カラー超伝導における非可換渦と準粒子との相互作用

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Quantum vortices in a color superconductor

- Color-flavor locked phase of QCD
  - Superconductor & Superfluid

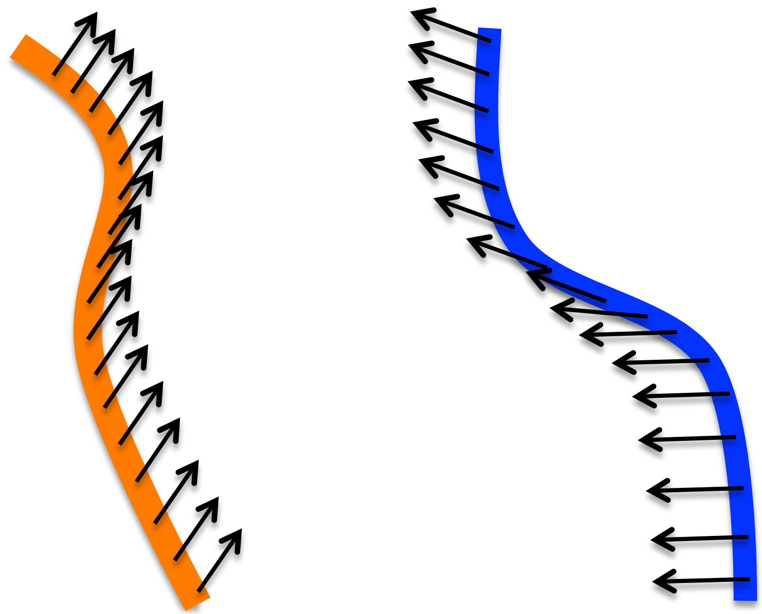
color magnetic flux & quantized circulation

[Balachandran, Digal and Matsuura (2006)]

- The existence of a vortex breaks the isotropy group **H**

Nambu-Goldstone mode on the vortex
"Orientational zero mode"

[Eto, Nakano and Nitta (2009)]



Arrows denote the values of the orientational mode.

Main purpose of this study

- We study

CFL vortex – quasiparticle interaction

- How the orientational zero modes couple with quasi-particles ?
- Method: dual transformation

Outline

- Introduction
  - What are quantum vortices?
  - Non-Abelian vortices in a color superconductor
- Dual transformation
- Vortex-quasiparticle interaction in the CFL phase
- Summary and outlook

Introduction

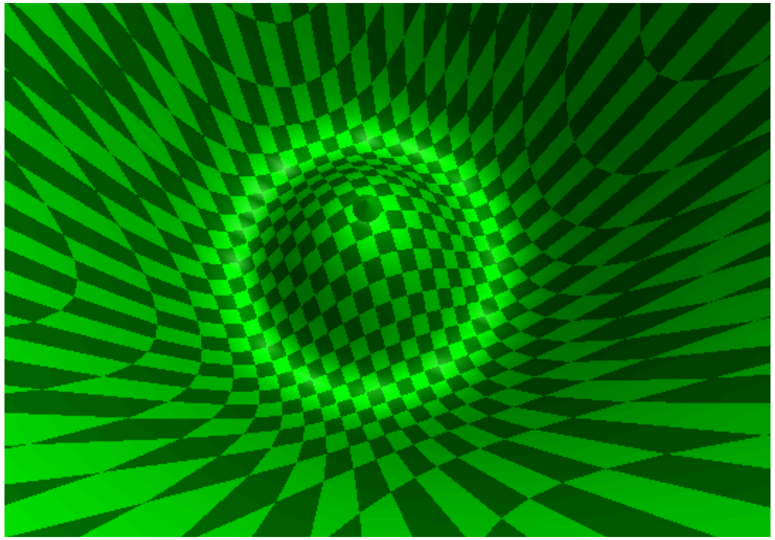
What are quantum vortices?

What are quantum vortices ?

- Topological defects (string-like in 3+1D)
- Symmetry breaking $G \to H$

$$\boxed{\pi_1(G/H) \neq \{e\}}$$

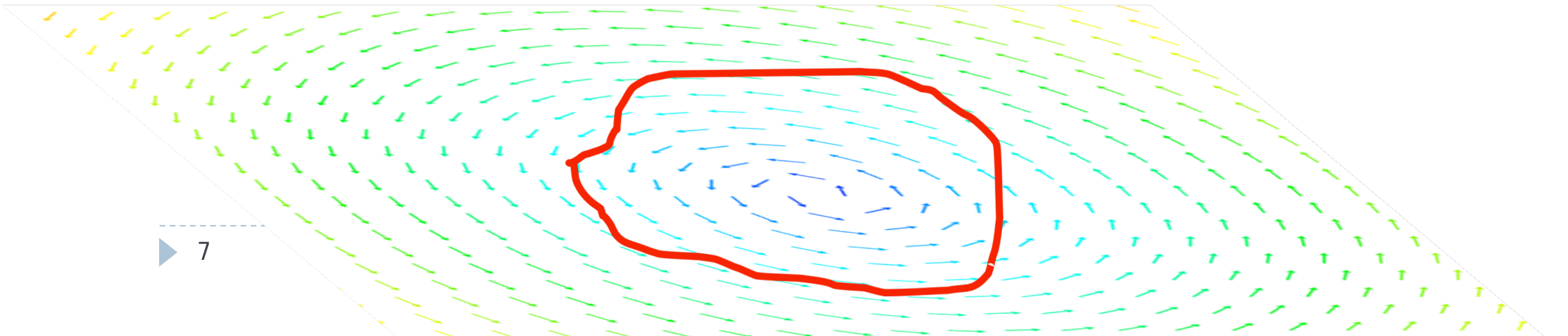
$\longrightarrow$ Topologically stable vortex exists



- Ex) Goldstone model

$$\mathcal{L} = |\partial_\mu \psi|^2 - \frac{\lambda}{4}\left(|\psi|^2 - v^2\right)^2 \qquad \psi(x)$$ complex scalar

$$\langle \psi \rangle = v e^{i\theta} \quad \pi_1(G/H) = \pi_1(U(1)) \simeq Z$$



Abelian vortex - "local" and "global"

- **Global Vortex**
  - breaking of global symmetry
- **Local Vortex**
  - breaking of local symmetry

Goldstone model

$$\mathcal{L}_{global} = |\partial_\mu \psi|^2 - \frac{\lambda}{4}\left(|\psi|^2 - v^2\right)^2$$

Abelian Higgs model

$$\mathcal{L}_{local} = |D_\mu \psi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{4}\left(|\psi|^2 - v^2\right)^2$$

Broken symmetry: Global $U(1)$ — Local $U(1)$

Ex) Superfluids — Superconductors

$\psi(x)$: wave function of the Bose condensate — wave function of condensed Cooper pairs

Abelian vortex - Quantization

- **Global Vortex**
- Quantized circulation

$$\Gamma_C = \oint_C \mathbf{v}_s \cdot d\mathbf{l} = \frac{h}{m} n$$

$$\psi = \sqrt{\rho} \mathrm{e}^{i\theta}$$

$$J_i = -2i \mathrm{Im} \psi^\dagger \partial_i \psi = \rho \frac{\partial_i \theta}{m} \equiv \rho (\mathbf{v}_s)_i$$

$$\mathbf{v}_s = \frac{\nabla \theta}{m}$$

- **Local Vortex**
- Quantized magnetic flux

$$\Phi_M = \oint_C A_i dx^i = \frac{h}{e} n$$

$$n \in Z$$

$$\Updownarrow$$

$$\pi_1(G/H) = \pi_1(U(1)) \simeq Z$$

Quantization Follow from the single-valuedness of the order parameter fields

Abelian vortex - Force btw vortices

- **Global Vortex**

$U(1)$ Phonon : Repulsive

Scalar mode : Attractive

- **Local Vortex**

Massive photon : Repulsive m_v

Scalar mode : Attractive m_s

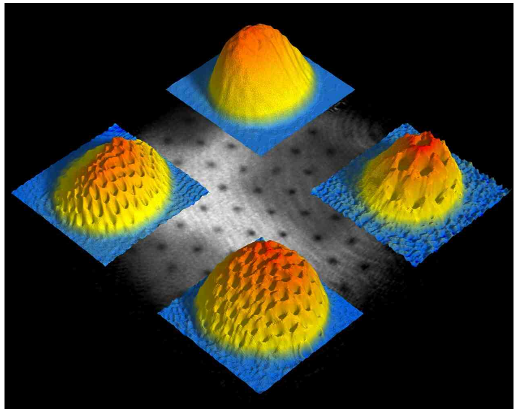
$m_v < m_s$ Type II SC

$m_v > m_s$ Type I SC

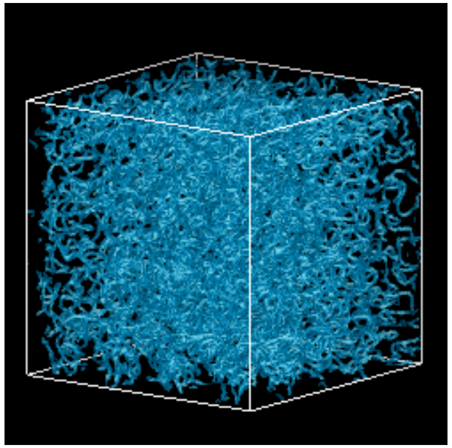
Interaction with quasi-particles is important to discuss the dynamics of vortices

Abelian vortex – Collective structure

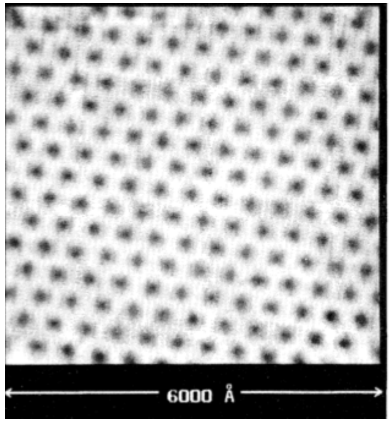
- **Global Vortex**
  - **vortex lattice**
  - **quantum turbulence**
- **Local Vortex**
  - **vortex lattice**
  - **vortex liquid**
  - **vortex glass**



[http://cua.mit.edu/ketterle_group/Nice_pics.htm]



[M. Kobayashi and M.Tsubota, J. Phys. Soc. Jpn. 74, 3248 (2005).]

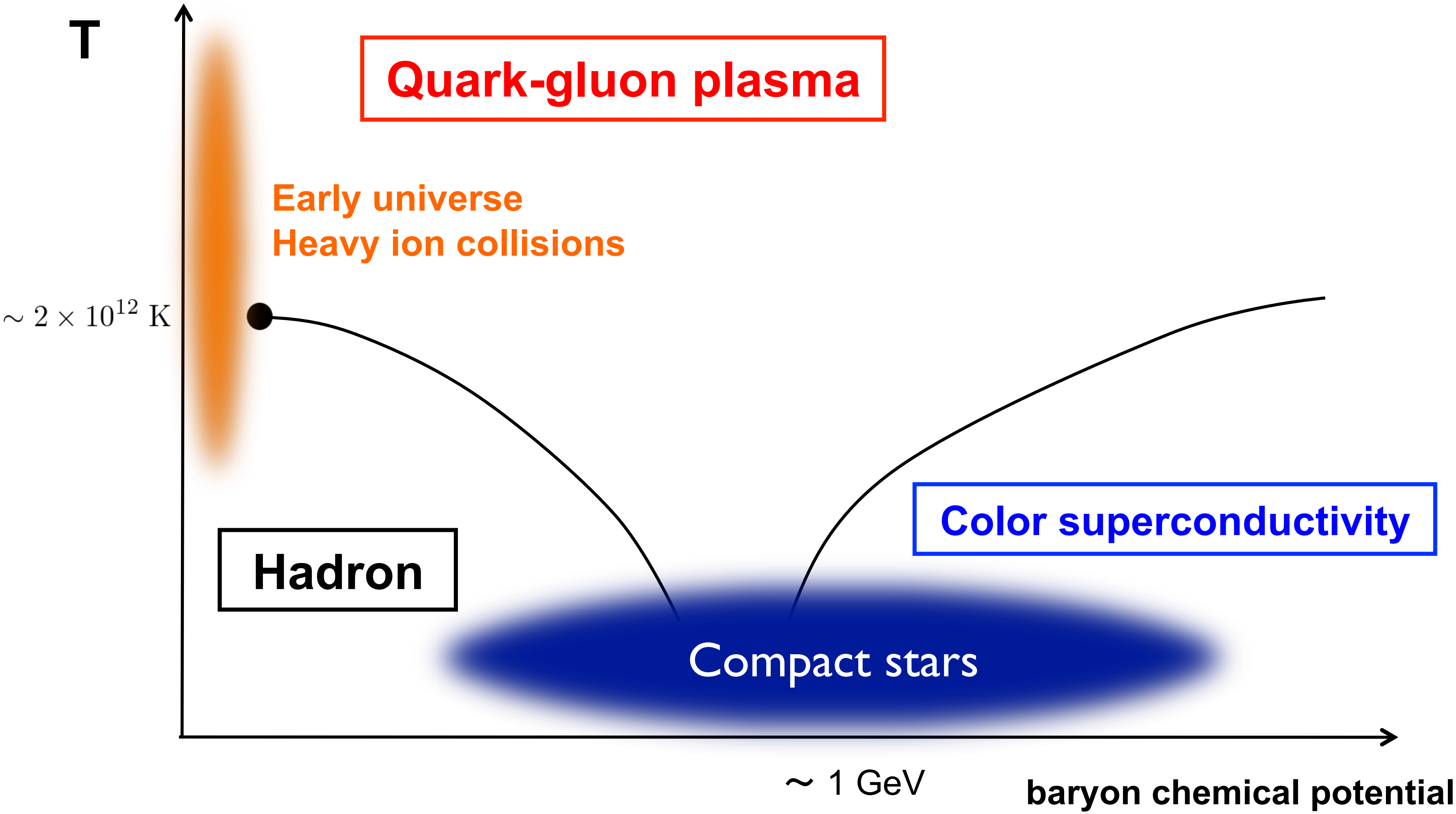


[Hess, et. al, Phys. Rev. Lett. 62 214.]

Possible collective structure can be investigated by making use of vortex-vortex interaction

Non-Abelian vortices in a color superconductor

Phase diagram of QCD



Color superconductivity

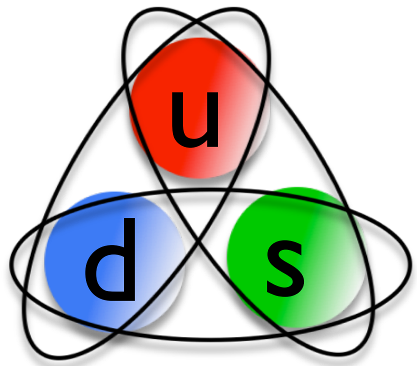
- Order Parameter

$$\Phi_{ai} = \epsilon_{abc}\epsilon_{ijk}\langle q_{bj} C \gamma_5 q_{ck} \rangle$$

Color $\bar{3}$ rep., Flavor $\bar{3}$ rep., $J^P = 0^+$

$\boxed{\textbf{diquark condensate}}$

a, b, c : color index
i, j, k : flavor index
C : charge conjugation matrix



- Color-Flavor Locked Phase

$$\Phi_{ai} = \Delta \delta_{ai}$$

- Expected to be realized at low T and high density

[Alford, Rajagopal and Wilczek (1998)]

Symmetry of the Ground States of CFL

$$\boxed{G = SU(3)_c \times SU(3)_F \times U(1)_B}$$

$$\Phi \to e^{i\theta} V_c \Phi V_F^T$$

$$(V_c, V_F, e^{i\theta}) \in G$$

$$V_c \in SU(3)_c,\ V_F \in SU(3)_F, e^{i\theta} \in U(1)_B$$

$$\Phi_{ai} = \Delta \delta_{ai}$$

$$\boxed{H = SU(3)_{c+F} \times (Z_3)_{F+B}}$$

$$\{h | h = (U, z^{-1}U^*, z), U \in SU(3), z \in Z_3\}$$

- Order parameter space

$$G/H \simeq \frac{U(1) \times SU(3)}{Z_3} \simeq U(3)$$

$$\pi_1(G/H) \simeq Z$$

Topologically stable vortex exists !

Vortex solution [Balachandran, Digal and Matsuura (2006)]

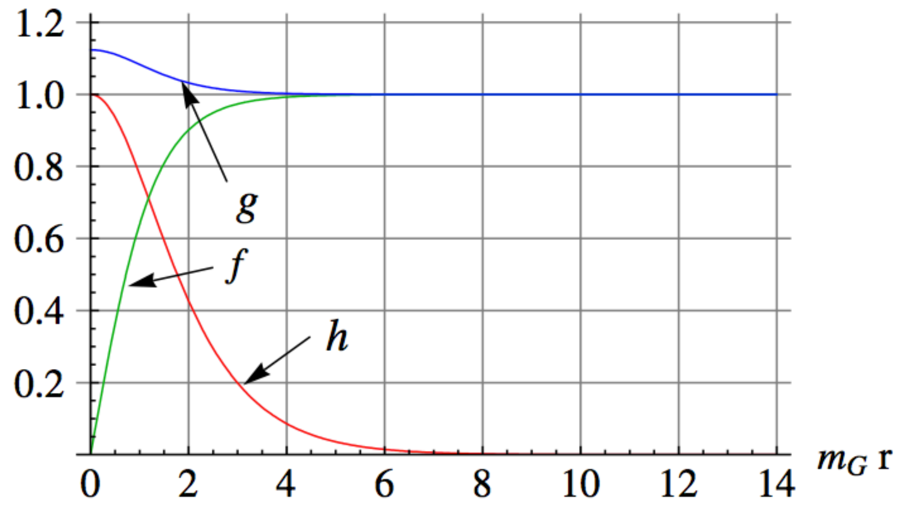
$$\Phi_0(x) = \begin{pmatrix} f(r)e^{i\theta} & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix} \qquad A_i(x) = \frac{1}{g}\frac{\epsilon_{ij}x^j}{r^2}[1-h(r)]\begin{pmatrix} -2/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}$$

At spatial infinity, $\Phi_0(x) \simeq |\Delta|\mathrm{diag}(e^{i\theta}, 1, 1)$

This winding is generated by

$$\frac{1}{3}\left(\underbrace{\mathrm{diag}(1,1,1)}_{\in U(1)_B} - \underbrace{\mathrm{diag}(-2,1,1)}_{\in SU(3)_c}\right)$$

- Global & Local vortex
  - quantized circulation
  - color magnetic flux



[Eto, Nakano and Nitta (2009)]

Orientational zero mode

Moduli space $\mathbb{C}$ $\mathbb{C} \times \underline{\mathbb{C}P^2}$

translational zero mode

orientational zero mode

Orientational zero mode (cont'd.)

- Existence of a vortex further breaks the symmetry

$$SU(3)_{c+F} \longrightarrow [U(1) \times SU(2)]_{c+F}$$

$$\boxed{\frac{SU(3)}{U(1) \times SU(2)} \simeq \mathbb{C}P^2}$$

Nambu-Goldstone mode

$$\Phi_0 \simeq \mathrm{e}^{i\theta/3}\mathrm{diag}(f, g, g) \equiv \Phi^*$$

$$\Phi = U\Phi^* U^{-1} = \mathrm{e}^{i\theta/N}\{F(f,g)\mathbf{1}_N + G(f,g)(\phi\phi^\dagger - \frac{1}{N}\mathbf{1}_N)\}$$

- Low effective theory on the vortex $\longrightarrow$ $\mathbb{C}P^2$ model

[Eto, Nakano and Nitta (2009)]

$$S_{\mathbb{C}P^2} = C \int dx_0 dx_3 \sum_{\mu=0,3} K_\mu \left[\partial^\mu \phi^\dagger \partial_\mu \phi + (\phi^\dagger \partial^\mu \phi)(\phi^\dagger \partial_\mu \phi)\right]$$

ϕ : complex 3 component $\qquad \phi^\dagger \phi = 1$

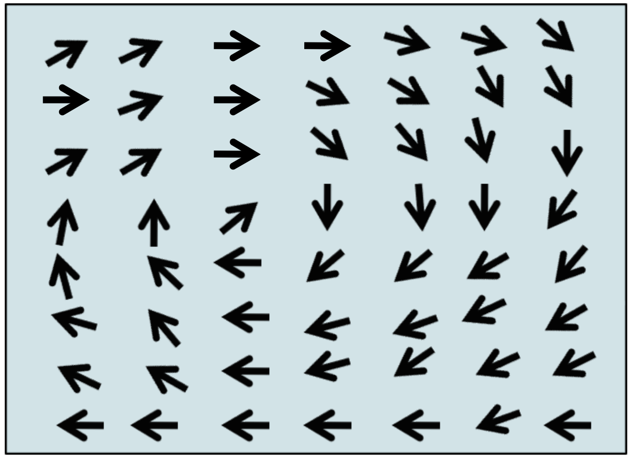
Dual Transformation

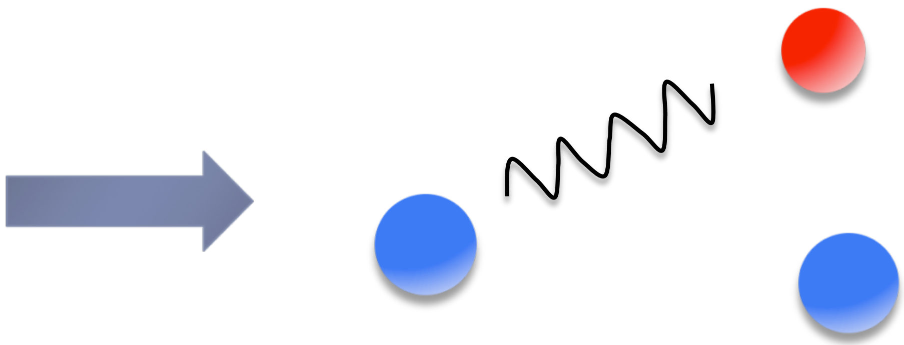
Dual Transformation

- Change of variables
- Particles and topological defects interchange their roles

Ex) Electric-Magnetic duality electron ⟷ magnetic monopole

Ex) XY spin model in 2+1 dim.





Spin wave: Real scalar ϕ ⟹ Vector particle A_μ

[José, Kadanoff, Kirkpatrick and Nelson (1977)]

Dual Transformation (cont'd.)

Advantages of the dual transfm.

Solitons look like particles

The method of ordinary field theories is applicable

Ex) Radiation of phonons from quantum vortices in superfluids
EOM of quantum vortices in superfluids & superconductors [M. Hatsuda,et.al, (1996)]
Radiation of axions from cosmic strings

Correspondence under the dual transfm.

					# of d.o.f
2+1 dim	massless scalar ϕ	$\longleftrightarrow$	massless vector	A_μ	1
3+1 dim	massless scalar ϕ	$\longleftrightarrow$	massless 2-form	$B_{\mu\nu}$	1
	massless vector A_μ	$\longleftrightarrow$	massless vector	A_μ	2
	massive vector A_μ	$\longleftrightarrow$	massive 2-form	$B_{\mu\nu}$	3

Vortex - quasiparticle interaction in the CFL phase

[Y. Hirono, T. Kanazawa and M. Nitta, PRD **83**, 085018 (2011) [arxiv:1012.6042]]

Start: Low energy effective Lagrangian for a color superconductor

- Massless N flavors & N colors
- Order Parameter

$$\Phi_{ai} = \epsilon_{abc}\epsilon_{ijk}\langle q_{bj}^T C\gamma_5 q_{ck}\rangle$$

- Ginzburg-Landau Effective Lagrangian

$$\mathcal{L}(x) = \frac{\epsilon}{2}\left(\mathbf{E}^a\right)^2 - \frac{1}{2\lambda}\left(\mathbf{B}^a\right)^2 + K_0 \mathrm{Tr}\left[\left(D_0\Phi\right)^\dagger D^0\Phi\right] + K_1 \mathrm{Tr}\left[\left(D_i\Phi\right)^\dagger D^i\Phi\right] + iK_0' \mathrm{Tr}\left[\Phi^\dagger D_0\Phi\right] - V(\Phi)$$

where $D_\mu \equiv \partial_\mu - igA_\mu^a T^a$

$$V(\Phi) = \mathrm{Tr}\left[\lambda_1(\Phi^\dagger\Phi)^2 - \lambda_2\Phi^\dagger\Phi\right] + \lambda_3\left(\mathrm{Tr}[\Phi^\dagger\Phi]\right)^2$$

- CFL phase $\Phi_{ai} = \Delta\delta_{ai}$

Quasi-particles in the CFL phase

gluon	CFL pion	scalar modes	$U(1)_B$ phonon

Consider the interaction of vortices
with gluons & $U(1)_B$ phonons

Dual transformation of $U(1)_B$ phonons

$U(1)_B$ phonons $\pi(x)$ → massless two-form field $B^0_{\mu\nu}$

$$\mathcal{L}^*_{\rm Ph} = -\frac{1}{12\tilde{K}_{\mu\nu\sigma}} H^0_{\mu\nu\sigma} H^{0,\mu\nu\sigma} \underline{- 2\pi m^0 B^{0,\mu\nu} \omega^0_{\mu\nu}}$$

Interaction btw. phonons & vortices

$$m^0 \sim |\Delta|$$

$$H^0_{\mu\nu\sigma} \equiv \partial_\mu B^0_{\nu\sigma} + \partial_\nu B^0_{\sigma\mu} + \partial_\sigma B^0_{\mu\nu}\ , \quad \tilde{K}_{\mu\nu\sigma} = \epsilon_{\rho\mu\nu\sigma} K^\rho\ , \quad K_\mu \equiv (K_0, K_1, K_1, K_1)^T$$

Vorticity tensor $\boxed{\omega^0_{\mu\nu} = \frac{1}{2\pi} \epsilon_{\mu\nu\rho\sigma} \partial^\rho \partial^\sigma \pi_{\rm MV}}$

$\omega^0_{\mu\nu} = \frac{1}{N}\frac{1}{2\pi} \epsilon_{\mu\nu\rho\sigma} \partial^\rho \partial^\sigma \theta$ for a static solution of the form $\Phi(x) = {\rm diag}(f(r)e^{i\theta}, g(r), g(r))$

- For a vortex of general shape,

$$\boxed{(\omega^0)^{\mu\nu}(x) = \frac{1}{N} \int d\tau d\sigma \frac{\partial(X^\mu, X^\nu)}{\partial(\tau,\sigma)} \delta^{(4)}(x^\mu - X^\mu(\tau,\sigma))}$$

$X^\mu(\tau,\sigma)$: vortex coordinate

Dual transformation of gluons

massive vector field A_μ^a ⇒ **massive two-form field $B_{\mu\nu}^a$**

- 1. Consider the partition function

$$Z = \int \mathcal{D}A_\mu^a(x)\mathcal{D}\Phi(x)\exp\left\{i\int d^4x\mathcal{L}(x)\right\}$$

Gauge fixing condition – imposed on $\Phi(x)$

- 2. Introduce $B_{\mu\nu}^a$ by Hubbard-Stratonovich transformation

$$\exp\left[i\int d^4x\left\{-\frac{1}{4}(F_{\mu\nu}^a)^2\right\}\right] \propto \int \mathcal{D}B_{\mu\nu}^a \exp\left[i\int d^4x\left\{-\frac{1}{4}\left[m^2(B_{\mu\nu}^a)^2 - 2m\tilde{B}_{\mu\nu}^a F^{a,\mu\nu})\right]\right\}\right]$$

[Seo, Okawa and Sugamoto (1979)]

- 3. Integrate out the original gauge fields A_μ^a

Dual transformation of gluons (cont'd.)

- Dual Lagrangian for gluons

$$\mathcal{L}_{\mathrm{G}}^{*} = -\frac{1}{12\tilde{K}_{\mu\nu\sigma}} H^{a}_{\mu\nu\sigma} H^{a,\mu\nu\sigma} \underbrace{- \frac{1}{4} m^2 \left(B^{a}_{\mu\nu}\right)^2}_{\text{mass term}} \underbrace{- \frac{m}{2g} B^{a,\mu\nu} \omega^{a}_{\mu\nu}}_{\text{Interaction}}$$

$$m \simeq g|\Delta|$$

$$H^{a}_{\mu\nu\sigma} \equiv \partial_\mu B^{a}_{\nu\sigma} + \partial_\nu B^{a}_{\sigma\mu} + \partial_\sigma B^{a}_{\mu\nu}\,,\ \tilde{K}_{\mu\nu\sigma} = \epsilon_{\rho\mu\nu\sigma} K^{\rho}$$

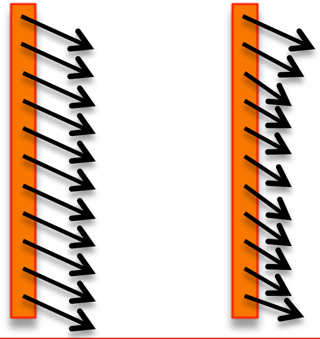
Vorticity tensor

$$\boxed{\omega^{a,\lambda\sigma} \equiv \epsilon^{\lambda\sigma\mu\nu} \left[\partial_\nu \left\{(\mathbf{\Phi}^{-1})(^{ab}_{\mu\rho}) J^{b,\rho}\right\} + J^{e,\alpha} (\mathbf{\Phi}^{-1})^{ec}_{\alpha\mu} f^{cda} (\mathbf{\Phi}^{-1})^{db}_{\nu\beta} J^{b,\beta}\right]}$$

where $\mathbf{\Phi}^{ab}_{\mu\nu} = \frac{1}{2} g_{\mu\nu} \sqrt{K_\mu K_\nu}\, \mathrm{Tr}\left[\Phi^\dagger T^a T^b \Phi\right]$, $J^a_\mu = -2K_\mu \mathrm{Im}\mathrm{Tr}\left[\Phi^\dagger \partial_\mu T^a \Phi\right]$

Vorticity tensor

- Express vorticity in terms of orientational zero modes ϕ
- If the orientation is z-independent



$$\omega^a_{\lambda\sigma} = -\frac{16}{N}\epsilon_{\lambda\sigma\mu\nu}\partial_\nu \left\{ \gamma(r) \left\{ \partial_\mu \theta + \frac{NK_0'}{2K_0}\delta_{\mu 0} \right\} \phi^\dagger T^a \phi \right\}$$

$$\boxed{\begin{aligned} &+ \epsilon_{\lambda\sigma\mu\nu}\Big\{ i\alpha(1+\beta)\left(\phi^\dagger T^a \partial_\mu \phi - \partial_\mu \phi^\dagger T^a \phi + 2\phi^\dagger T^a \phi \partial_\mu \phi^\dagger \phi\right) \\ &\qquad - \frac{4}{N}\alpha\gamma(1+\beta)\left\{\partial_{[\mu}\phi^\dagger T^a \phi + \phi^\dagger T^a \partial_{[\mu}\phi\right\}\left\{\partial_{\nu]}\theta + \frac{NK_0'}{2K_0}\delta_{\nu]0}\right\} \\ &- \frac{i}{2}\alpha^2(1+\beta^2)\left[\phi^\dagger T^a \phi \partial_{[\mu}\phi^\dagger \partial_{\nu]}\phi + \partial_{[\mu}\phi^\dagger T^a \partial_{\nu]}\phi + \phi^\dagger T^a \partial_{[\mu}\phi \partial_{\nu]}\phi^\dagger \phi + \partial_{[\mu}\phi^\dagger T^a \phi \partial_{\nu]}\phi^\dagger \phi\right]\Big\} \end{aligned}}$$

Correction due to the modulation in the $\mathbb{C}P^{N-1}$ space

- Localized around the vortex
- Gluons interact with orientational zero modes

Orientation dependence of interaction of two vortices via gluon exchanges

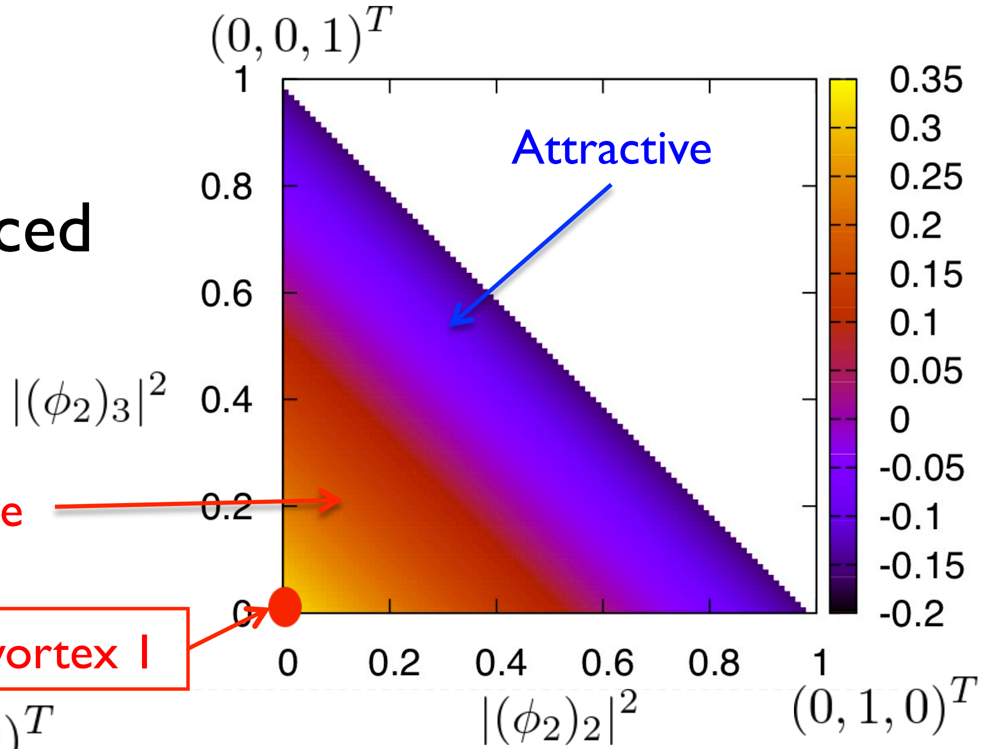
- Interaction energy of two vortices

$$\mathrm{E} \sim \int d\vec{X}_1 d\vec{X}_2 \underbrace{\phi_1^\dagger T^a \phi_1 \; \phi_2^\dagger T^a \phi_2}_{\text{Orientation dependence}} \frac{\exp(-M_G|\vec{X}_1-\vec{X}_2|)}{\sqrt{|\vec{X}_1-\vec{X}_2|}}$$

$\phi_i \equiv \phi(\vec{X}_i)$ $\quad \vec{X}_i$: Vortex coordinate $i = 1, 2 \quad \mathrm{M}_G = g|\Delta|$

- Attractive
  when the color flux is reduced
- Repulsive
  if their orientations are close

Repulsive

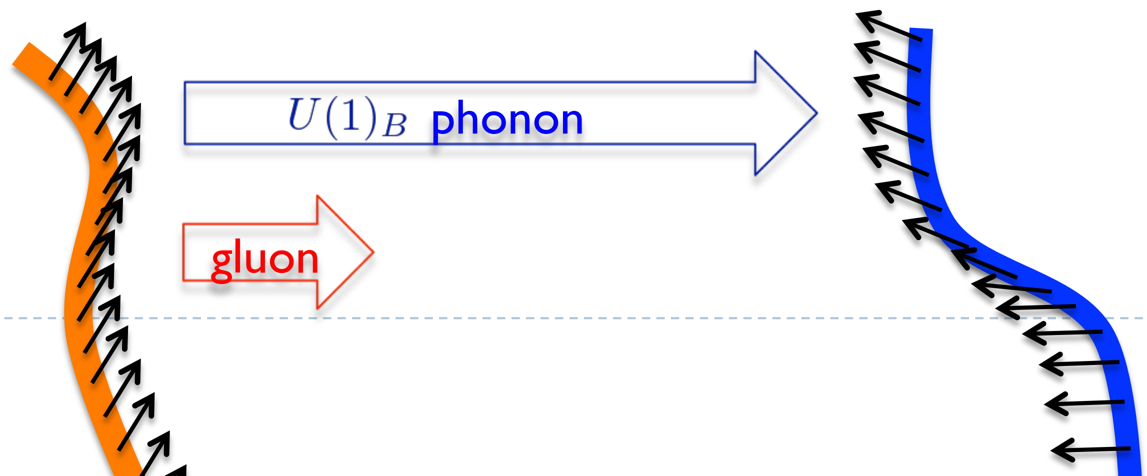


Orientation of the vortex 1

$\phi_1 = (1,0,0)^T$

Summary

- Interaction of non-Abelian vortices with quasi-particles is derived in terms of a dual transformation
  - $U(1)_B$ phonons are described by massless two-form fields and interact with translational zero modes
  - Gluons are described by massive two-form fields and interact with translational & orientational zero modes
    - Spatial modulation of orientations gives additional contribution
  - We have investigated the orientation dependence of interaction of two vortices via gluon exchanges



Outlook

- Possible applications
  - Effect of massive scalar modes
  - Radiation of phonons / gluons
  - Reconnection
  - Low energy effective theory of a vortex
  - Collective structure of vortices (lattice, turbulence, …)
  - Interaction with fermionic zero modes
  - Play some role in glitch phenomena in neutron stars?

- Other interesting topics
  - Fermionic zero modes trapped in the vortices
  - Novel non-Abelian statistics of fermionic zero modes
  - Monopoles in the vortices as a kink of (1+1)D effective theory
  - …