p

$$p^2 = \sum_i p_i^2$$

Results with extended op.

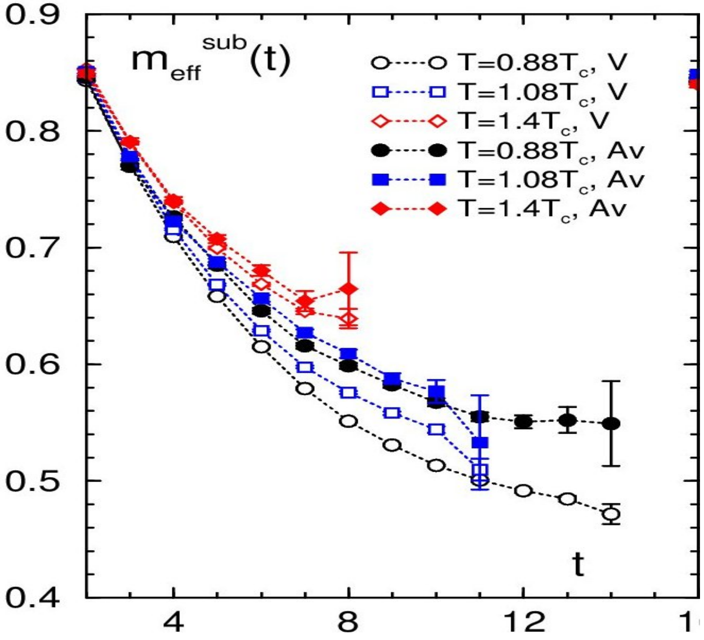

usual effective mass


subtracted effective mass

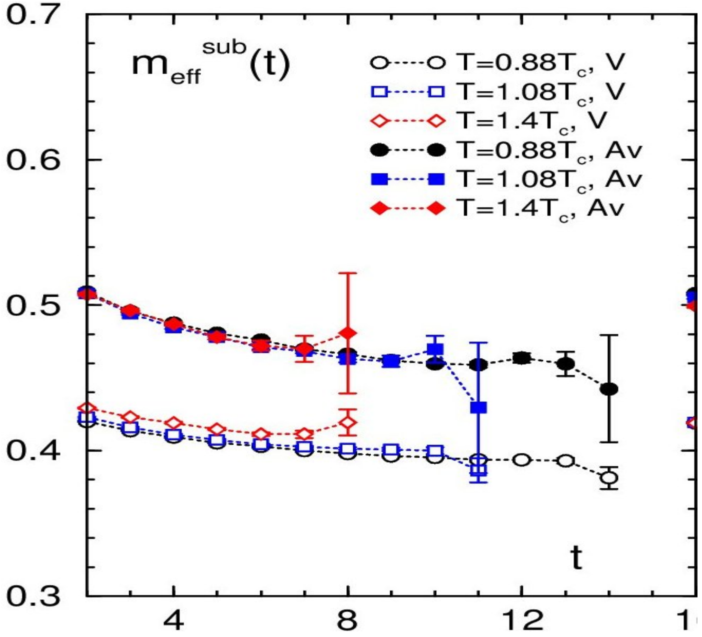
- extended op. enhances overlap with const. mode
- small constant effect is visible in V channel
- no large change above T_c in $m_{eff}^{sub}(t)$

Discussion

point operators



extended operators



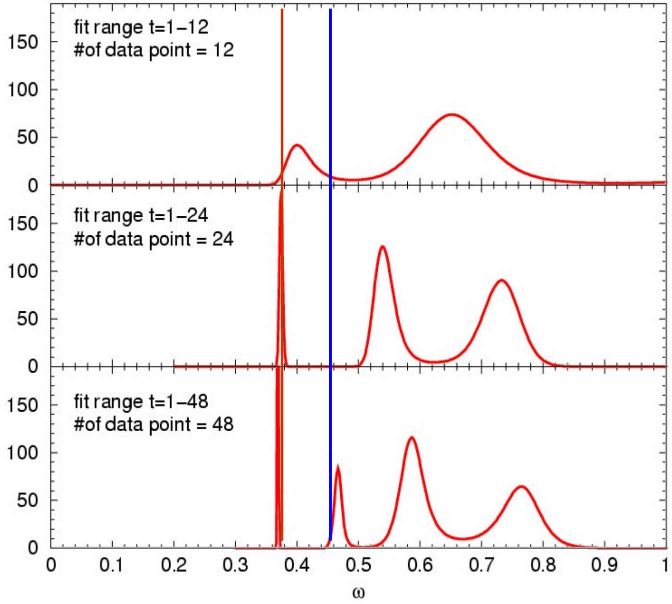
The drastic change of P−wave states is due to the const. contribution.

→ There are small changes in SPFs (except for ω=0 peak).

Why several MEM studies show the dissociation of χ_c states ?

Difficulties in MEM analysis

MEM test using T=0 data



\# data
for T/T_c=1.2

\# data
for T/T_c=0.6

\# data
for T/T_c=0

MEM analysis sometimes fails if data quality is not sufficient

Furthermore P−wave states have larger noise than that of S−wave states

Conclusion

- There is the constant mode in charmonium correlators above T_c
- The drastic change in χ_c states is due to the constant mode

➔ the survival of χ_c states above T_c, at least $T=1.4T_c$.

The result may affect the scenario of J/ψ suppression.

In the MEM analysis,

one has to check consistency of the results at $\omega \gg T$

using, e.g., midpoint subtracted correlators.

$$\bar{C}(t) = C(t) - C(N_t/2)$$

$$\bar{C}(t) = \int_0^\infty d\omega \rho_\Gamma(\omega) K^{sub}(\omega, t),$$

$$K^{sub}(\omega,t) = \frac{\sinh^2(\frac{\omega}{2}(N_t/2 - t))}{\sinh(\omega N_t/2)}$$