Chiral Magnetic Effect and the QCD Phase Transitions

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Typical Scales in QCD Physics

What happens

at $T \sim \Lambda_{QCD} \sim 200\text{MeV}$?

→ Quark-Gluon Plasma

What happens

at $\rho_B \sim (\Lambda_{QCD})^3 \sim 1\text{fm}^{-3}$?

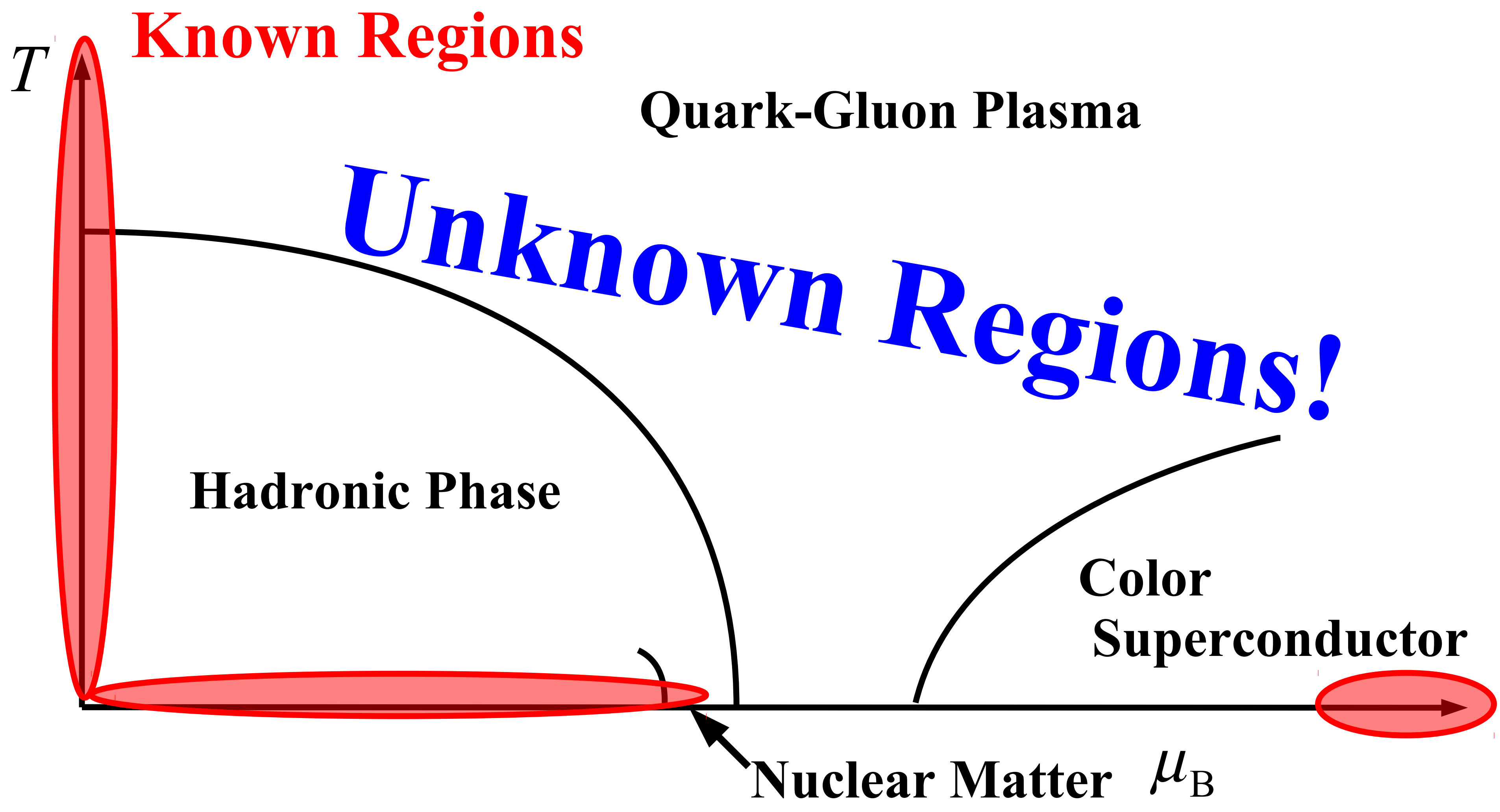
→ Color Superconductor

What happens

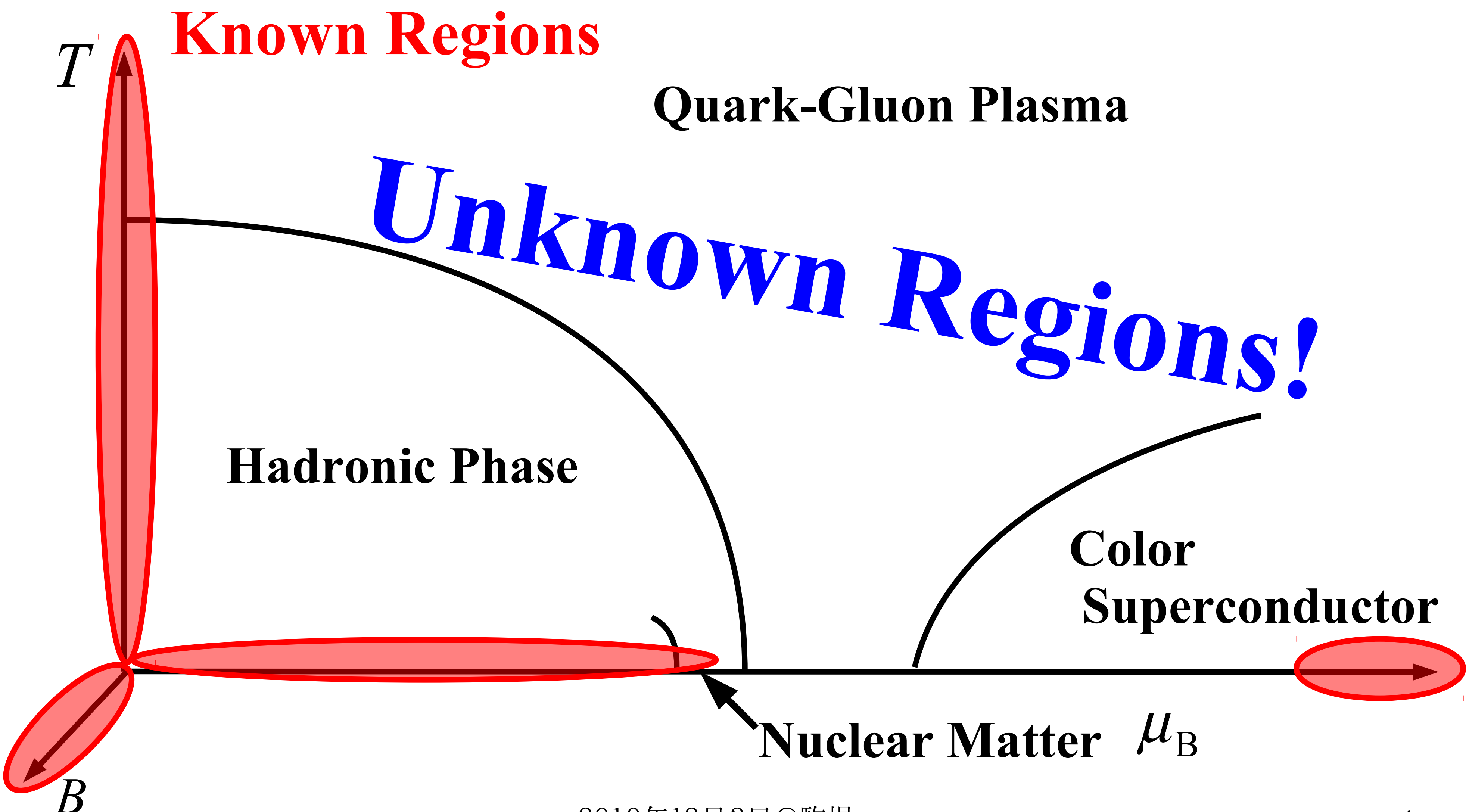
at $eB \sim (\Lambda_{QCD})^2 \sim 6.8 \times 10^{18}\text{G}$?

→ ?????

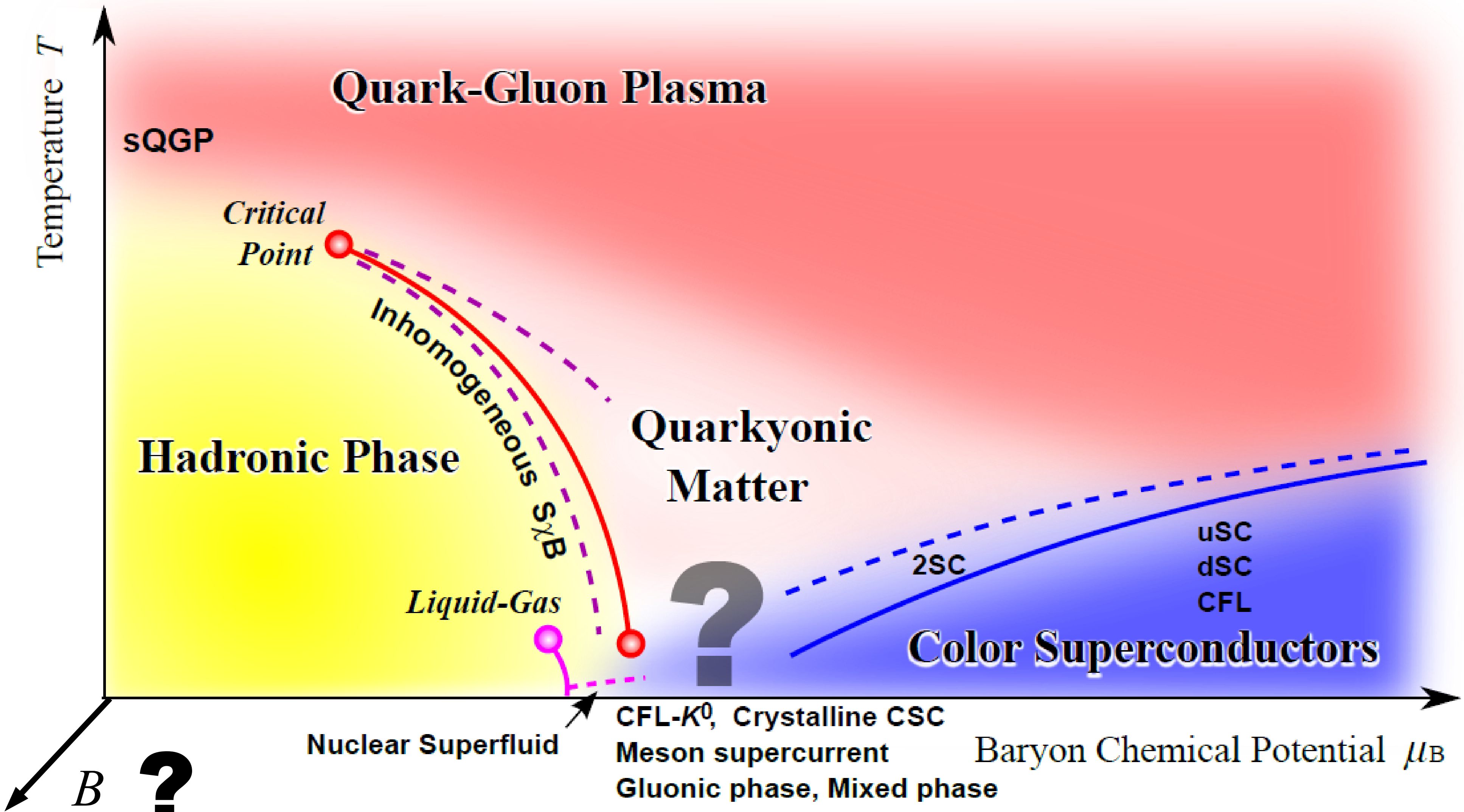
QCD Phase Diagram



QCD Phase Diagram



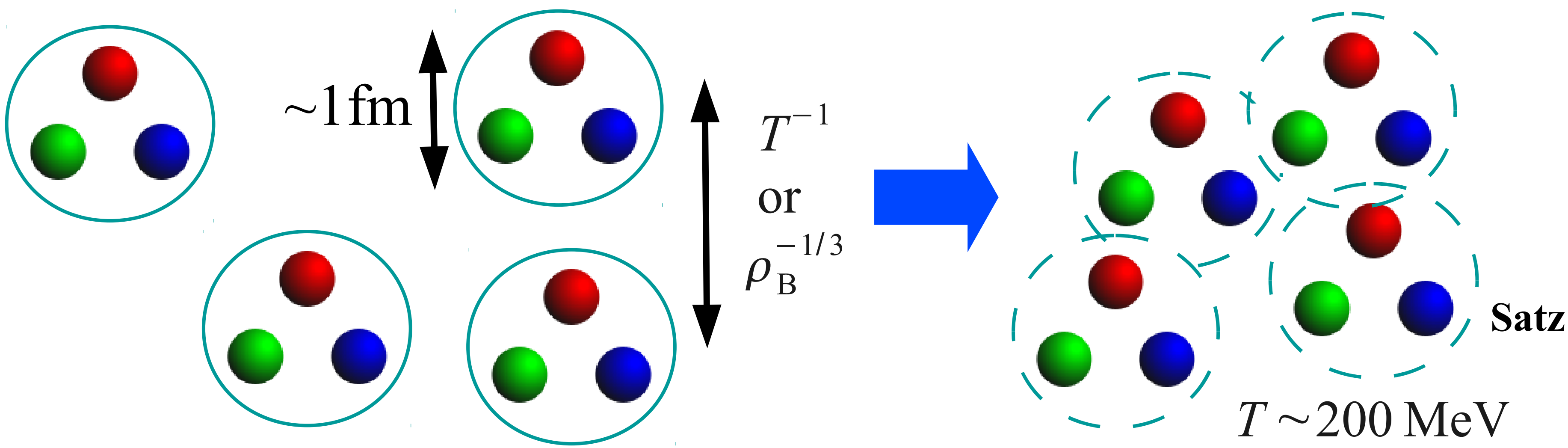
QCD Phase Diagram



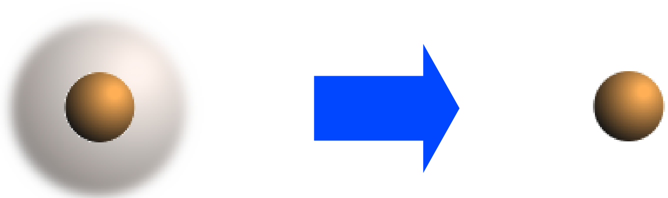
KF-Hatsuda (2010)

Two QCD Phase Transitions

Quark Deconfinement Transition (Polyakov Loop)



Chiral Phase Transition (Chiral Condensate)



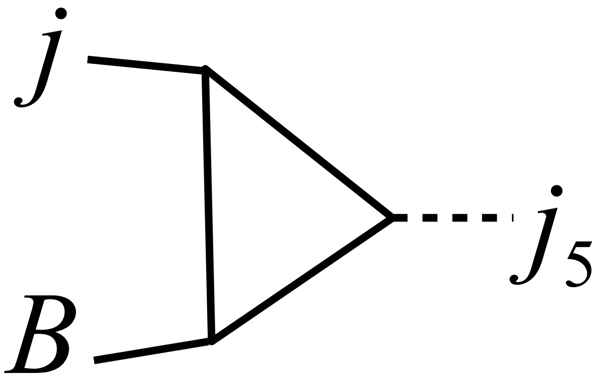
"Bare" Quarks
$m_q \sim$ 3-6 MeV

"Constituent" Quarks
$M_q \sim$ 350 MeV

Nambu-Jona-Lasinio

Most Likely $T_{ch} > T_{dec}$

Anomaly Matching Condition ('t Hooft)

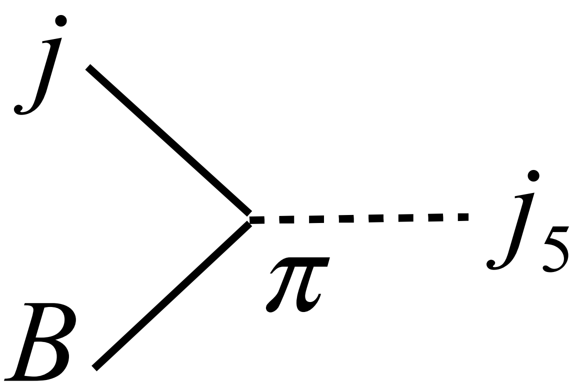


$$K^{AB}_{\mu\nu}(q) = i\int d^4x\, e^{iq\cdot x}\langle T\, j^A_{5\mu}(x)\, j^B_\nu(0)\rangle_B$$

$$\sim -N_c \frac{e_A B}{2\pi^2}\frac{q_\mu \tilde{q}_\nu}{q^2}\frac{1}{2}\delta^{AB}$$

c.f. Chiral Magnetic Effect $j_3 \sim B\, j_{50}$ $j_{53} \sim B\, j_0$

If quarks are confined, how this IR singularity appears?



In the confined world there must be massless NG bosons (chiral broken).

For $N_f = 2$, massless proton and neutron can saturate the IR singularity...

In real world $N_f > 2$ but *s* quarks are massive !?

Most Likely $T_{ch} > T_{dec}$

Potential Model Argument (Casher)

Assume that massless quarks are confined in a bag by *s*-wave potential (which does not change the spin).

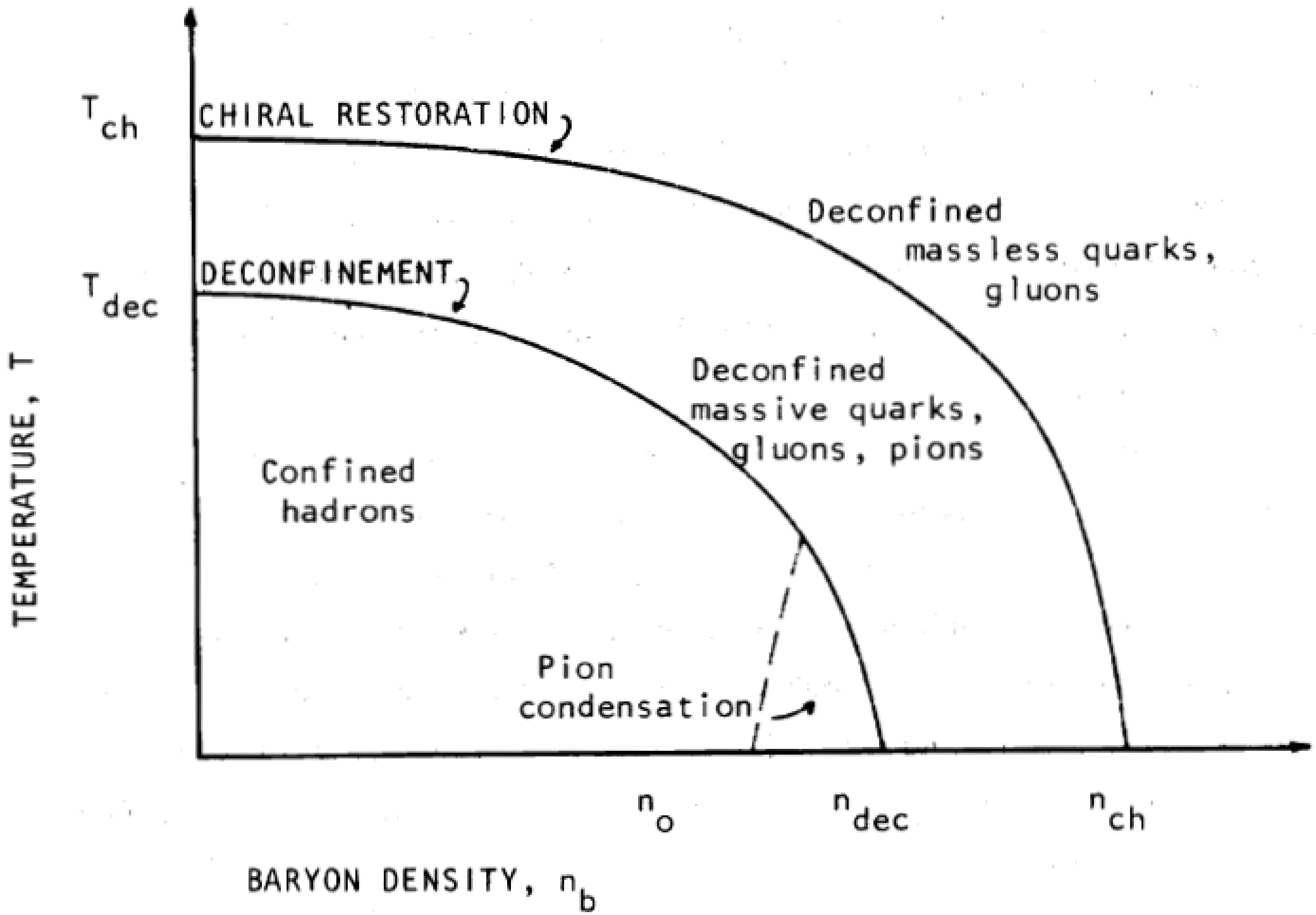
When quarks turn back, they change the chirality, which should be compensated by chiral condensate.

Confinement requires chiral symmetry breaking

Just arguments... No proof...

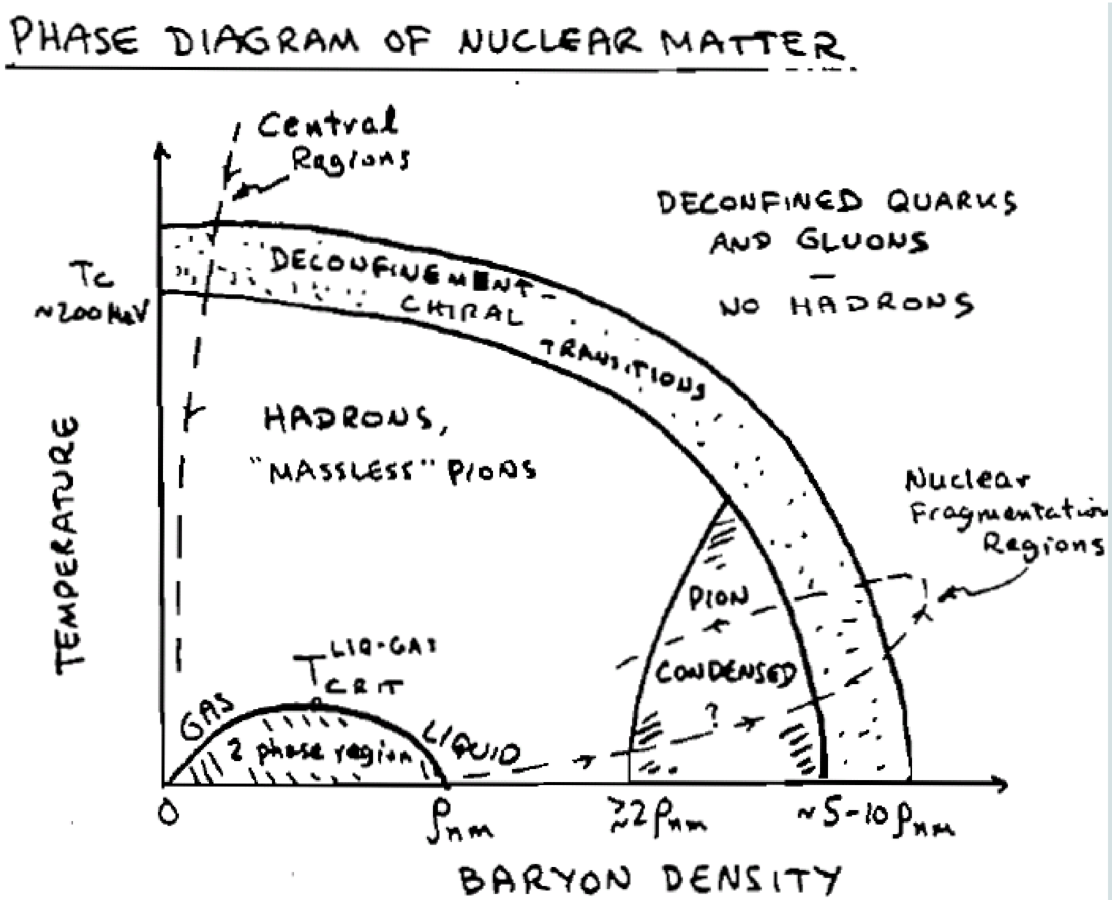
Two Separate Transitions

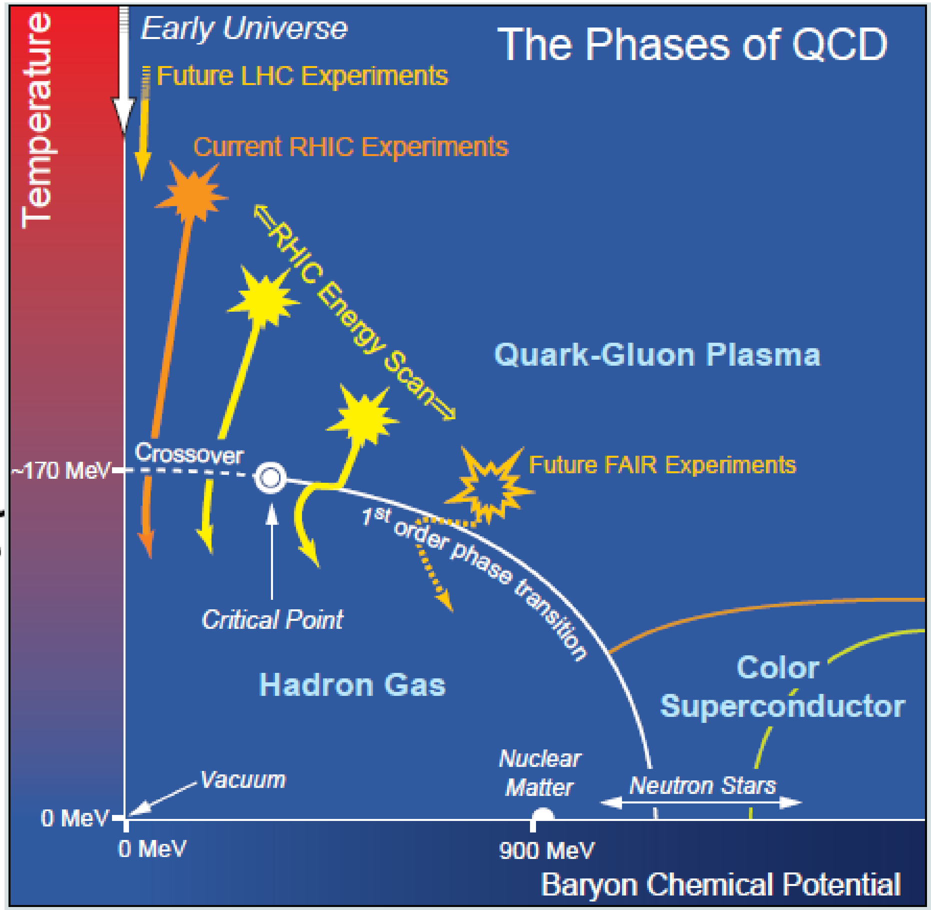
Gordon Baym (1982)



"A Long Range Plan"

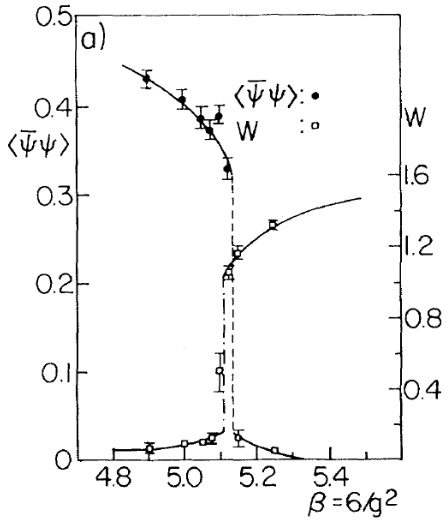
Taken from 1983 and Taken from 2007



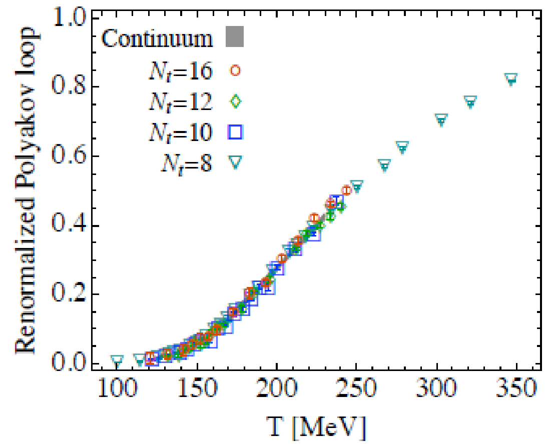


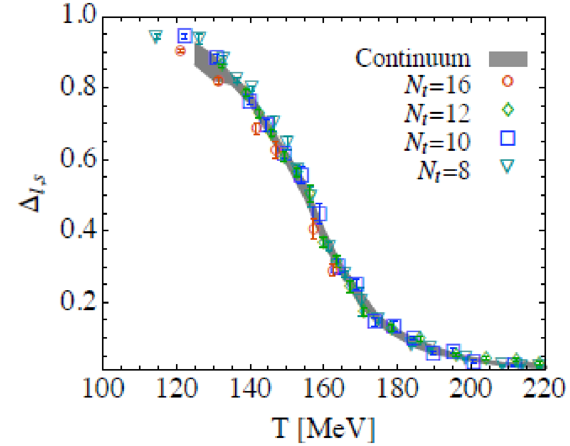
Some Highlights

Kogut et al (1983)

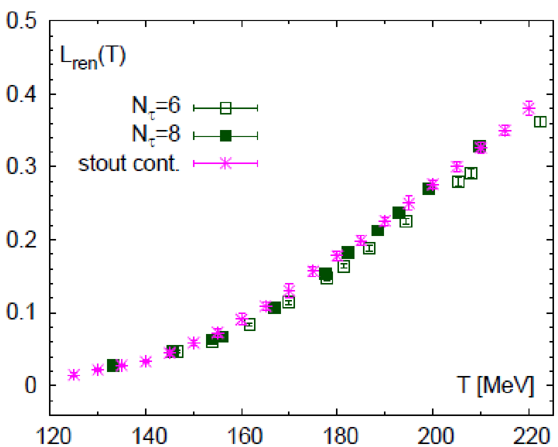


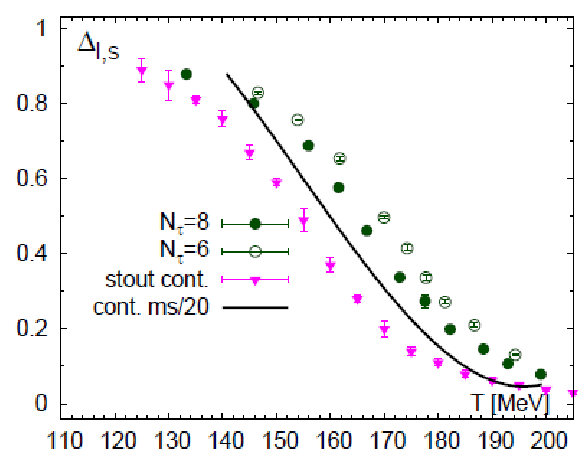
Wuppertal-Budapest (2010)





Bazavov et al (2010)

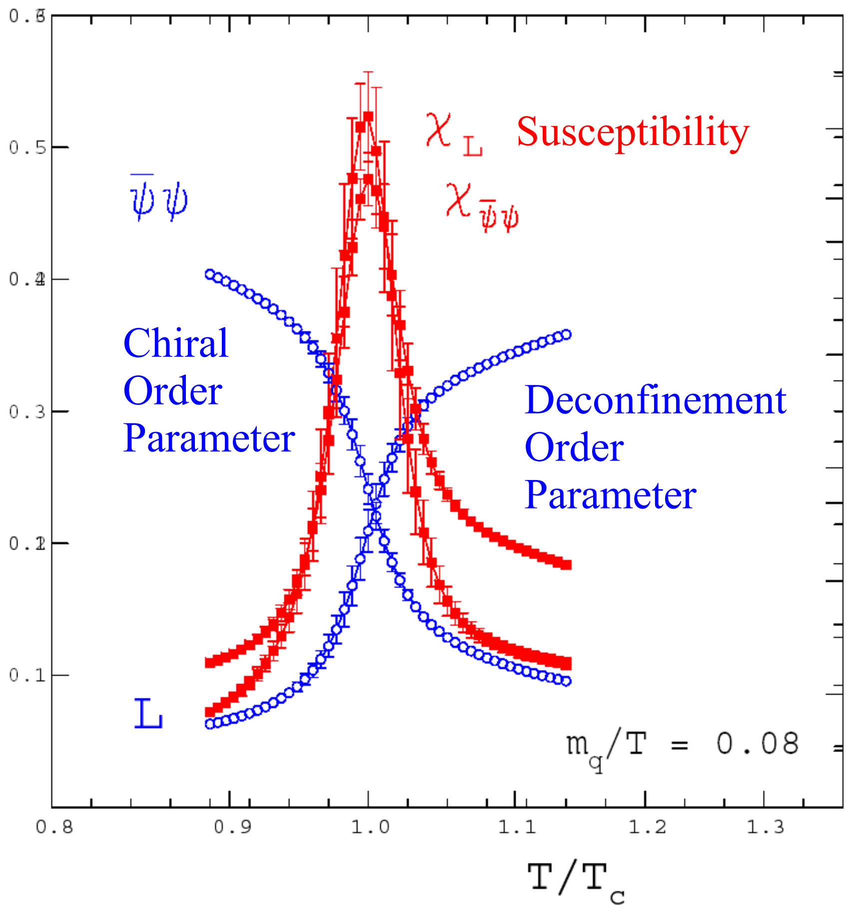




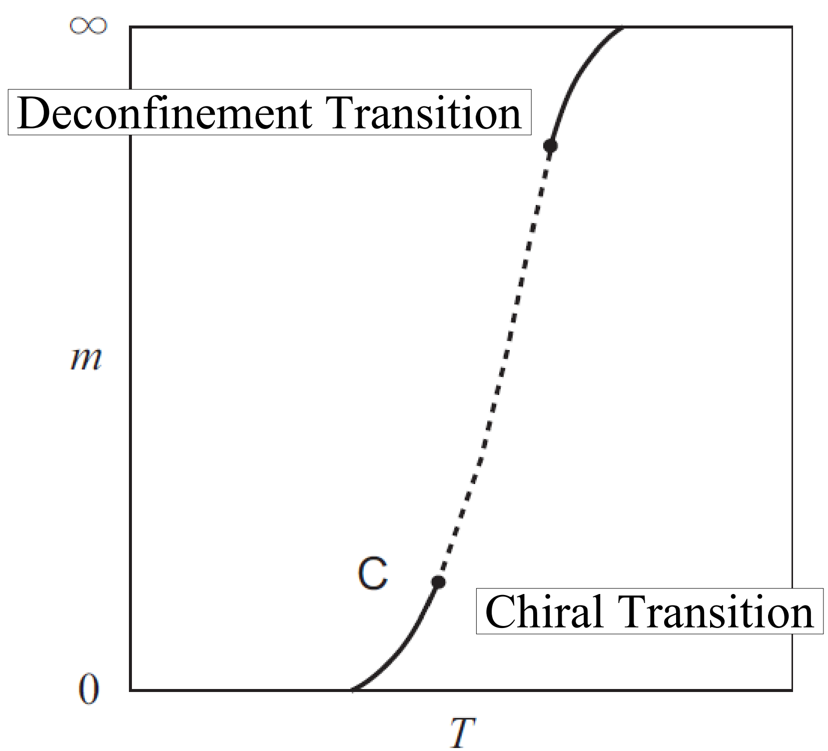
Deconfinement and Chiral Transitions (Nearly) Coincide

Coincidence for Any Quark Mass

Old Lattice-QCD Data
Hands (physics/0105022)



Two-crossovers connected
(Fukushima-Hatta 2003)



(Nearly) Simultaneous crossover for any quark mass

Coincidence for Any Magnetic Field

Recent Lattice-QCD Simulation (M.D'Elia et al)

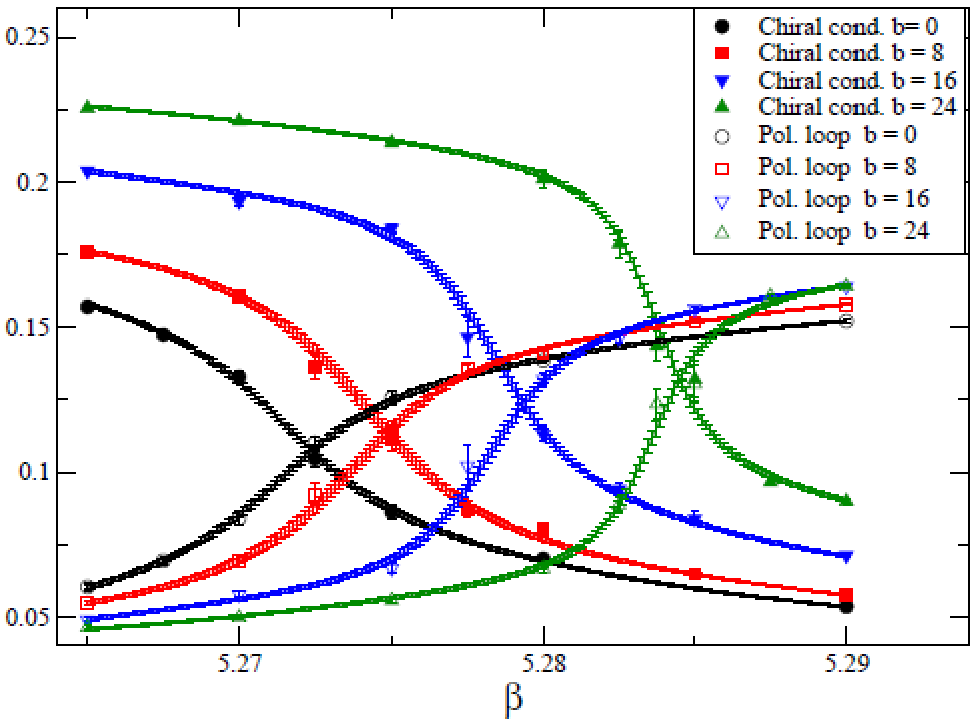


FIG. 3: Same as in Fig. 1 for $am = 0.01335$

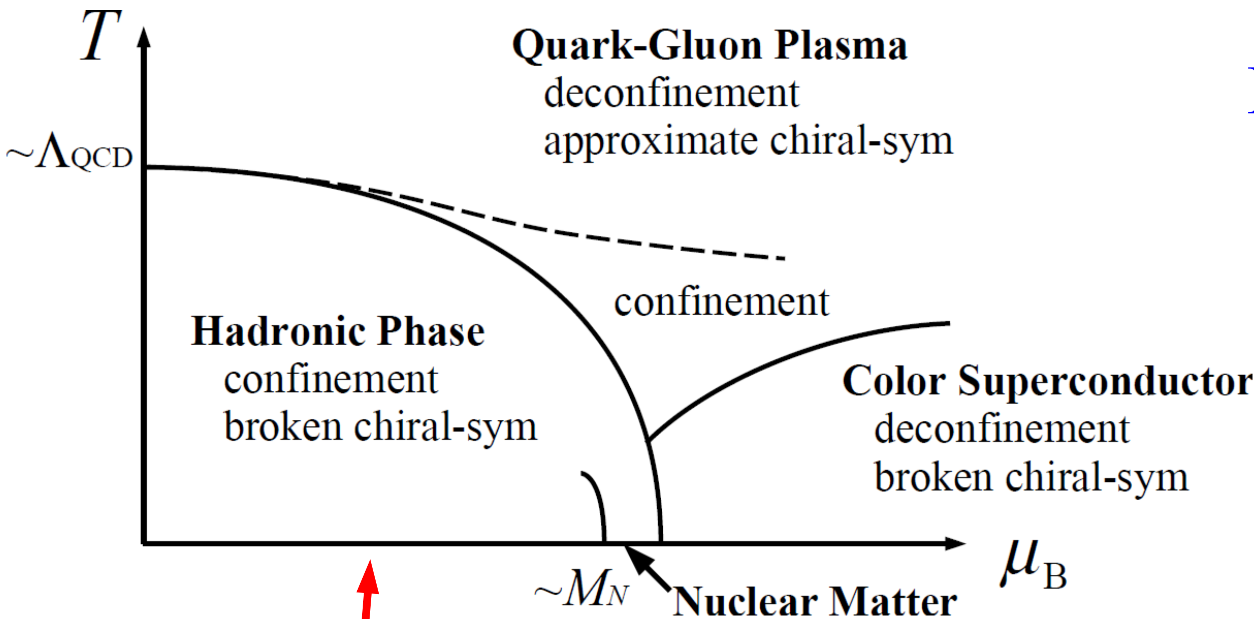
Change in T_c is 10~20%

Magnetic field $eB \sim 0.75\text{GeV}^2$ should be large enough to affect QCD physics.

Why locked together??

Phase diagram in the *T*-*B* plane.

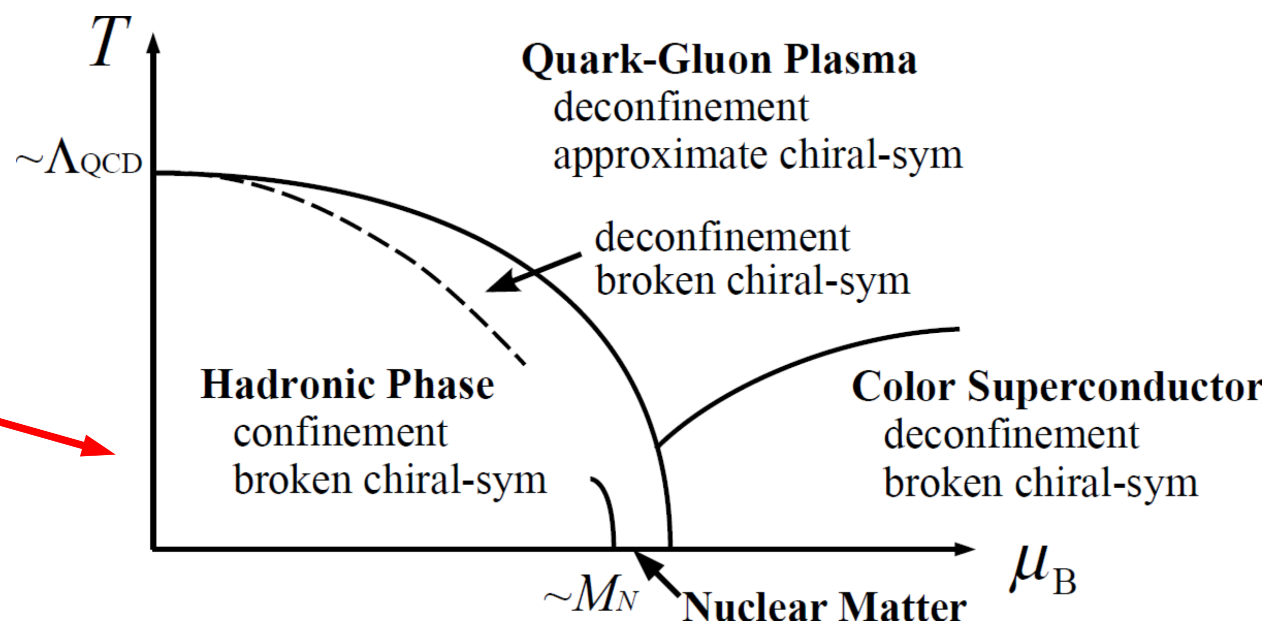
Finite Density Extension



Deconfinement becomes smoother crossover at higher baryon density

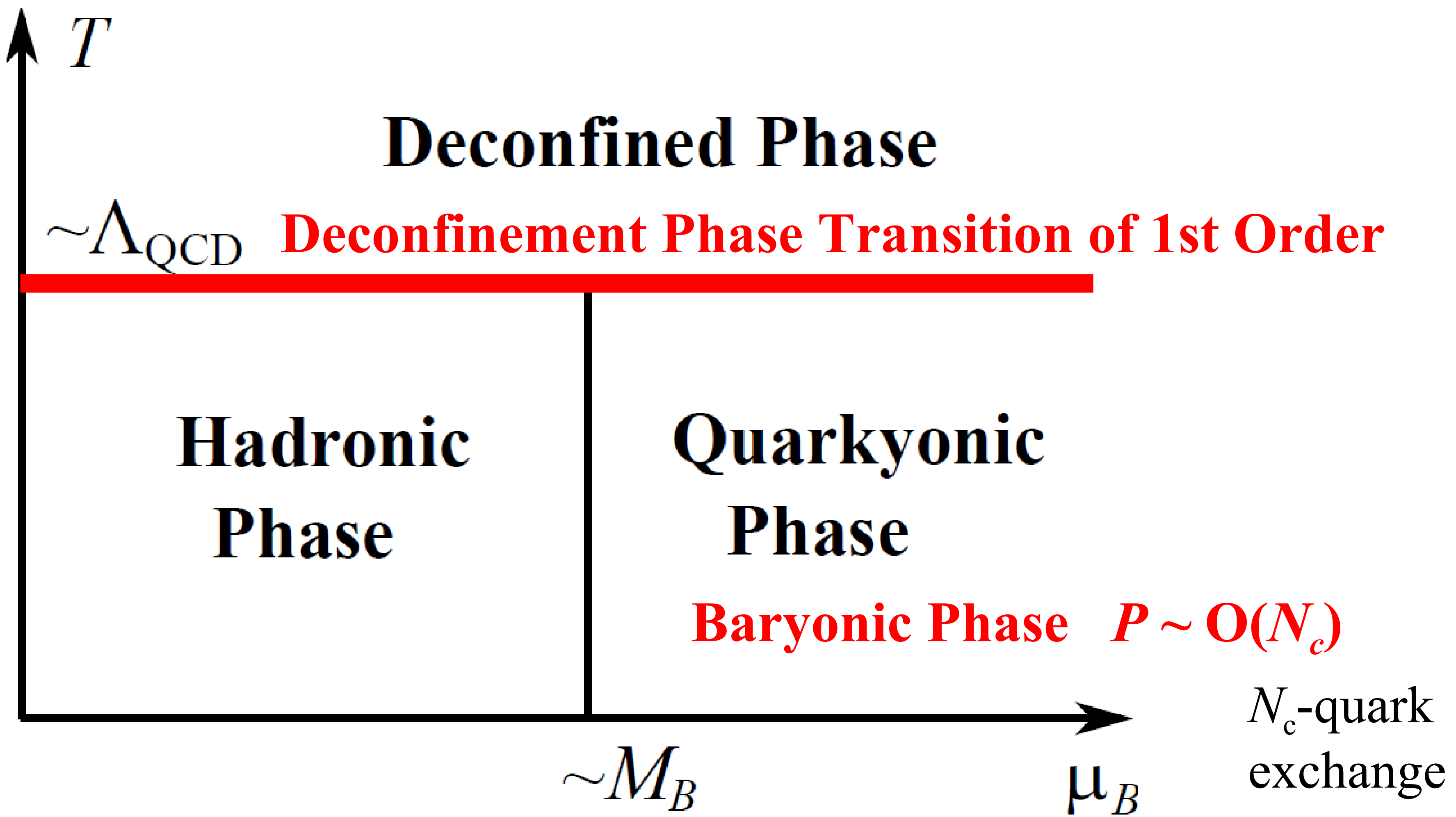
Is this possible ?

Implied by Casher – 't Hooft



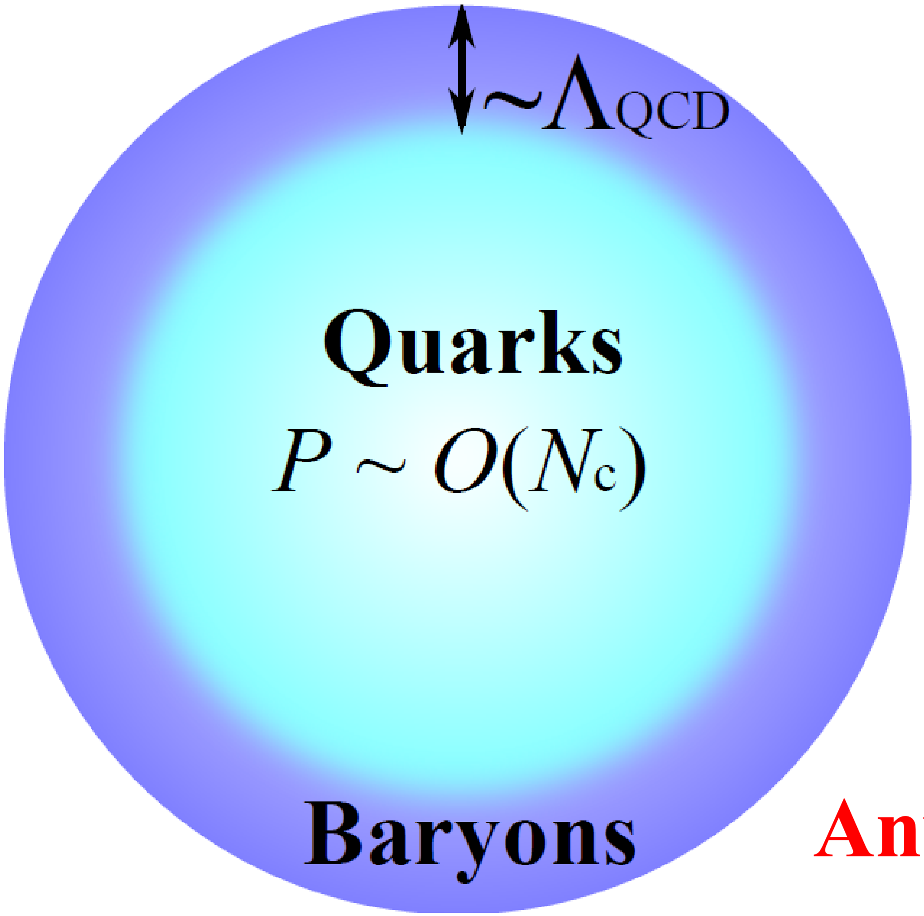
Possibility implied by BNL People

Phase Diagram of Large-N_c QCD



Interpretation – Quarkyonic Matter

Structure of the Fermi Sphere



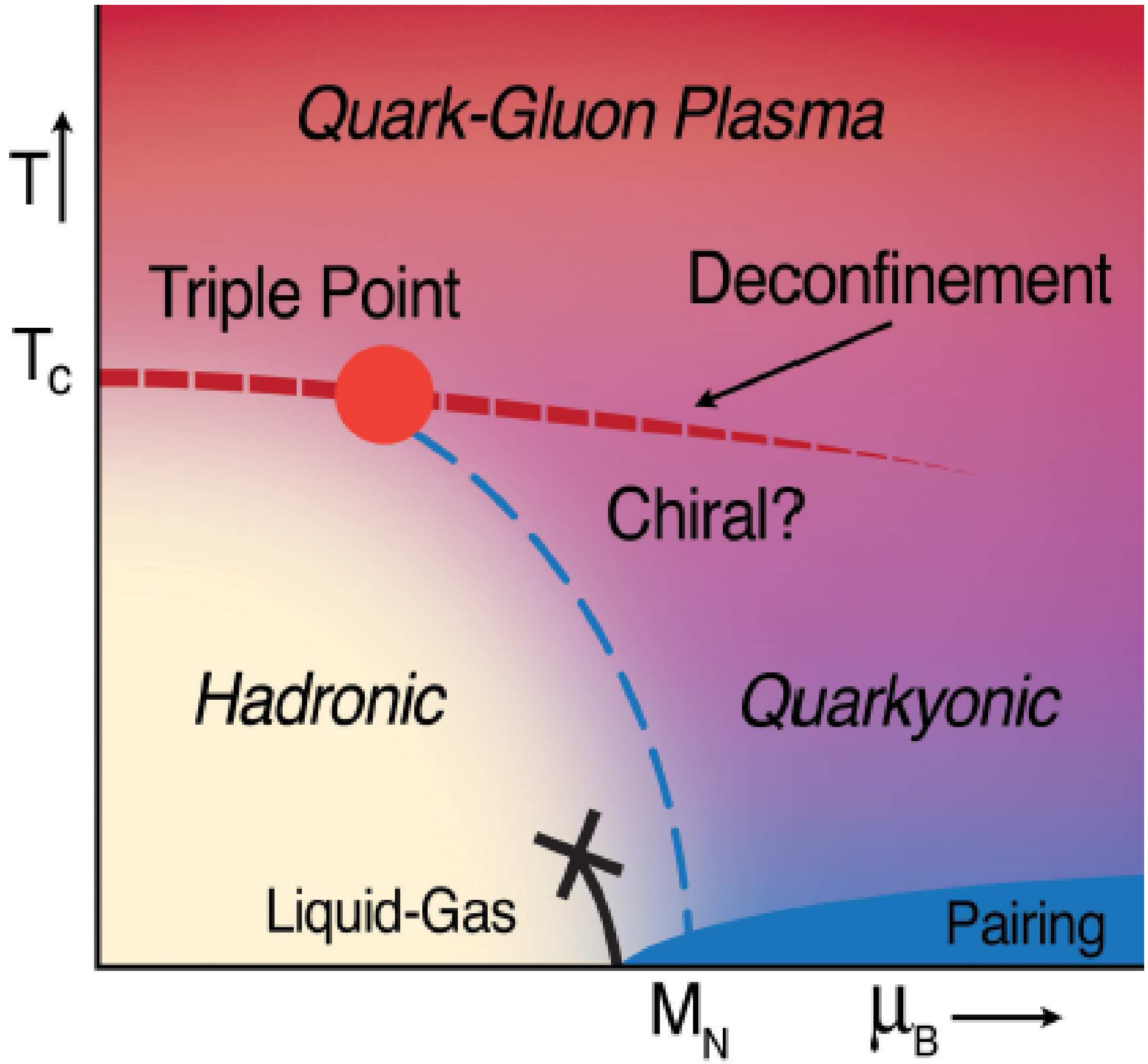
Ground state of large-N_c quark matter at $\mu_q \gg \Lambda_{QCD}$

McLerran, Pisarski
Hidaka, Kojo

Any remnant in N_c=3 world?

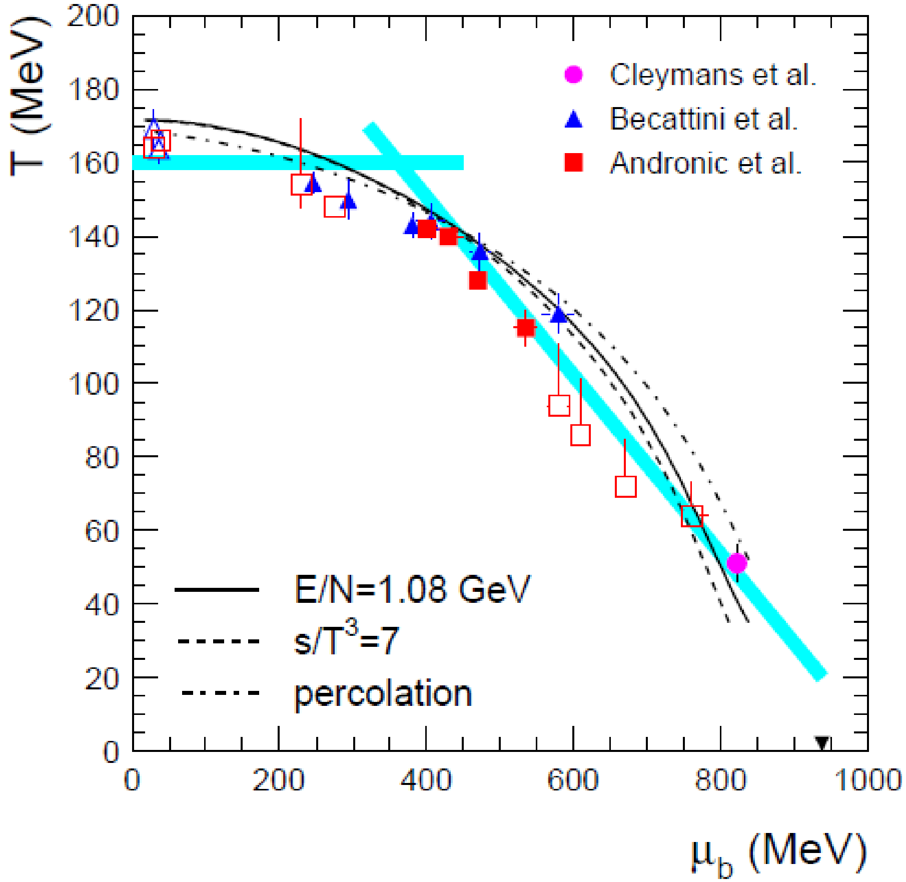
Large-N_c-inspired Phase Diagram

NPA837, 65 (2010)
Andronic, Blaschke,
Braun-Munzinger,
Cleymans, KF,
McLerran, Oeschler,
Pisarski, Redlich,
Sasaki, Satz, Stachel



Experiment Data

Chemical Freeze-out Points from the Statistical Model



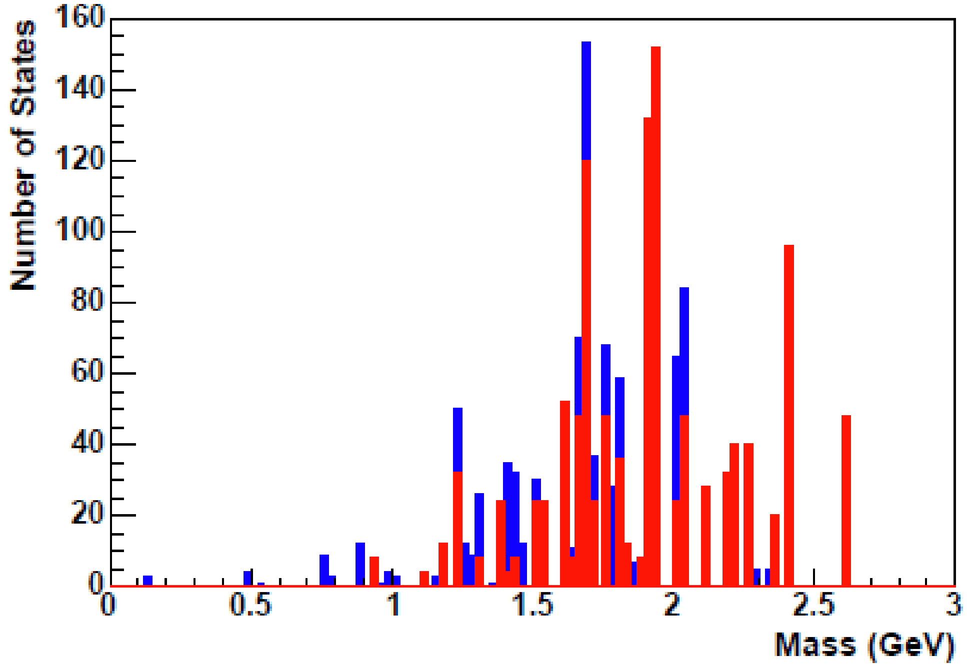
NPA837, 65 (2010)
Andronic, Blaschke, Braun-Munzinger, Cleymans, KF, McLerran, Oeschler, Pisarski, Redlich, Sasaki, Satz, Stachel

Only available information at finite baryon density

Most important data to analyze the phase diagram

Statistical Model

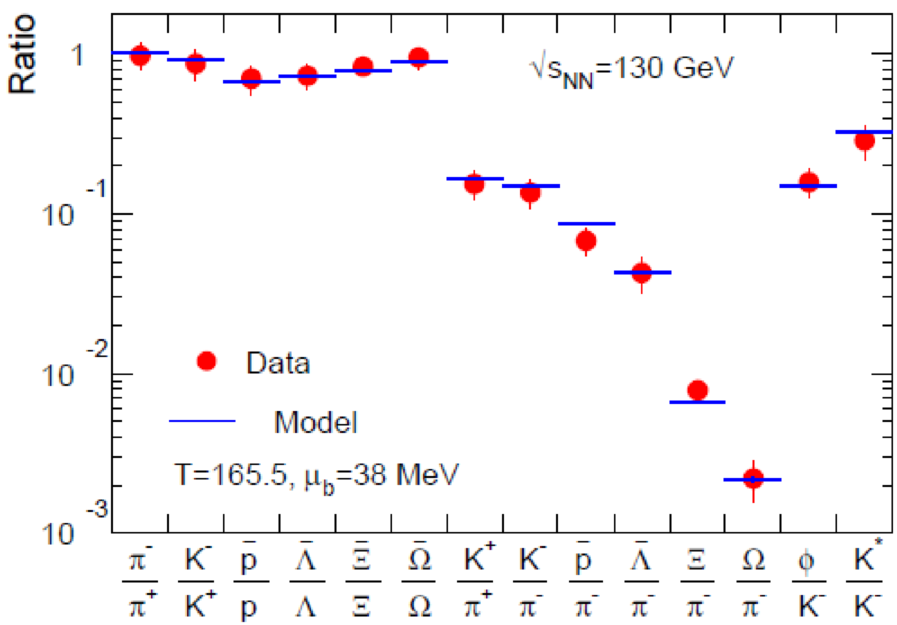
Free stable and unstable hadrons (resonances) at temperature $\boldsymbol{T}$, baryon chemical potential $\boldsymbol{\mu}_{\mathbf{B}}$, strangeness chemical potential μ_{s}, electric chemical potential μ_{Q}, where μ_{s} and μ_{Q} are constrained by the collision condition.

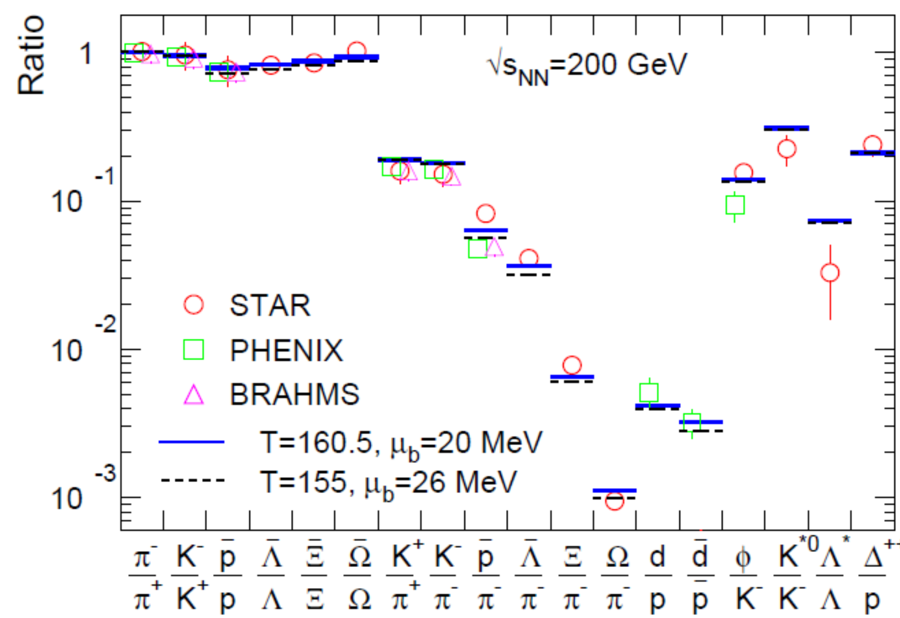


Contained mesons (blue) and baryons (red): figure from THERMUS2.1 Manual

Success of the Statistical Model

Thermal Weight = Particle Yield Ratio



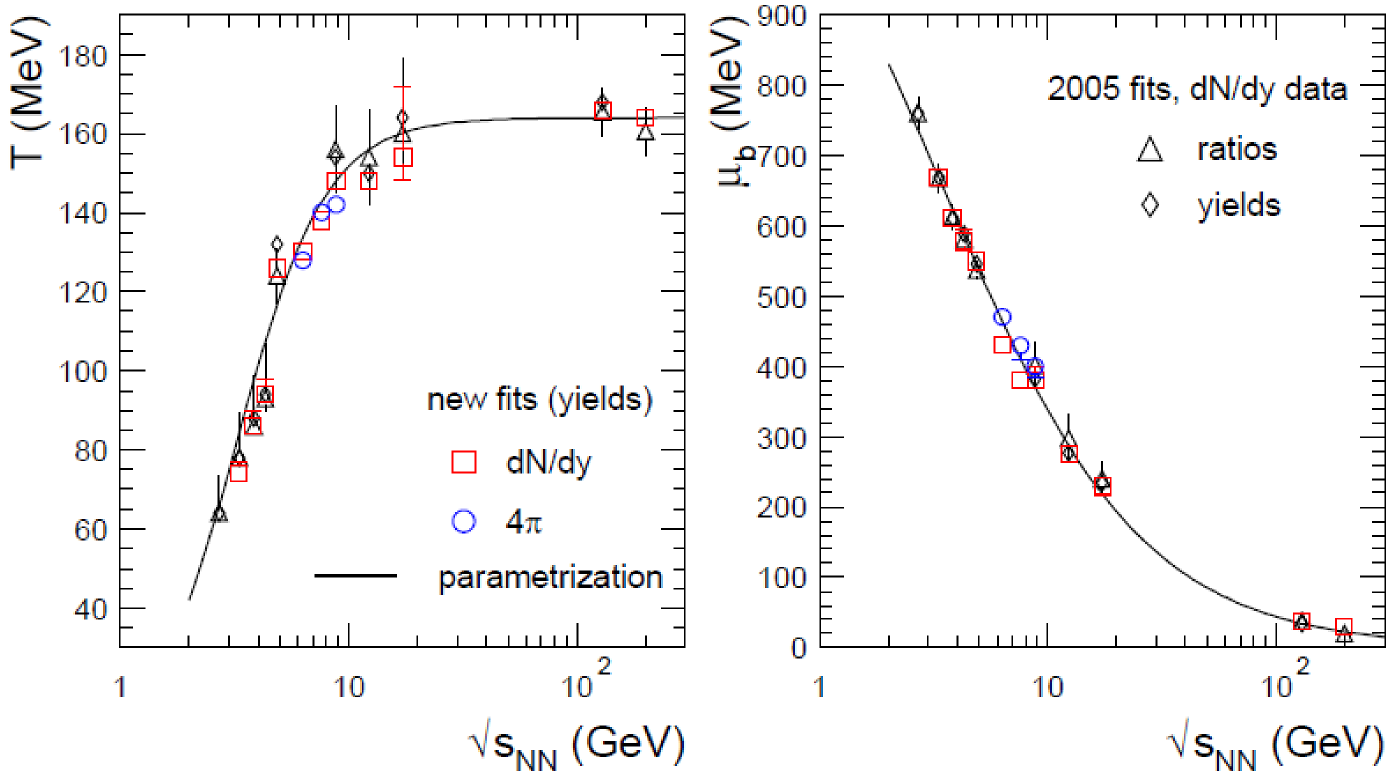


Braun-Munzinger-Magestro-Redlich-Stachel (2001)
Andronic-Braun-Munzinger-Stachel (2006)

$$\text{Different } \sqrt{s_{NN}} \quad \rightarrow \quad \text{Different } T \text{ and } \mu_B$$

Freeze-out T and μ_B

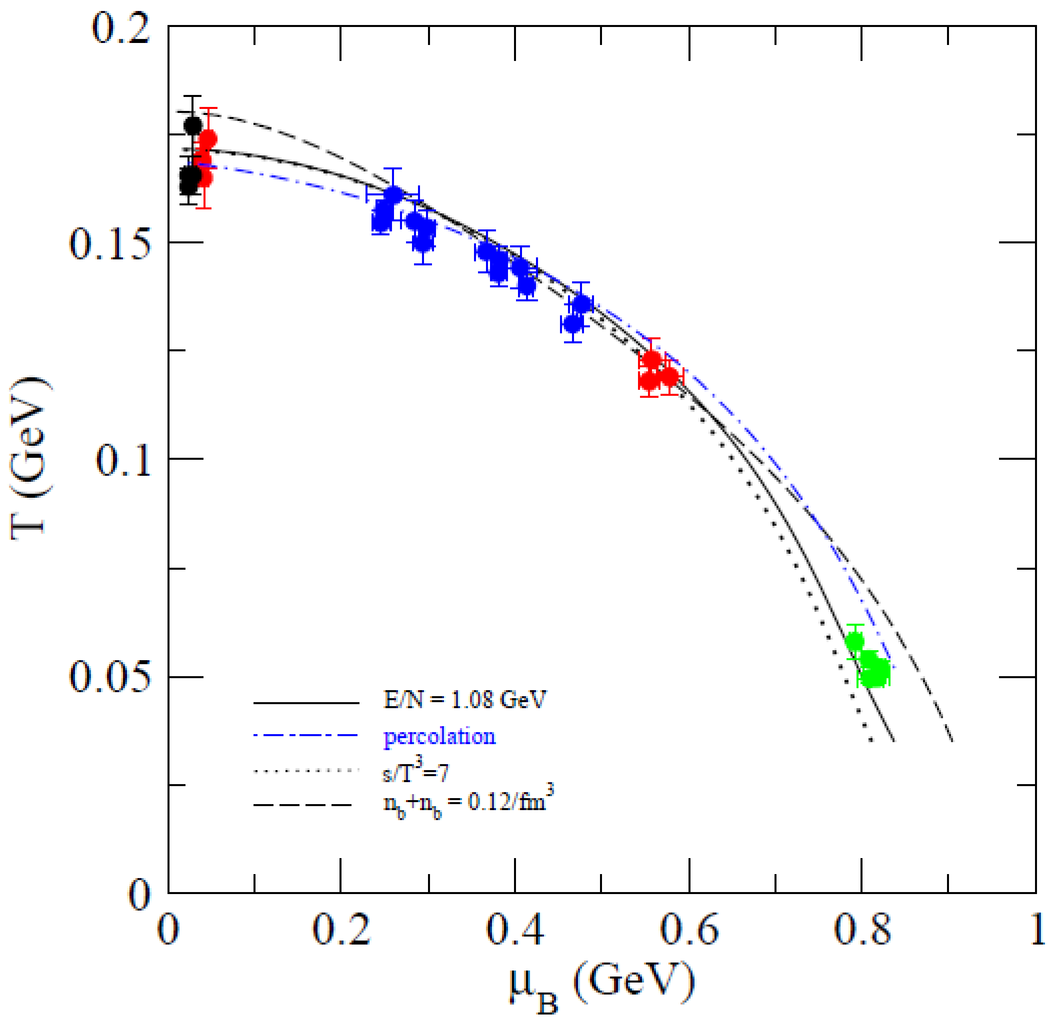
Andronic-Braun-Munzinger-Stachel



$$T[\mathrm{MeV}] = T_{lim}\left(1 - \frac{1}{0.7 + (\exp(\sqrt{s_{NN}}(\mathrm{GeV})) - 2.9)/1.5}\right)$$

$$\mu_b[\mathrm{MeV}] = \frac{a}{1 + b\sqrt{s_{NN}}(\mathrm{GeV})},\qquad \begin{aligned} a &= 1303 \pm 120\ \mathrm{MeV} \\ b &= 0.286 \pm 0.049\ \mathrm{GeV}^{-1} \end{aligned}$$

Freeze-out Criteria



$$\langle E \rangle / \langle N \rangle \simeq 1 \text{ GeV}$$

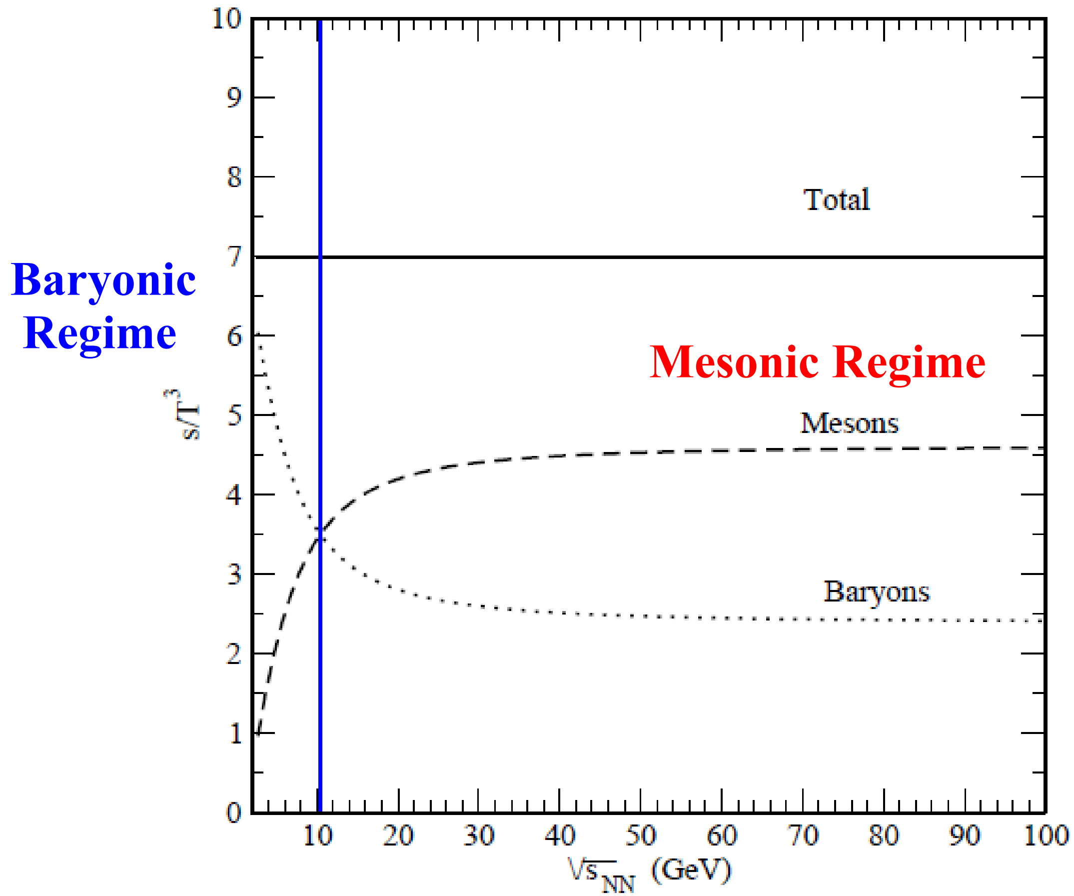
$$\frac{s}{T^3} \simeq 7$$

$$n_B + n_{\bar{B}} \simeq 0.12 \,\text{fm}^{-3}$$

Thermodynamic quantities give some conditions that can fit the freeze-out line in the Statistical Model.

Meson-Baryon Crossover

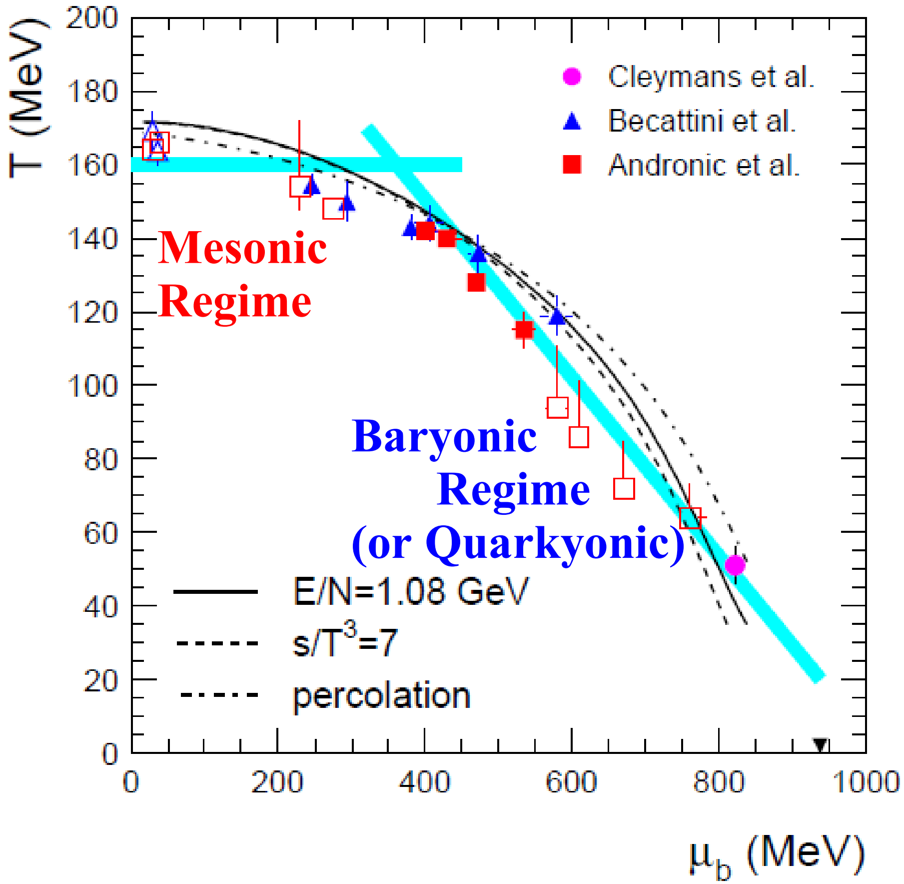
What made BNL people excited



Cleymans-Oeschler-Redlich-Weaton (2005)

Triple-point-like Region

Along the freeze-out curve (phase boundary) there is a transitional change from **meson-dominant** to **baryon-dominant** regimes.



Mesonic Hagedorn Transition

$$Z \sim \int dm\, \rho(m)\, e^{-m/T}$$

$$\rho(m) \sim e^{m/T_H}$$

$$T_c = T_H$$

Baryonic Hagedorn Transition

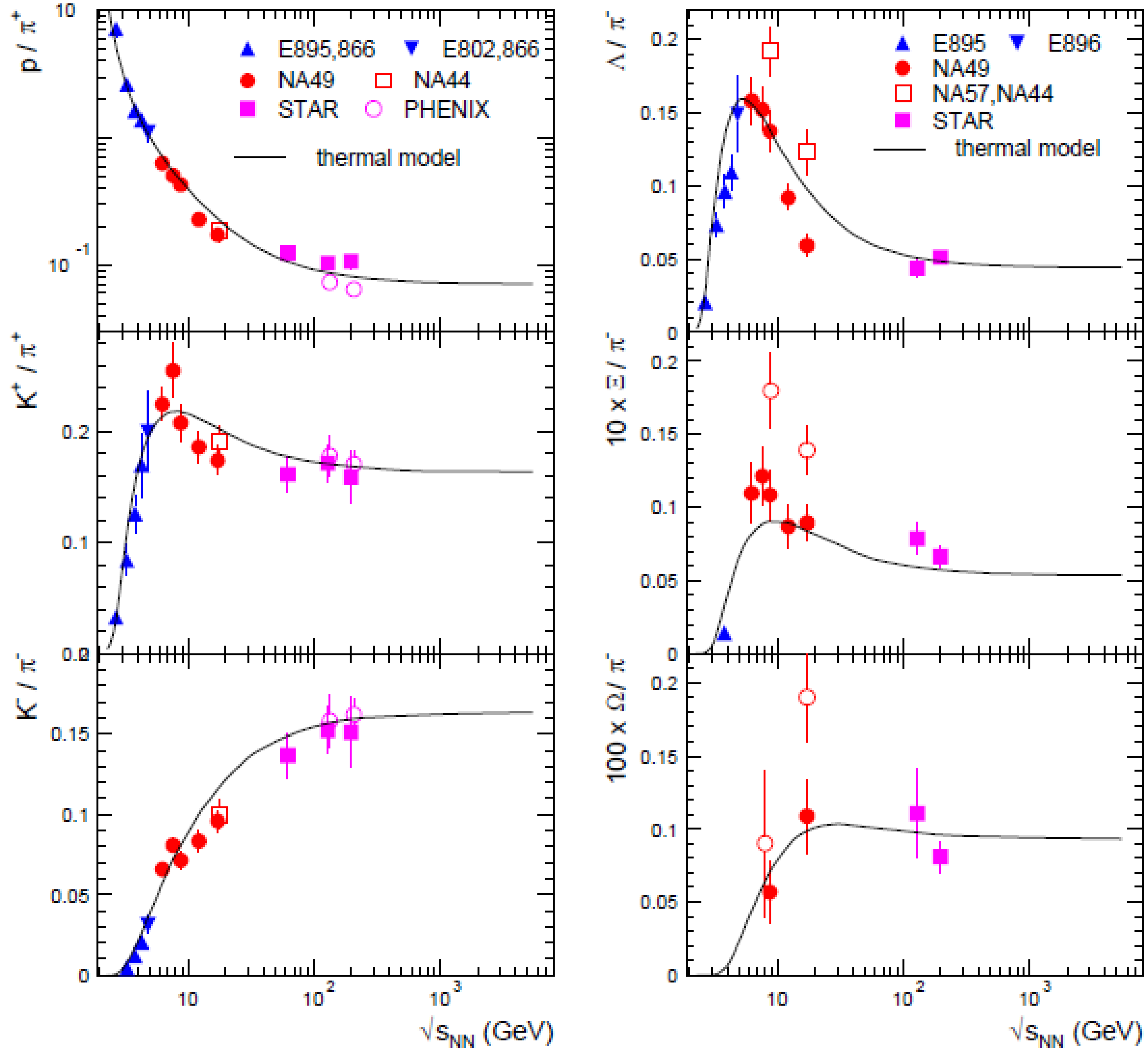
$$Z \sim \int dm\, \rho_B(m)\, e^{-(m_B-\mu_B)/T}$$

$$\rho(m) \sim e^{m_B/T_B}$$

$$T_c = (1-\mu_B/m_B)\, T_B$$

Something at Crossover ?

Marek's Horn



Chiral Model Study

Advantages

- Phase diagram can be drawn by order parameters.
- P-models (*PNJL*, *PQM*) can describe the full thermodynamic properties.

Caveats

- Model parameters have uncertainty.
- Approximations (mean-field) – can be overcome by the (functional) renormalization group improvement.

Pure-gluonic Model $(m_q \gg \Lambda_{QCD})$

Ansatz (Polyakov loop ~ A_0)

$$V(\ell) = -\frac{1}{2}a(T)\ell\bar{\ell} + b(T)\log\left[1-6\ell\bar{\ell}+4(\ell^3+\bar{\ell}^3)-3(\ell\bar{\ell})^2\right]$$

Vandermonde determinant
= SU(3) Haar measure

Parameters

$$a(T) = T^4(3.51-2.47t^{-1}+15.2t^{-2})$$

$$b(T) = -1.75t^{-3}\cdot T^4$$

$$t = T/T_0$$

3 (+1) parameters
Ratti-Thaler-Weise
Ratti-Roessner-Weise

$\ell(T)$
$p(T)$

c.f. strong-coupling ansatz

$$a(T) = T\cdot b\cdot 54e^{-a/T}$$

$$b(T) = T\cdot b$$

2 parameters
Fukushima

Chiral Model $(m_q = m_{phys})$

Nambu–Jona-Lasinio (NJL) Model

$$L = \bar{\psi}(i\gamma\cdot\partial - m_f) + \frac{g_S}{2}[(\bar{\psi}\lambda\psi)^2 + (\bar{\psi}i\gamma_5\lambda\psi)^2] + g_D[\det\bar{\psi}(1-\gamma_5)\psi + \mathrm{h.c.}]$$

Parameters

$\Lambda = 631.4\,\mathrm{MeV}$
$m_{ud} = 5.5\,\mathrm{MeV}$
$m_s = 135.7\,\mathrm{MeV}$
$g_S\Lambda^2 = 3.67$
$g_D\Lambda^5 = -9.29$

← m_π, f_π, m_K, $m_{\eta'}$, one more? M_{ud} or $\langle\bar{\psi}\psi\rangle$

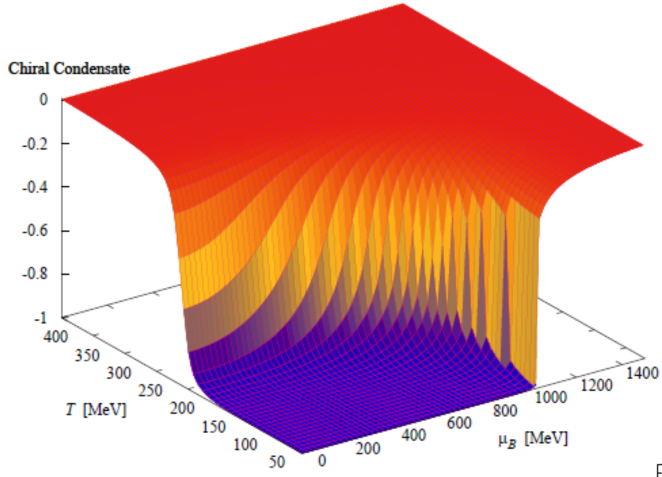
c.f. PQM model Schaefer, Wambach

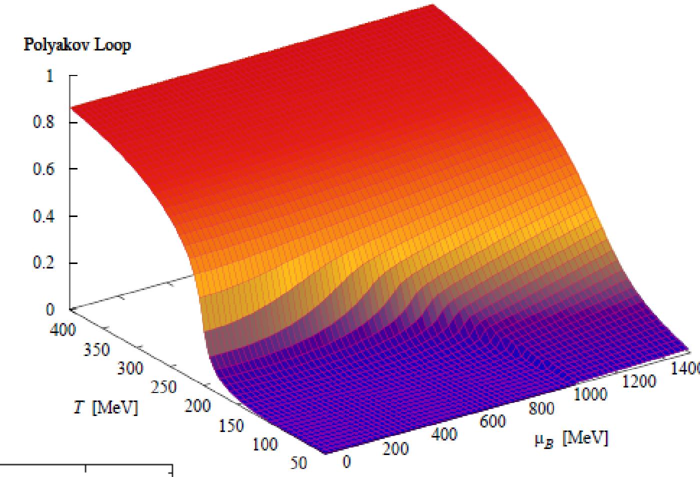
Coupling (covariant derivative for A_0)

$$2N_f\int\frac{d^3k}{(2\pi)^3}\mathrm{tr}\left[\log\left(1+L\,\mathrm{e}^{-(E-\mu)/T}\right)+\log\left(1+L^\dagger\,\mathrm{e}^{-(E+\mu)/T}\right)\right]$$

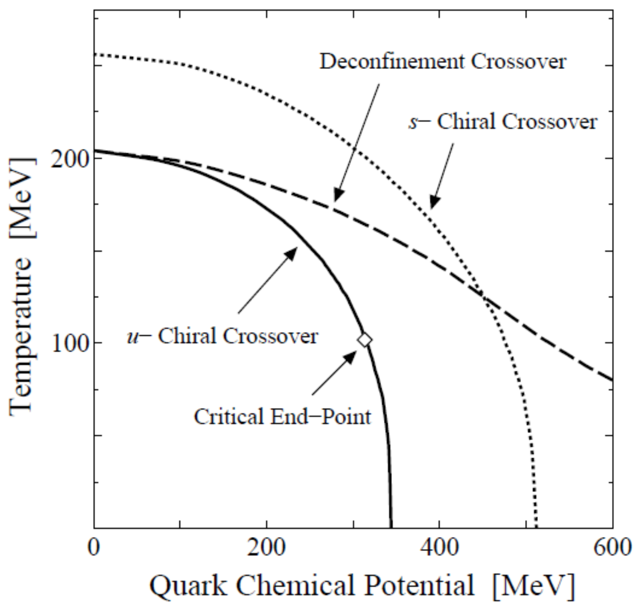
Typical Phase Structure from PNJL

Chiral Condensate and Polyakov Loop





Consistency with the argument in the Statistical Model ?

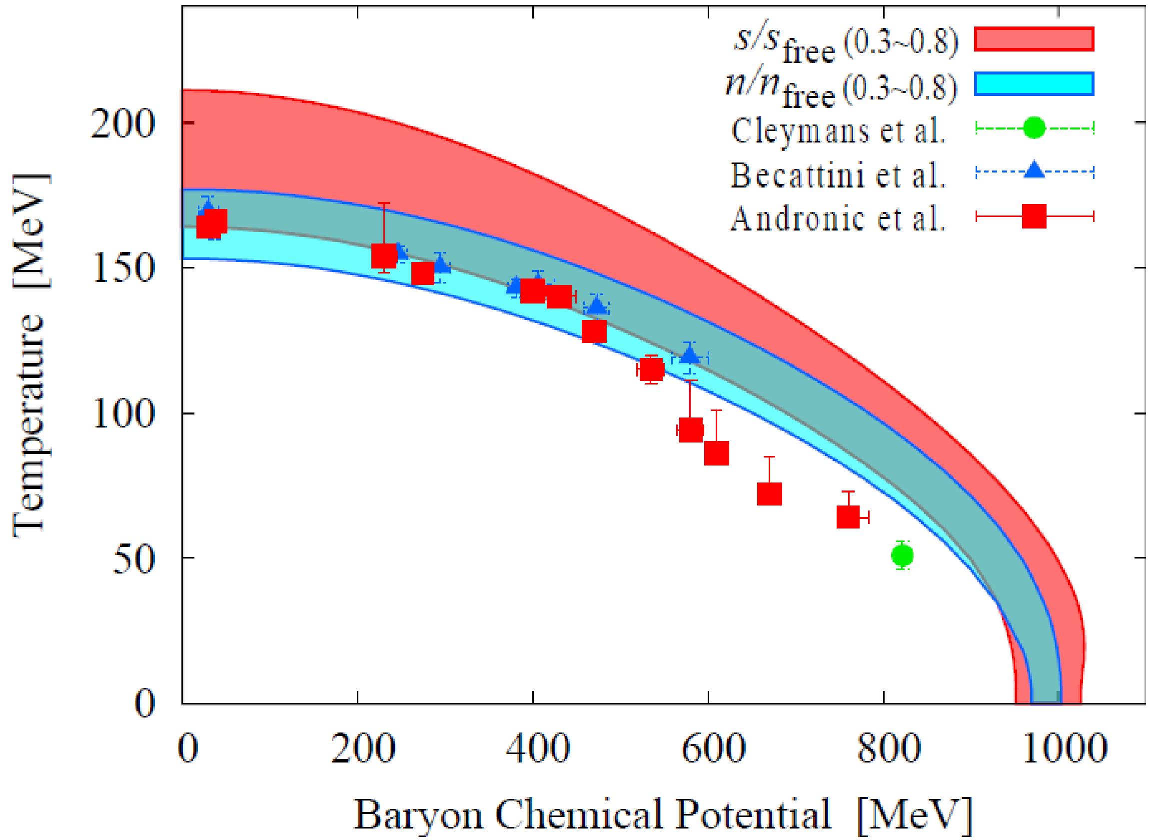


Looks like the large-N_c phase diagram...?

2-flavors – KF (2004)
3-flavors – KF (2008)

Thermodynamics from the SM

Results from THERMUS (Wheaton and Cleymans) KF (2010)



$$s_{\text{free}} = \left((N_c^2 - 1) + \frac{7}{4} N_c N_f \right) \frac{4\pi^2}{45} T^3 + \frac{N_c N_f}{3} \mu^2 T$$

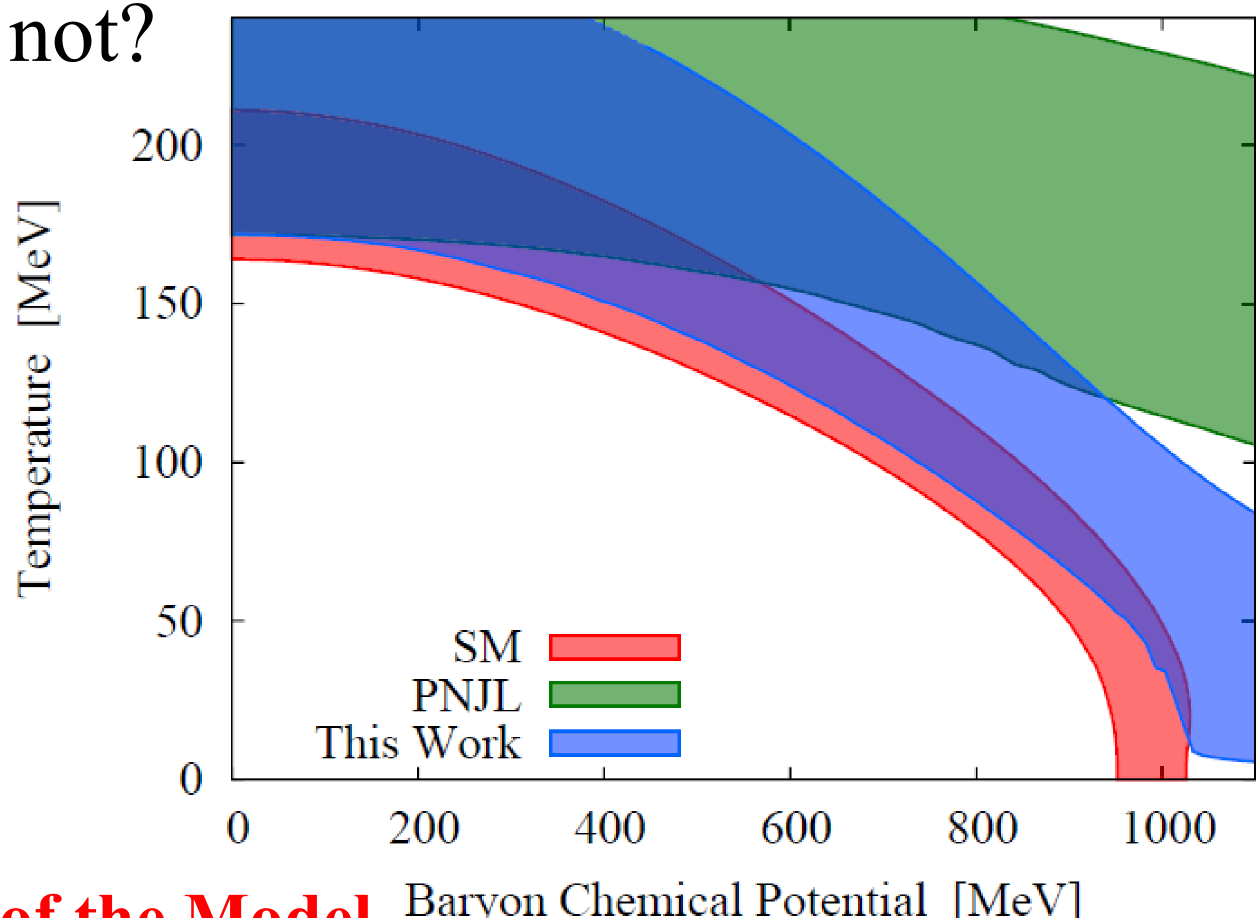
$$n_{\text{free}} = N_f \left(\frac{\mu^3}{3\pi^2} + \frac{\mu T^2}{3} \right)$$

To discuss the phase diagram, the neutrality conditions are intentionally relaxed (w.r.t. strangeness and electric charge), which are minor effects to the entropy.

Blow-up in entropy should be regarded as "*deconfinement*" intuitively.

Entropy Density

Energy density, baryon density, etc may exceed the free quark-gluon (SB) limit, but the entropy density may not?



Failure of the Model

$T_o(\mu_B)$ – *Simple Ansatz*

Parametrization for Chemical Freezeout Curve

$$T_f(\mu_B) = a - b\mu_B^2 - c\mu_B^4$$

$$a = 166(2)\,\mathrm{MeV} \quad b = 1.39(16)\times 10^{-4}\,\mathrm{MeV}^{-1} \quad c = 5.3(21)\times 10^{-11}\,\mathrm{MeV}^{-3}$$

Cleymans-Oeschler-Redlich-Wheaton (2006)

Assumption for $T_0(\mu)$ (Polyakov-loop behavior)

$$\frac{T_0(\mu_B)}{T_0(0)} = 1 - (bT_0)\left(\frac{\mu_B}{T_0}\right)^2 = 1 - 2.78\times 10^{-2}\left(\frac{\mu_B}{T_0}\right)^2$$

c.f Perturbative Matching

$$T_0(\mu_B) = T_\tau e^{-1/(\alpha_0 b(\mu))} \qquad \frac{T_0(\mu_B)}{T_0(0)} \simeq 1 - 2.1\times 10^{-2}\left(\frac{\mu_B}{T_0}\right)^2$$

Schaefer-Pawlowski-Wambach

Lattice QCD

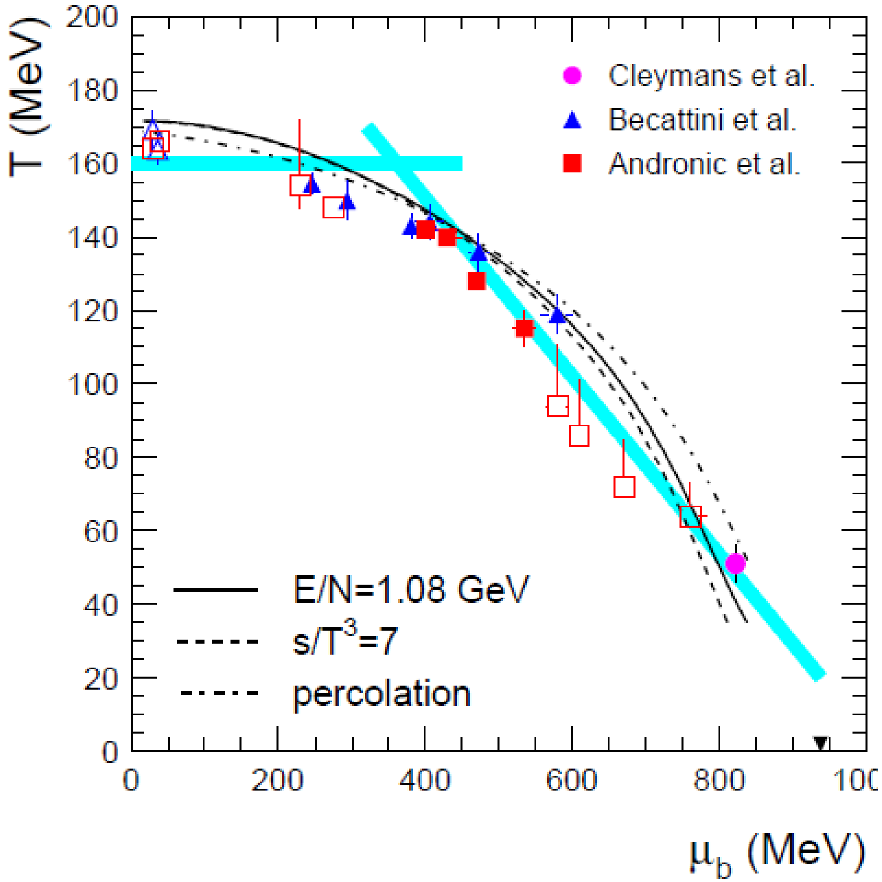
$$\frac{T_c(\mu_B)}{T_c(0)} \simeq 1 - 6.8\times 10^{-3}\left(\frac{\mu_B}{T_0}\right)^2$$

Curvature ~ 1/3 **de Forcrand**

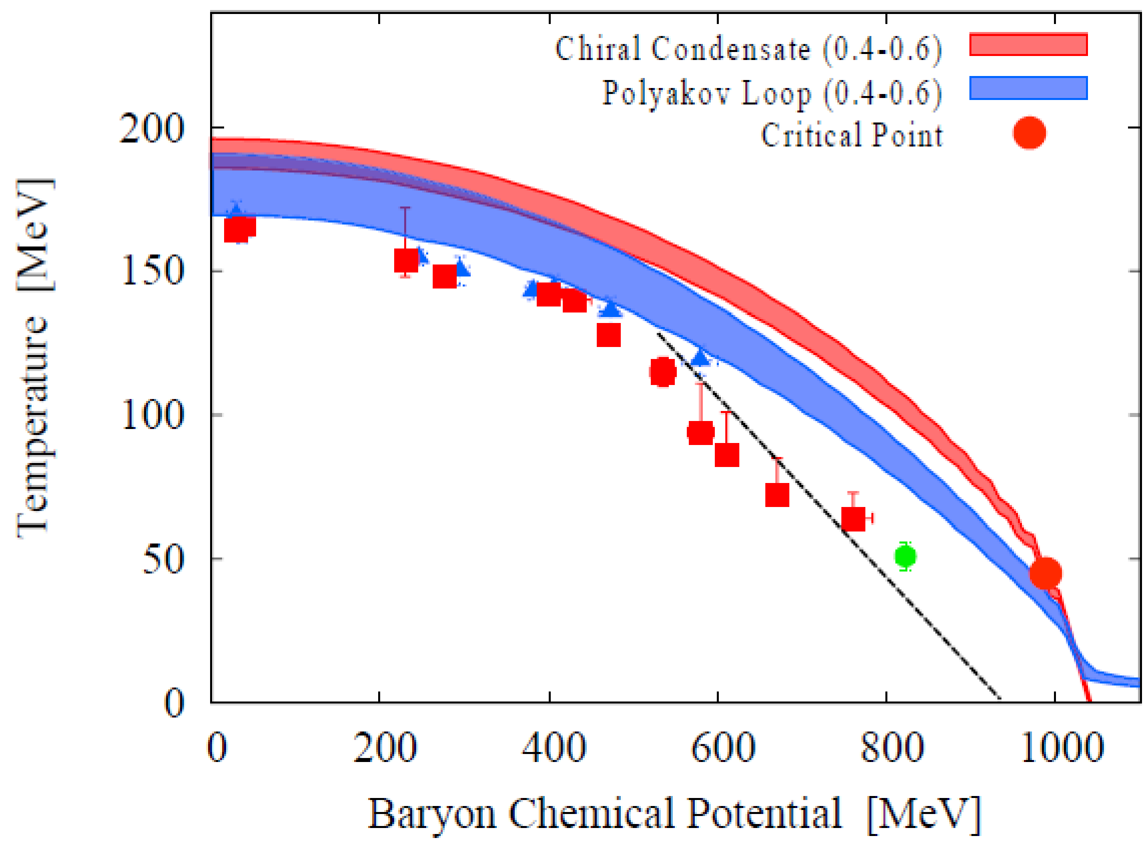
Phase Boundaries

Matching in Thermodynamics

Chemical Freeze-out



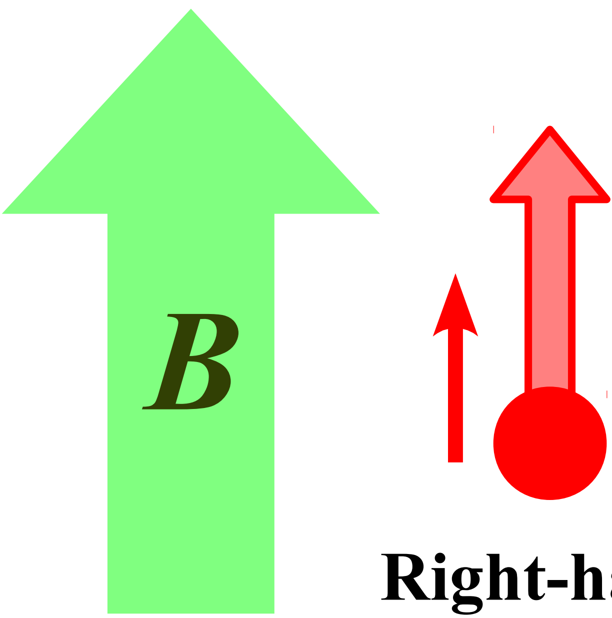
Phase boundaries from a model that is consistent with SM



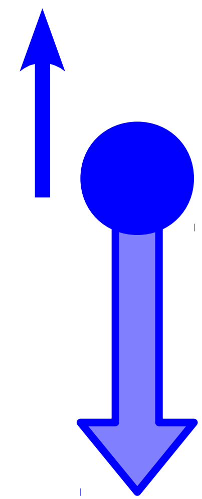
Fukushima (2010)

Under a Strong Magnetic Field

Classical Picture



Left-handed Quark
= momentum ⇨
anti-parallel to
spin →



Right-handed Quark
= momentum ⇨
parallel to
spin →

Finite θ Effect
($\mathcal{P}$ breaking)

$$J \neq 0 \quad \text{if} \quad N_5 = N_R - N_L \neq 0$$

Kharzeev-McLerran-Warringa (2007)

$\mathcal{P}$ and $C\mathcal{P}$ Violation in the YM Theory

Gauge Actions

$\mathcal{P}$- and $C\mathcal{P}$- even ($\mathcal{T}$-even) terms

$$F_{\mu\nu}F^{\mu\nu} = 2F_{0i}F^{0i} + F_{ij}F^{ij}$$

Even w.r.t. spatial and temporal indices

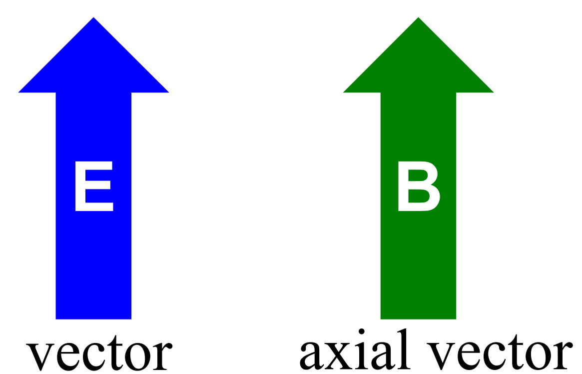
$\mathcal{P}$- and $C\mathcal{P}$- odd ($\mathcal{T}$-odd) terms

$$F_{\mu\nu}\widetilde{F}^{\mu\nu} = 2F_{01}F^{23} + 2F_{02}F^{31} + 2F_{03}F^{12}$$

Odd w.r.t. spatial and temporal indices

Parallel $\boldsymbol{E}$ and $\boldsymbol{B}$

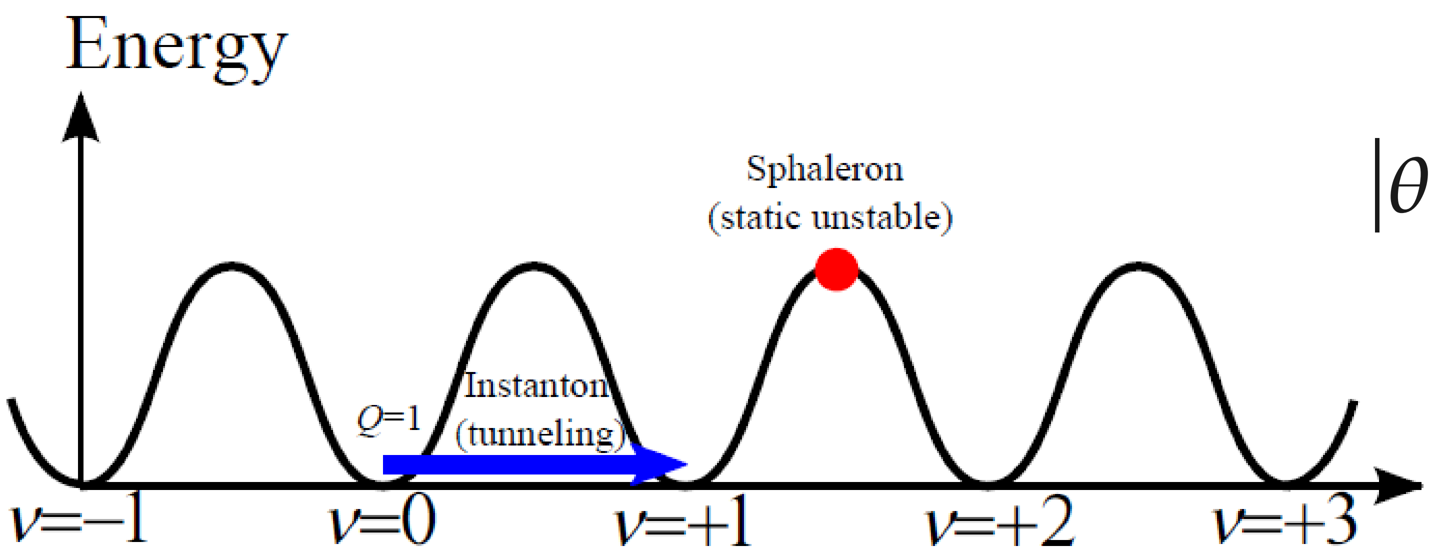
$$F_{\mu\nu}\widetilde{F}^{\mu\nu} = 2\boldsymbol{E}\cdot\boldsymbol{B}$$



θ -Vacuum and Strong CP Problem

Topological Structure and θ -Vacuum

Manton
Faddeev
Jackiw-Rebbi



$$|\theta\rangle = \sum_{\nu} e^{i\theta\nu}|\nu\rangle$$

(Bloch state)

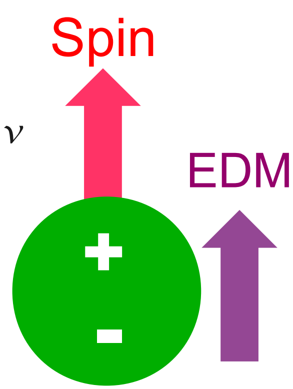
Strong CP Problem

$$\langle\theta|\theta\rangle \rightarrow \quad S_{\mathrm{QCD}} = -\frac{1}{2g^2}\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + \theta\frac{1}{16\pi^2}\mathrm{tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$|d_n| \sim (e\, m_q / m_N^2)\theta$$

$$|\theta| < 0.7\times 10^{-11}$$

No CP breaking (Why?)



Finite-θ Hadronic World

θ can be eliminated by $U(1)_A$ rotation

One solution to the strong CP problem is the presence of massless quarks (almost excluded...)

Effect of strong θ-angle to hadron physics

$$U \rightarrow e^{i\theta} U$$

Scalar meson

$$\sigma \sim \bar{\psi}\psi \rightarrow \bar{\psi} e^{i\gamma_5\theta}\psi \sim \sigma\cos(\theta) + \eta_0 \sin(\theta)$$

Pseudo-scalar meson

$$\eta_0 \sim \bar{\psi} i\gamma_5 \psi \rightarrow \bar{\psi} e^{i\gamma_5\theta} i\gamma_5\psi \sim \eta_0\cos(\theta) - \sigma\sin(\theta)$$

η_0 may condense in addition to the chiral σ condensate

Possibility for Finite η_0 Condensate

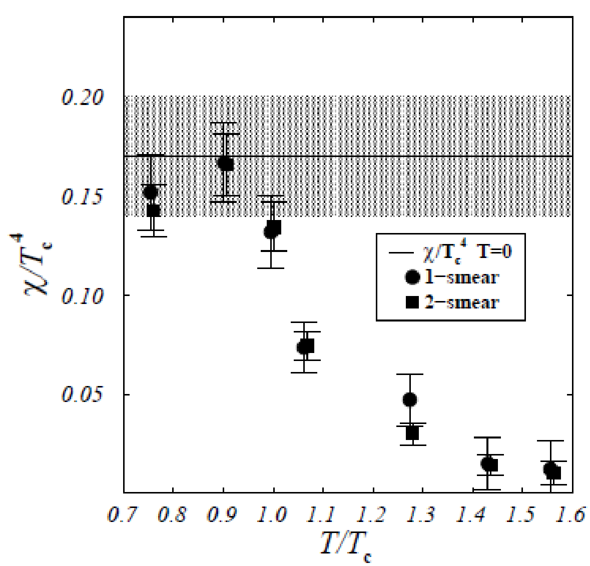
If $U(1)_A$ symmetry is NOT broken

σ and η are degenerate (η may have a chance as much as the σ condensate develops)

$U(1)_A$ is broken but can be "effectively" restored

$U(1)_A$-breaking effective interaction is induced by the topological susceptibility

Veneziano-Di Vecchia



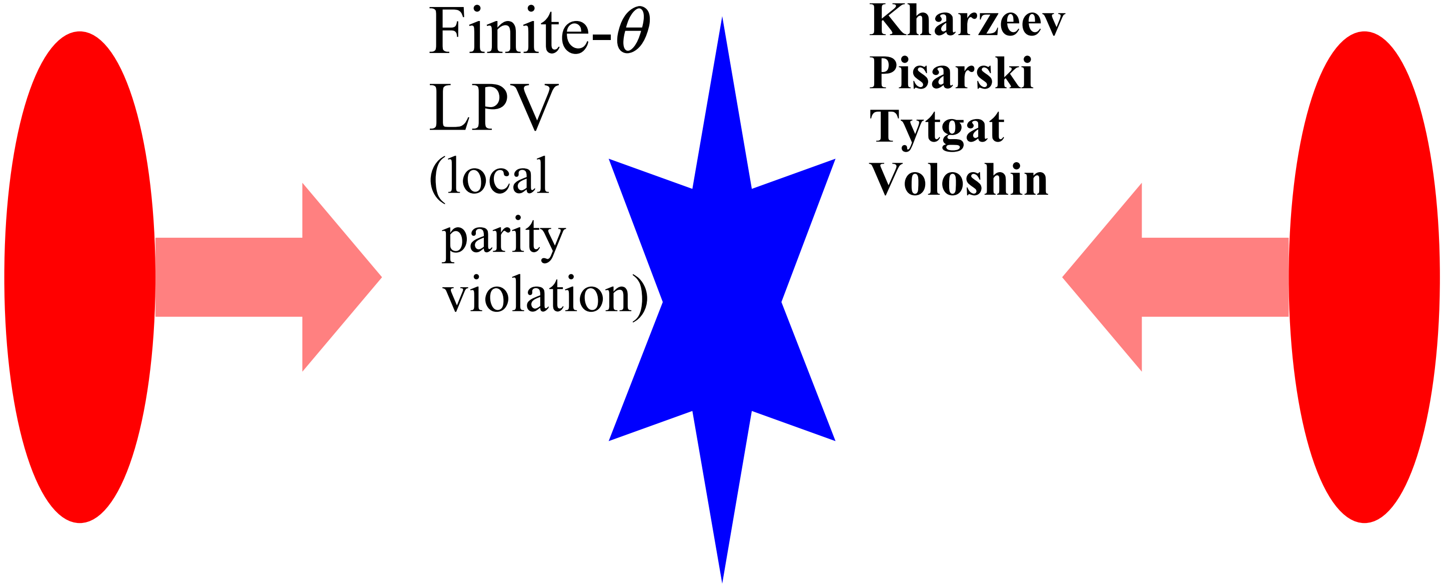
Lattice Simulation
Alles et al (1996)

Susceptibility drops off at high temperature $T \sim T_c$

$$n(T) = \left(\frac{8\pi^2}{g^2} \rho^{-5} e^{-8\pi^2/g^2(\rho)} \right) \exp\left[-\pi^2 \rho^2 T^2 \left(\frac{2N_c + N_f}{3} \right) \right]$$

Gross-Pisarski-Yaffe

Relativistic Heavy-Ion Collisions



Heavy-Ion (nucleus)

Au, Pb, Cu, ...

$\sqrt{s_{NN}} = 200\,\text{GeV}, 62\,\text{GeV}, \ldots$

Quark-Gluon Plasma

Baym
Shuryak

Direct photon measurement
(not from π^0, η' etc)
$\rightarrow$ Initial $T \sim 4\times10^{12}\,\text{K} \sim \text{GeV}$

c.f. $T_c \sim \Lambda_{\text{QCD}}$ $\rho \sim 0.3\,\text{fm}$

Topological Excitations in high-T QCD

Instantons in Euclidean (θ-vacuum)

Instantons are exponentially suppressed at high T.

$$n(T) = \left(\frac{8\pi^2}{g^2} \rho^{-5} e^{-8\pi^2/g^2(\rho)} \right) \exp\left[-\pi^2 \rho^2 T^2 \left(\frac{2N_c + N_f}{3} \right) \right]$$

Sphalerons in Minkowskian

Sphalerons are parametrically enhanced at high T.

$$\Gamma \sim \alpha_s^5 T^4$$

QCD sphalerons are abundant in hot and dense matter created in the relativistic heavy-ion collisions

Arnold-McLerran (1987)

Topological Diffusion Rate

$$\Gamma = \frac{1}{2} \lim_{t\to\infty} \lim_{V\to\infty} \int d^4x \langle q(x)q(0) + q(0)q(x) \rangle$$

$$\langle Q^2 \rangle = 2\Gamma V t$$

Random Walk at Finite T

In the strong-coupling AdS/CFT by Son and Starinets (hep-th/0205051)

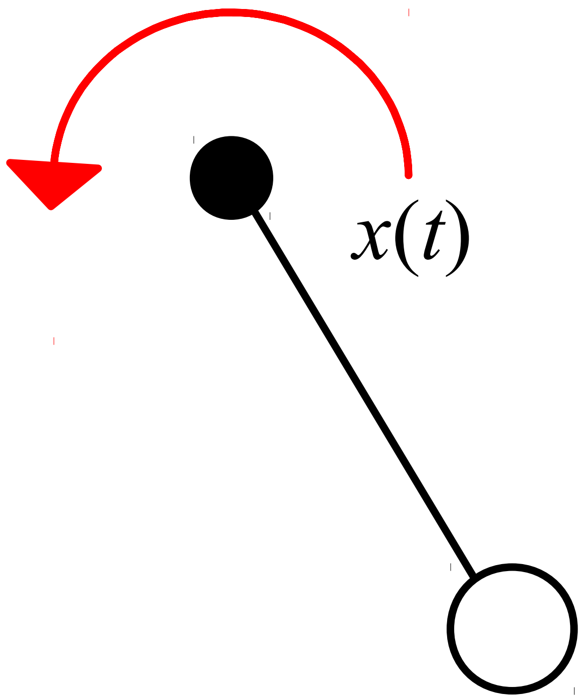
$$\Gamma = \frac{(g_{\mathrm{YM}}^2 N)^2}{256\pi^3} T^4$$

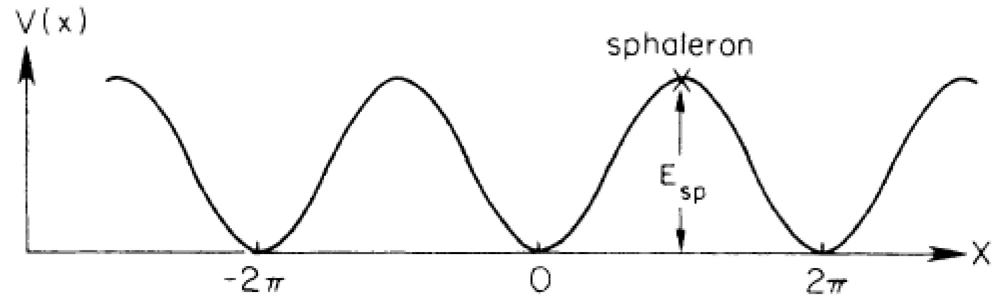
In the weak-coupling perturbation by Arnold, Son, Yaffe, Bodeker, Moore, etc

$$\Gamma = \mathrm{const}\cdot (g_{\mathrm{YM}}^2 N)^5 \ln \frac{1}{g_{\mathrm{YM}}^2 N} T^4$$

Sphaleron in Pendulum

Simplest Example



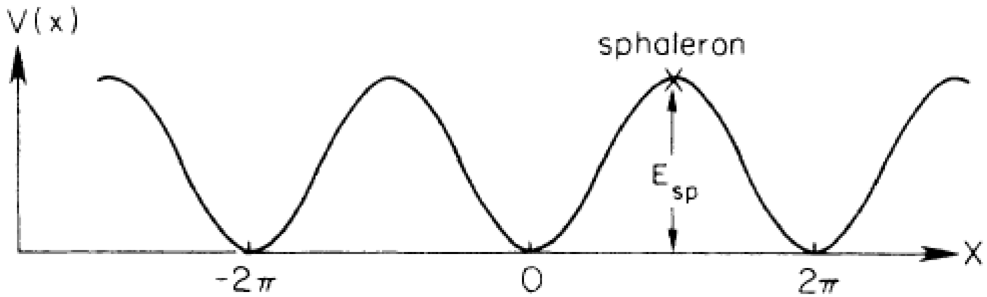


Chern-Simons number $x(t)/2\pi$

Topological charge $Q = \frac{1}{2\pi}\int_0^{\beta} d\tau\, \dot{x}$

Topological Rate in Real- and Imaginary-Time

Pendulum **(Arnold-McLerran)**



Chern-Simons number $x(t)/2\pi$

Topological charge $Q = \frac{1}{2\pi}\int_0^\beta d\tau\, \dot{x}$

Finite-T Euclidean Action $S_E = \int_0^\beta d\tau \left(\frac{1}{2}\dot{x}^2 + i\frac{\theta}{2\pi}\dot{x} \right)$

Topological Susceptibility (Diffusion Rate) $A(t) = \left\langle T\left(\frac{x(t) - x(0)}{2\pi} \right)^2 \right\rangle$

Real-time (classical approx.) $A(t) \simeq \frac{t^2}{4\pi^2}\bar{v}^2 = \frac{t^2}{4\pi^2\beta}$

Imaginary-time $A(-i\beta) = \langle Q^2 \rangle \simeq 2\exp\left(-2\pi^2/\beta\right)\cos\theta$

Analytical Continuation

Diffusion Rate at High T

$$A(t)=\left\langle T\left(\frac{\delta x(t)-\delta x(0)}{2\pi}\right)^2\right\rangle+O(e^{-2\pi^2/\beta})$$

$$=\frac{1}{4\pi^2 m}\left(\frac{\exp(-i m|t|)-1}{\exp(-\beta m)-1}-\frac{\exp(i m|t|)-1}{\exp(\beta m)-1}\right)+O(e^{-2\pi^2/\beta})$$

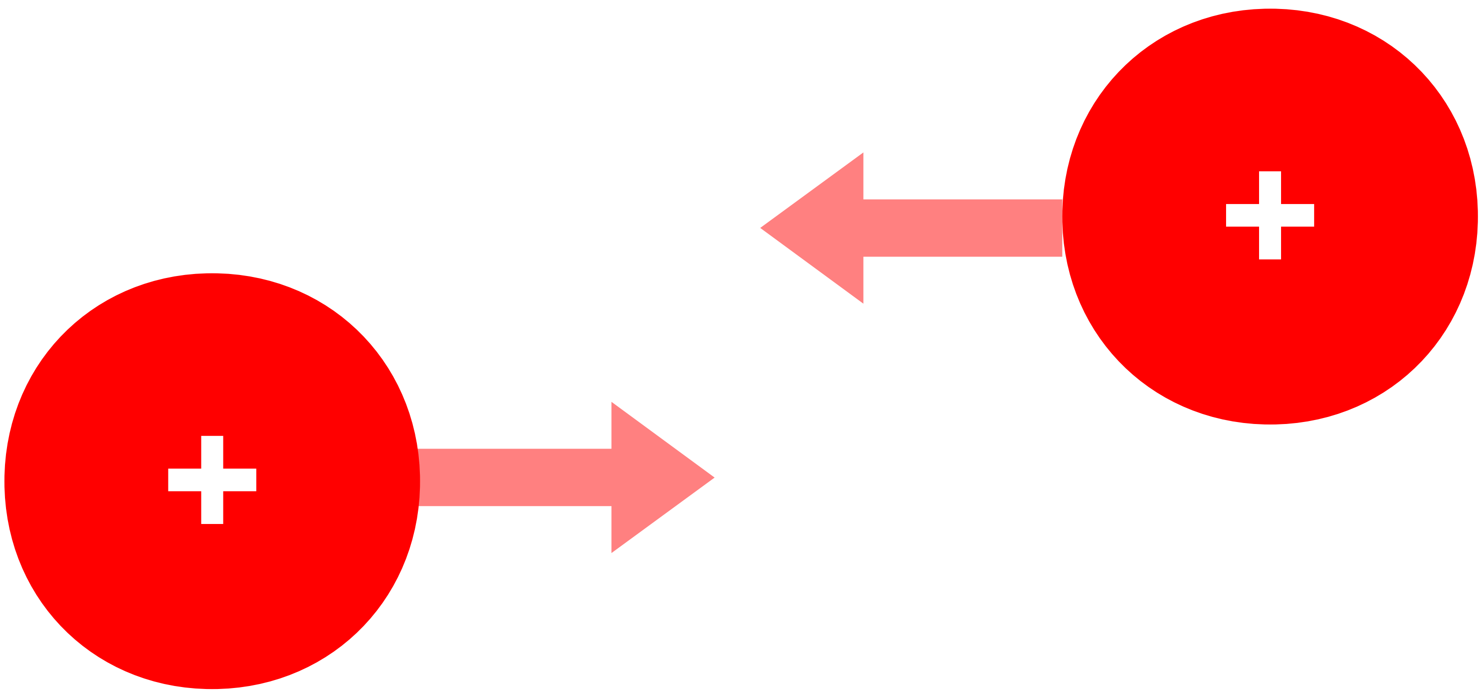
$$\rightarrow\frac{1}{4\pi^2}\left(\frac{t^2}{\beta}+i t\right)+O(e^{-2\pi^2/\beta})$$

$$A(t=-i\beta)\rightarrow O(e^{-2\pi^2/\beta})$$

Instantons (Euclidean windings) are suppressed at high T but communications in real time are not and dominated by the contribution from the zero-winding sector.

Non-Central Collision

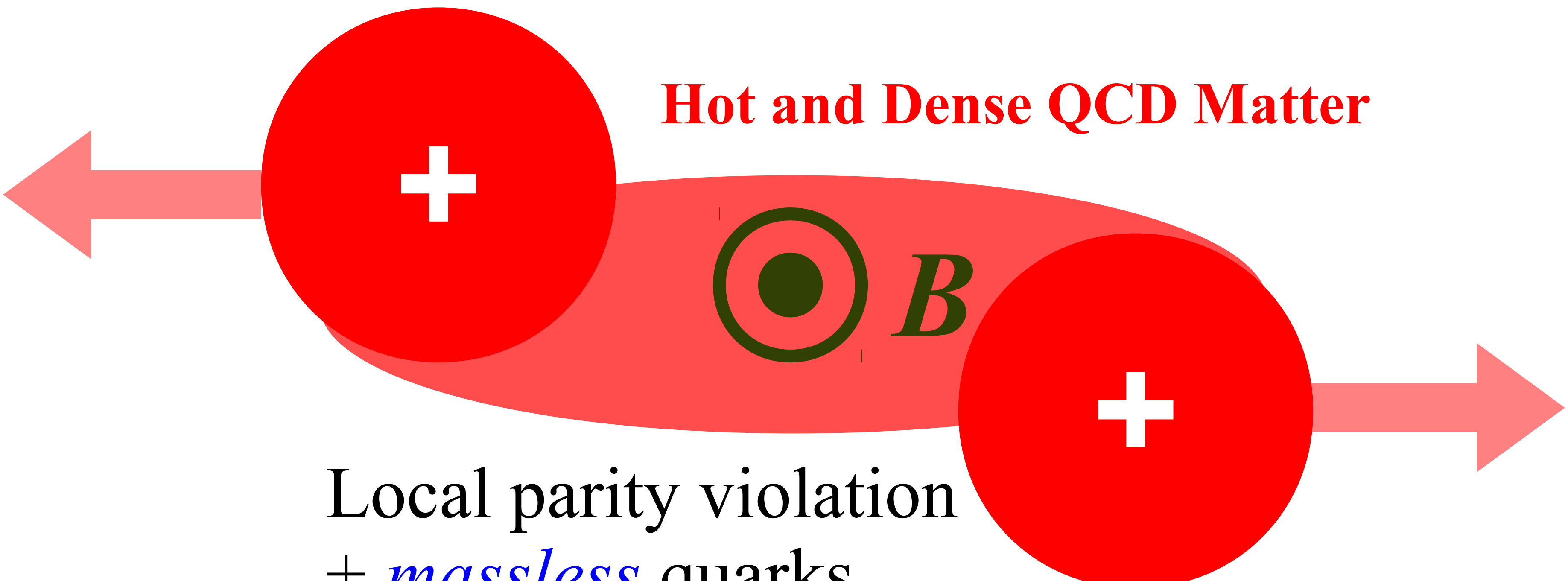
Before Collision (seen from above)



Centrality is determined by N_{part}

Non-Central Collision

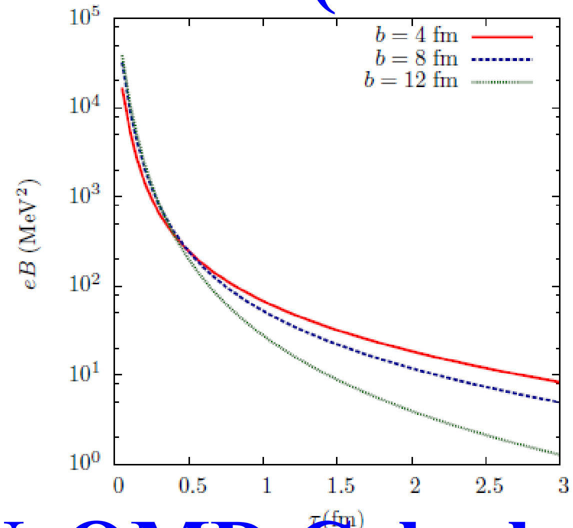
After Collision

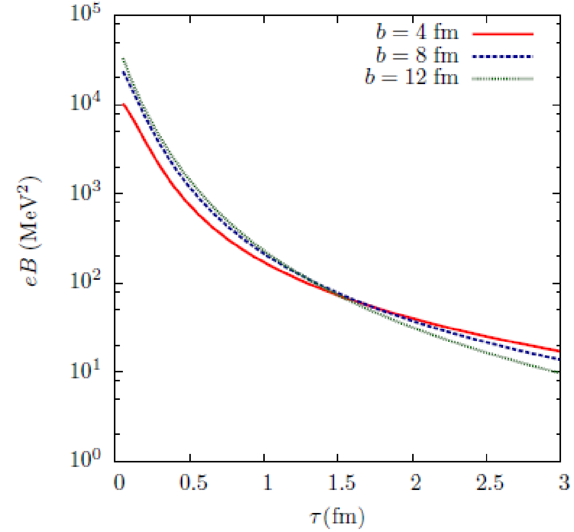


Local parity violation
\+ *massless* quarks

Estimated Magnetic Fields

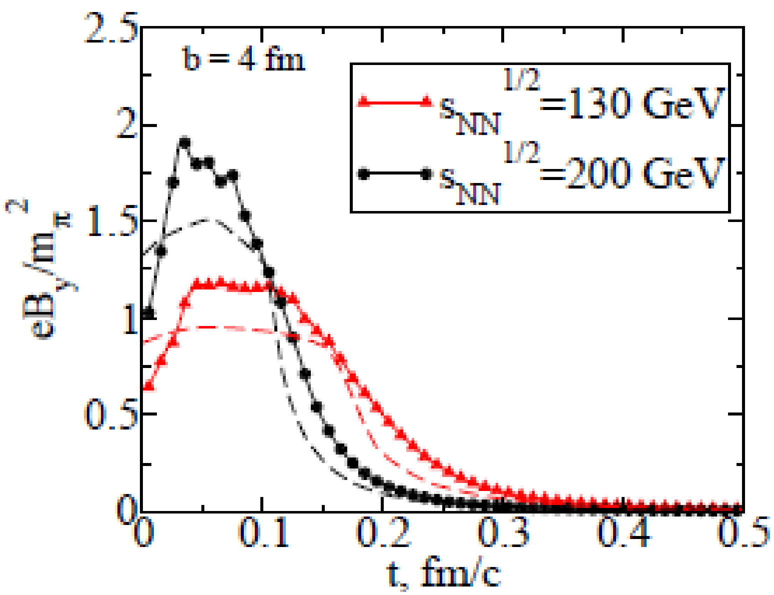
Classical (Pancake) Calcs (Kharzeev-McLerran-Warringa)

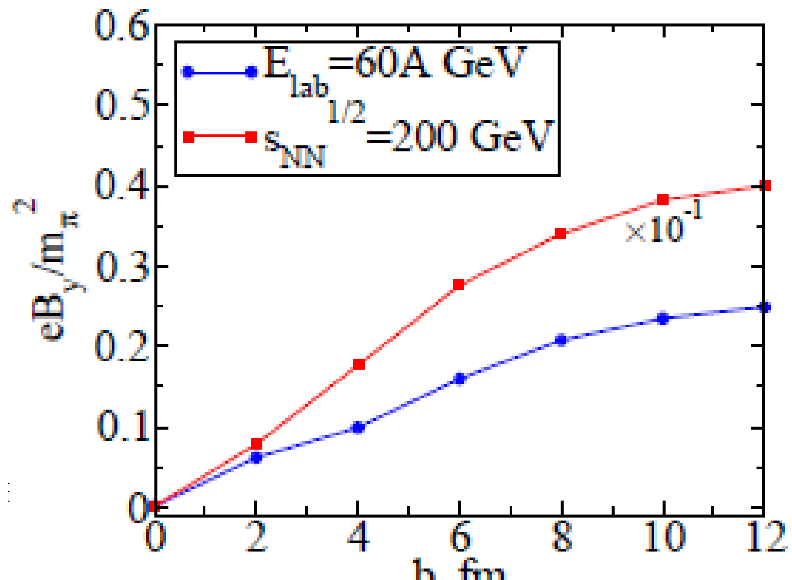




$$eB = 1\,[\,\mathrm{MeV}^2\,]$$
$$\rightarrow B \simeq 1.7 \times 10^{14}\,[\,\mathrm{Gauss}\,]$$

UrQMD Calculations (Skokov-Illarionev-Toneev)

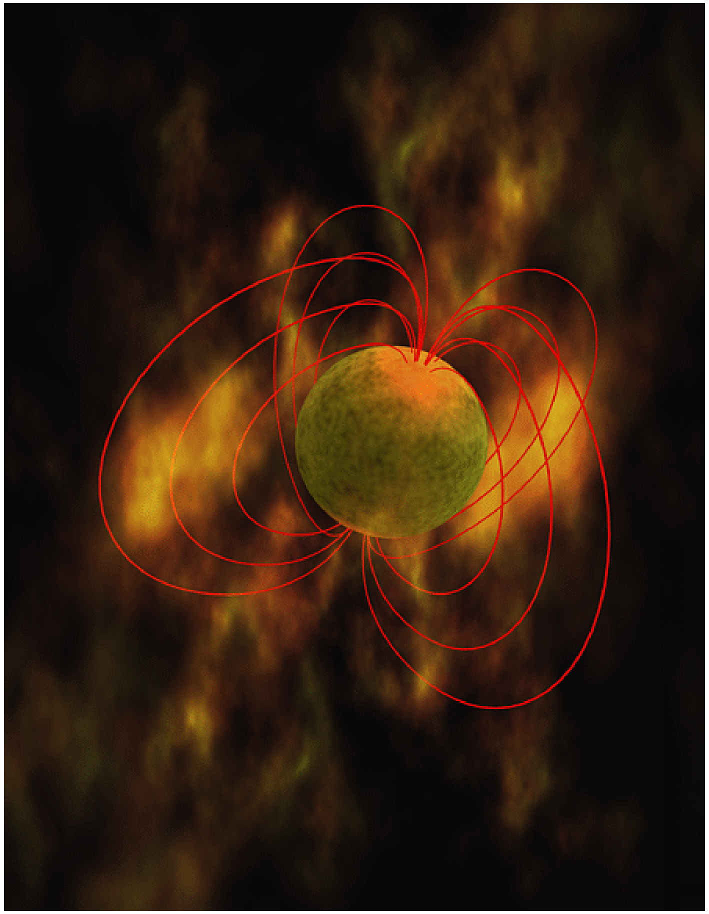




How Big?

$$eB \sim m_\pi^2 \;\rightarrow\; 10^{18}\ \text{Gauss}$$

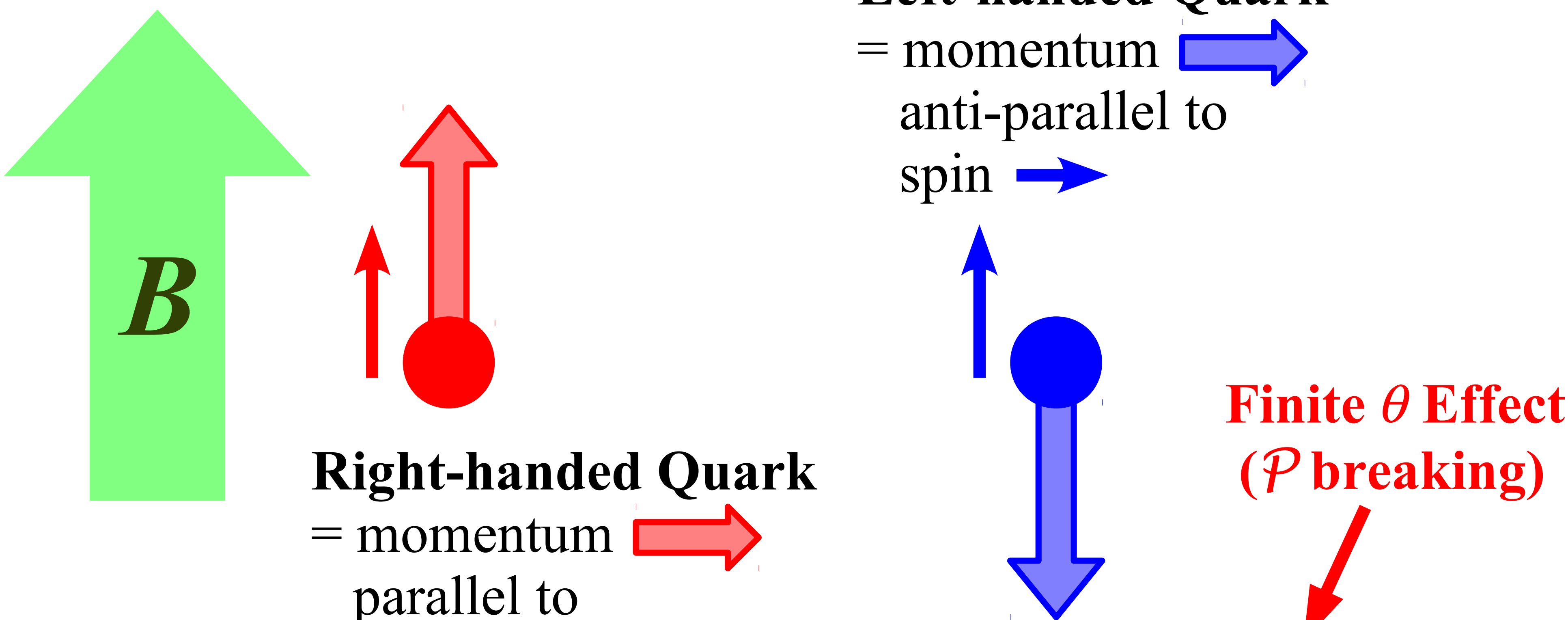
$$10^3 \sim 10^6 \times$$



Neutron Star
(Magnetar)

Chiral Magnetic Effect

Classical Picture



$$\boldsymbol{J} \neq 0 \quad \text{if} \quad N_5 = N_R - N_L \neq 0$$

Kharzeev-McLerran-Warringa (2007)

Anomaly Relations

Induced N_5 by topological effects

$$\frac{dN_5}{dt} = -\frac{g^2 N_f}{8\pi^2}\int d^3x\ \mathrm{tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

QCD Anomaly Relation

Grand canonical ensemble with μ_5 to describe induced N_5

Induced J by the presence of N_5 and B

$$\boldsymbol{j} = \frac{e^2\mu_5}{2\pi^2}\boldsymbol{B}$$

QED Anomaly Relation

$$\left(\boldsymbol{j} = \sum_{i=\mathrm{flavor}} \frac{q_i^2\mu_5}{2\pi^2}\boldsymbol{B} \quad \text{in QCD}\right)$$

Metlitski-Zhitnitsky (2005)
Fukushima-Kharzeev-Warringa (2008)

Derivation (naïve calculation)

Thermodynamic Potential *(UV divergent)*

$$\Omega = -V N_c \sum_f \frac{|q_f B|}{2\pi} \sum_{s=\pm} \sum_{n=0}^{\infty} \alpha_{n,s}^f \int \frac{dp_3}{2\pi} \left[\omega_{n,s} + 2T \ln\left(1 + e^{-\beta\omega_{n,s}}\right) \right]$$

$$\omega_{n,s}^2 = \left(\sqrt{p_3^2 + 2|q_f B| n} + \mathrm{sgn}(p_3) s \mu_5 \right)^2 + m^2 \qquad \alpha_{n,s} = \begin{cases} 1 & n > 0, \\ \delta_{s+} & n = 0, \quad eB > 0 \\ \delta_{s-} & n = 0, \quad eB < 0 \end{cases}$$

Current *(UV finite)*

$$j_3 = \left. \frac{\partial \Omega}{\partial A_3} \right|_{A_3 = 0} \longleftarrow \quad \frac{\partial}{\partial A_3} = e \frac{\mathrm{d}}{\mathrm{d}p_3}$$

Only surface terms!

$$j_3 = e \frac{|eB|}{4\pi^2} \sum_{s,n} \alpha_{n,s} \left[\omega_{n,s}(p_3 = \Lambda) - \omega_{n,s}(p_3 = -\Lambda) \right]$$

$$= e \frac{|eB|}{2\pi^2} \sum_{s,n} \alpha_{n,s} s \mu_5 = \frac{e^2 B \mu_5}{2\pi^2}$$

Derivation (energy conservation)

Energy Conservation (Nielsen-Ninomiya 1983)

Electric field E → Energy shift (Fermi energy)

$$p_F^R = eEt \qquad p_F^L = -p_F^R$$

Landau Levels

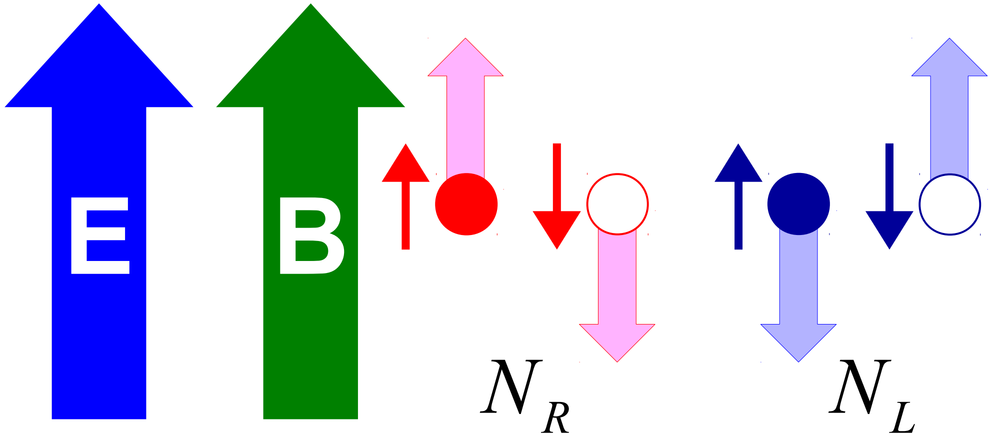
Density of states $\mathrm{d}n/\mathrm{d}z = p_F^R/2\pi \quad \mathrm{d}^2 n/\mathrm{d}x\,\mathrm{d}y = eB/2\pi$

$$\frac{p_F^R}{2\pi}\cdot\frac{eB}{2\pi} = \frac{e^2}{4\pi^2}\,\boldsymbol{E}\cdot\boldsymbol{B}\,t \;\;\Rightarrow\;\; \frac{\mathrm{d}^4 N_5}{\mathrm{d}t\,\mathrm{d}^3x} = \frac{e^2}{2\pi^2}\,\boldsymbol{E}\cdot\boldsymbol{B}$$

Energy cost

$$\int \mathrm{d}^3x\,\boldsymbol{j}\cdot\boldsymbol{E} = \mu_5\frac{\mathrm{d}N_5}{\mathrm{d}t}$$

$$= \frac{e^2\mu_5}{2\pi^2}\int \mathrm{d}^3x\,\boldsymbol{E}\cdot\boldsymbol{B}$$

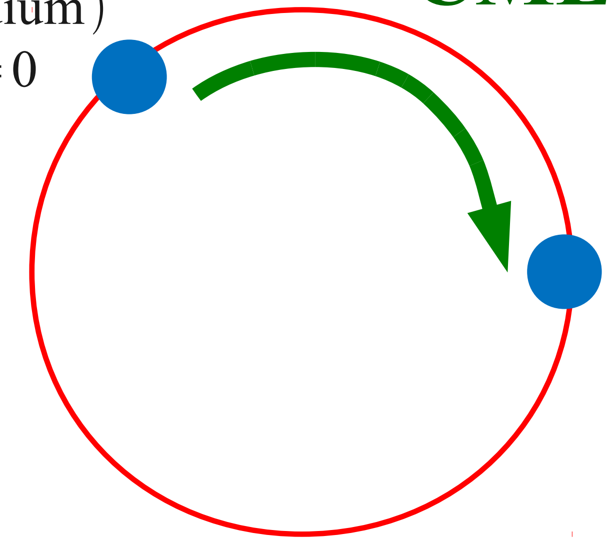


Derivative → Chiral Chemical Potential

Schematic picture

$\mathcal{P}$-breaking $\theta \neq 0$ (in-medium)
$\langle \sigma \rangle \neq 0 \quad \langle \eta_0 \rangle \neq 0$

CME



Kharzeev, Pisarski, Tytgat, Krasnitz, Venugopalan, Voloshin, ...

$\theta = 0$ (vacuum)
$\langle \sigma \rangle \neq 0 \quad \langle \eta_0 \rangle = 0$

Our Vacuum

Space-time dependent θ-angle

$$\gamma\cdot\partial\psi \rightarrow \gamma\cdot\partial\left(e^{i\gamma_5\theta/2N_f}\psi\right) = e^{i\gamma_5\theta/2N_f}\left(\gamma\cdot\partial + i\left(\gamma\cdot\partial\theta/2N_f\right)\gamma_5\right)\psi$$

$$\partial_0\theta/2N_f = \mu_5 \neq 0 \quad \Longrightarrow \quad N_5 = N_R - N_L \neq 0$$

Coupling between B and θ

Maxwell-Chern-Simons theory

$$L = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\theta}{4} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Gauss' law

$$\vec{\nabla} \cdot \vec{E} = \rho + \vec{\nabla}\theta \cdot \vec{B}$$

Ampere's law

$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j} + \left[(\partial_0 \theta) \vec{B} - (\vec{\nabla}\theta) \times \vec{E} \right]$$

Faraday's law

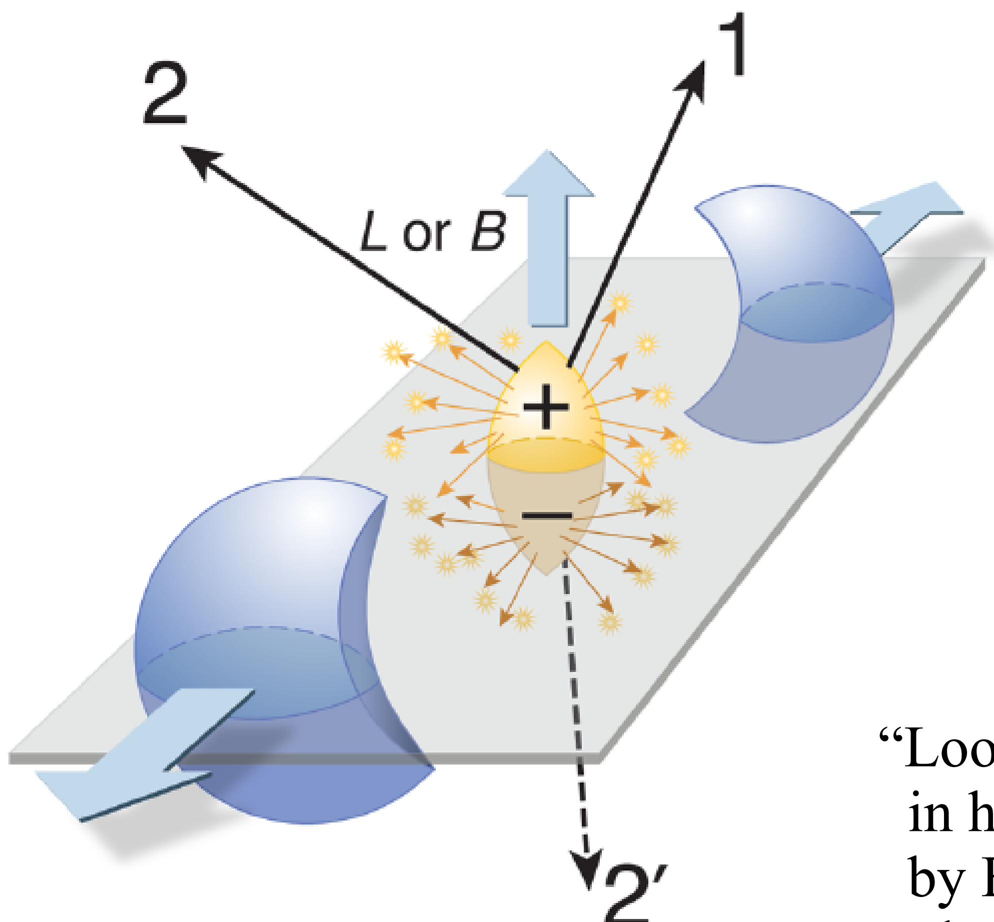
$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

Gauss' law for magnetism

$$\vec{\nabla} \cdot \vec{B} = 0$$

***B* (from QED) + *θ* (from QCD)**

Charge Separation

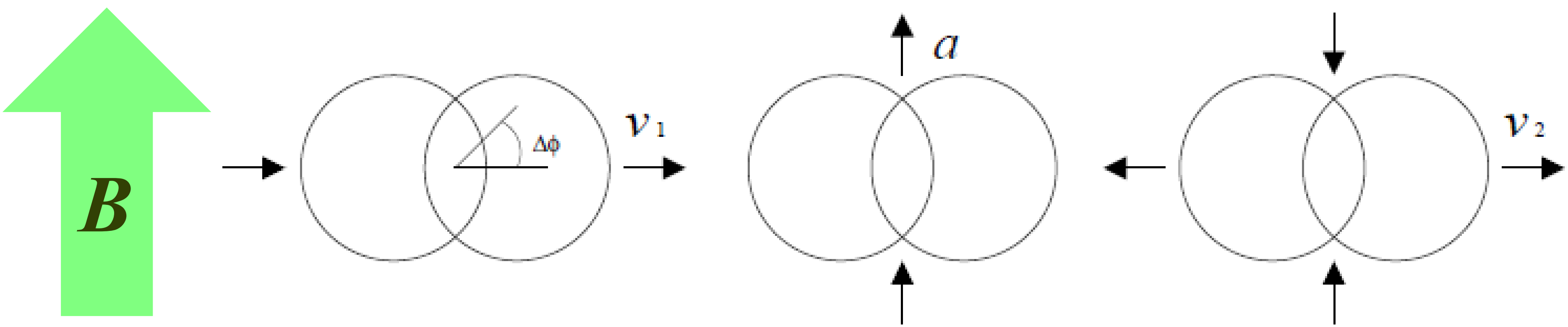


"Looking for parity violation
in heavy-ion collisions"
by Berndt Müller
Physics 2, 104 (2009)

Charge Asymmetry in HIC

Measured multiplicity

$$\frac{dN_{\pm}}{d\phi} \propto 1+2v_{1\pm}\cos(\Delta\phi)+2a_{\pm}\sin(\Delta\phi)+2v_{2\pm}\cos(2\Delta\phi)+\ldots$$



ϕ : Azimuthal Angle

v_1 : Directed Flow

v_2 : Elliptic Flow

a : Charge Asymmetry ← Vanishing on Average

Observable by STAR

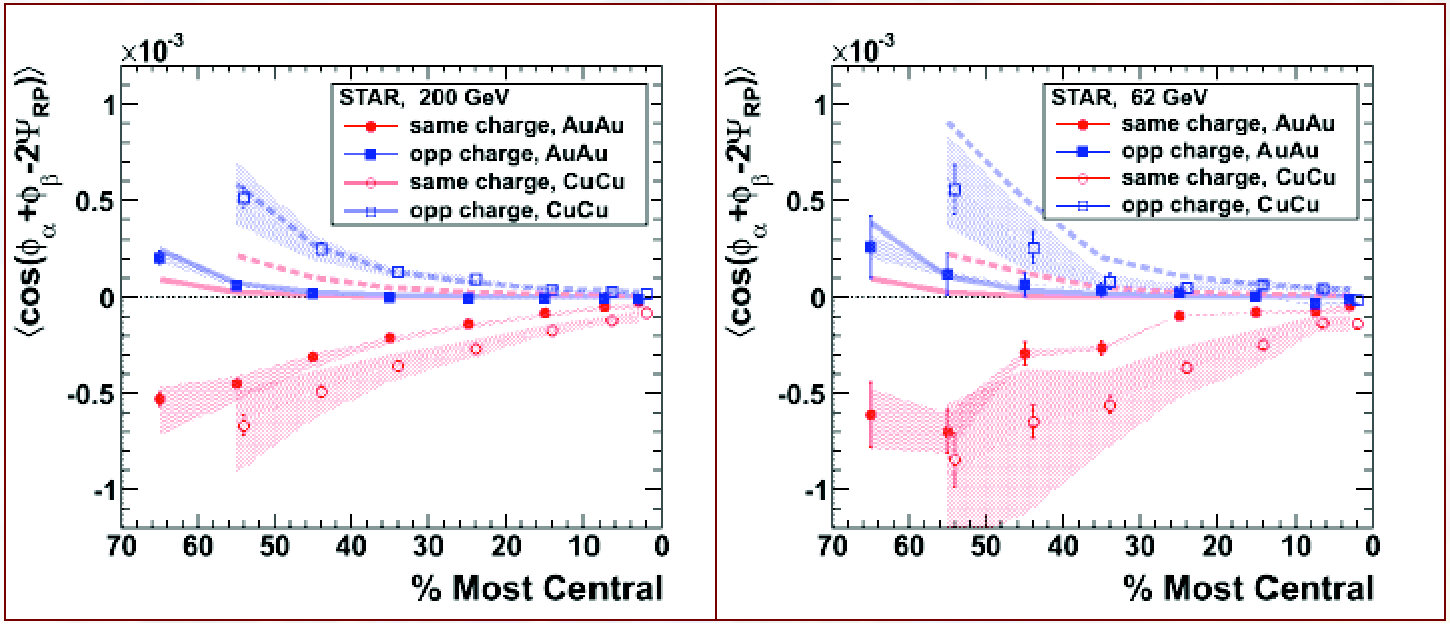
Observable (fluctuation measurement)

$$\langle\langle \cos(\Delta\phi_\alpha + \Delta\phi_\beta)\rangle\rangle \equiv \left\langle\left\langle \frac{1}{N_\alpha N_\beta}\sum_{i=1}^{N_\alpha}\sum_{j=1}^{N_\beta}\cos(\Delta\phi_{\alpha,i} + \Delta\phi_{\beta,j})\right\rangle\right\rangle$$

$$= \langle\langle \cos\Delta\phi_\alpha \cos\Delta\phi_\beta\rangle\rangle - \langle\langle \sin\Delta\phi_\alpha \sin\Delta\phi_\beta\rangle\rangle$$

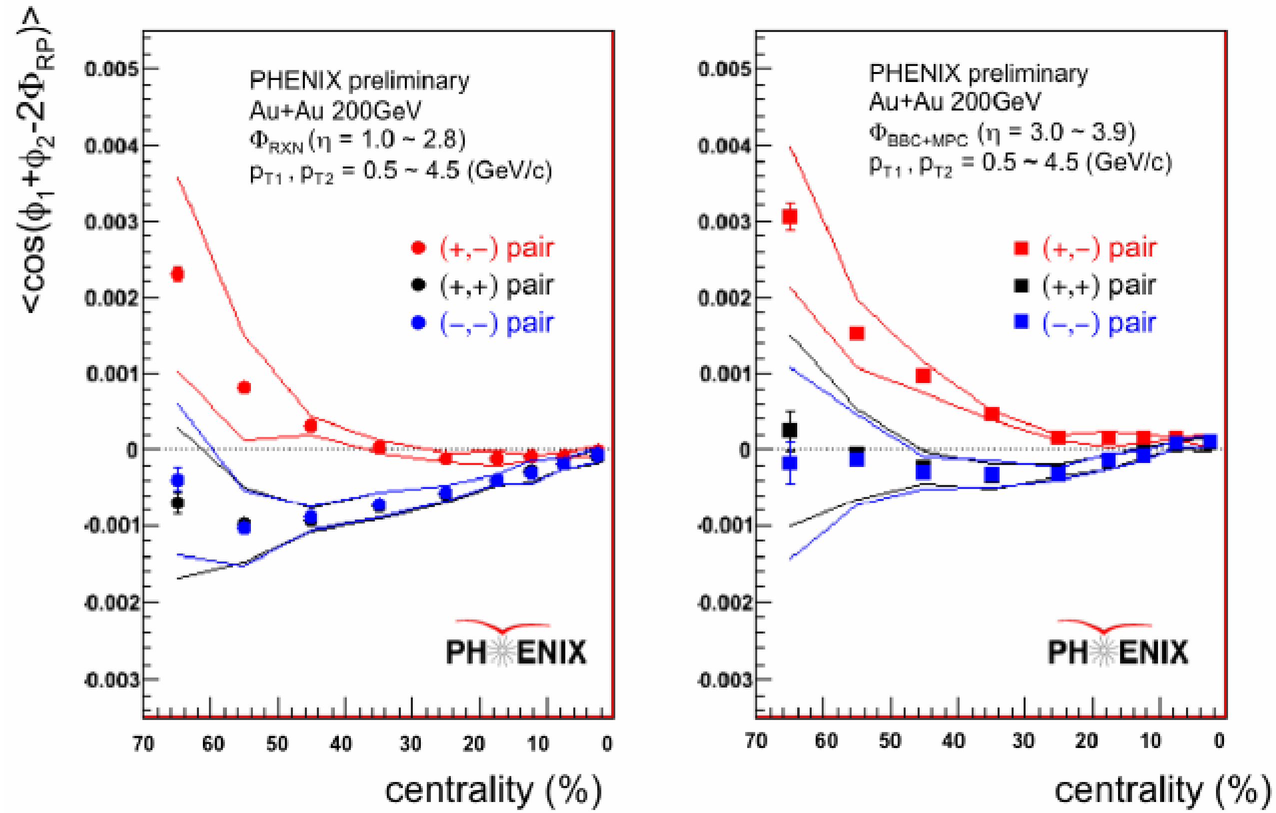
$$= \left(\langle\langle v_{1,\alpha}v_{1,\beta}\rangle\rangle + B^{\rm in}_{\alpha\beta}\right) - \left(\langle\langle a_\alpha a_\beta\rangle\rangle + B^{\rm out}_{\alpha\beta}\right).$$

STAR Results



Confirmation by PHENIX

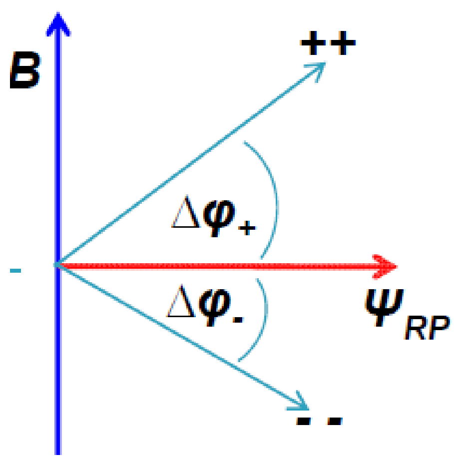
Good agreement between STAR and PHENIX



Not conclusive – Backgrounds from Flow, Decay, etc

Multi-Particle Correlation

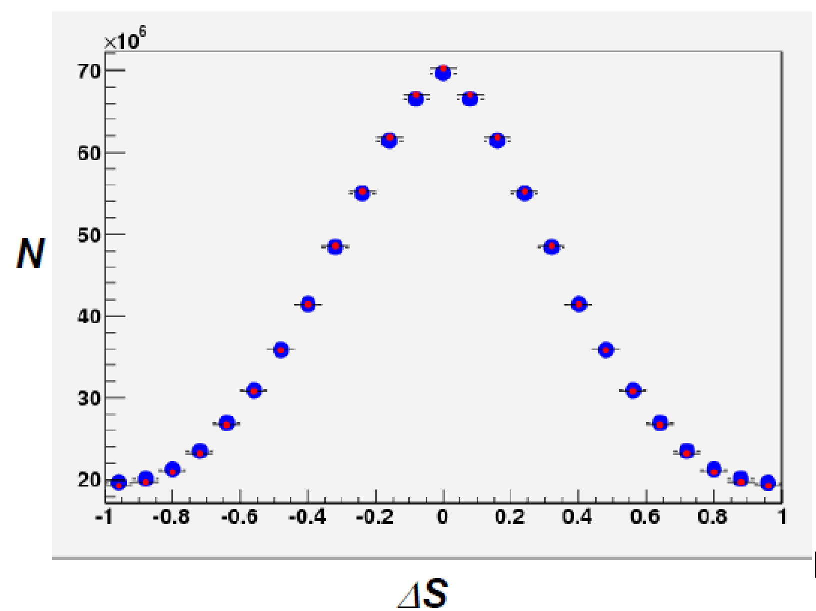
Two Distribution Functions



$$\langle S_p^{h+} = \sin\Delta\phi_{p+} \rangle \quad \langle S_n^{h-} \rangle \quad \langle S_p^{h\mp} \rangle \quad \langle S_n^{h\mp} \rangle$$

$$C_p(\Delta S) = \frac{N\left(\langle S_p^{h+} \rangle - \langle S_n^{h-} \rangle\right)}{N\left(\langle S_p^{h\mp} \rangle - \langle S_n^{h\mp} \rangle\right)}$$

PHENIX
Talk by R.Lacey



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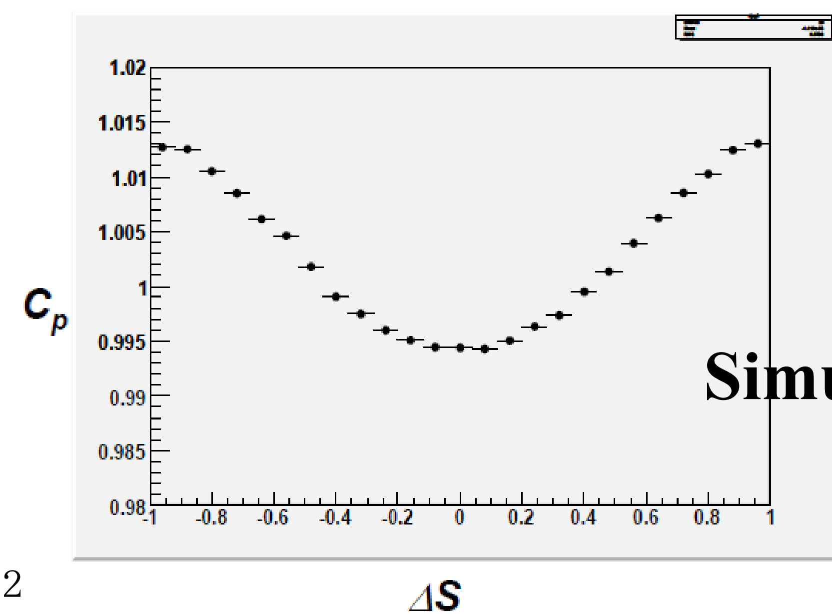
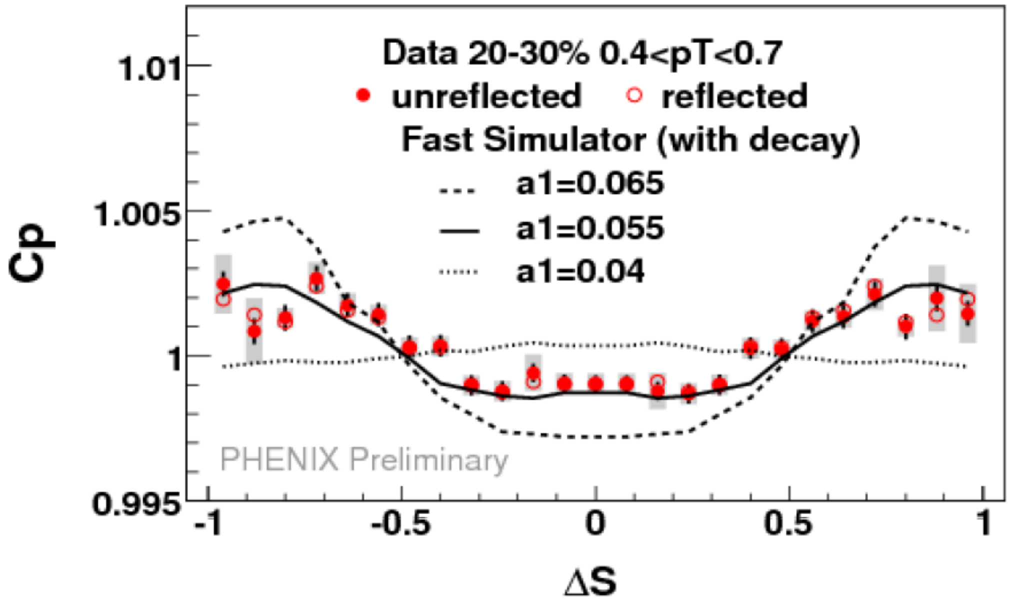
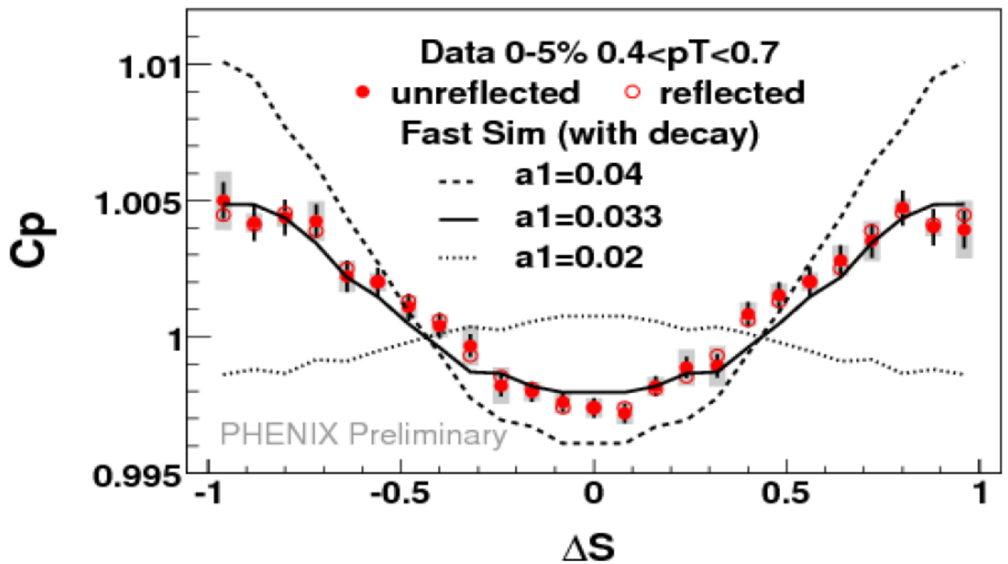


Simulation

Preliminary Data (in favor of CME)





Charge Asymmetry from Current

Induced charge asymmetries

$$\sum_{i=1}^{N_+} \cos \Delta\phi_{+,i} \approx -\sum_{i=1}^{N_-} \cos \Delta\phi_{-,i} \propto J_x$$

$$\sum_{j=1}^{N_+} \sin \Delta\phi_{+,j} \approx -\sum_{i=1}^{N_-} \sin \Delta\phi_{-,i} \propto J_z$$

Induced charge asymmetry fluctuations

Experimentally observed quantity

$$\boxed{\cos(\Delta\phi_\alpha + \Delta\phi_\beta)} \propto \frac{\alpha\beta}{N_\alpha N_\beta}\left(J_\perp^2 - J_\parallel^2\right)$$

$$\langle J_\parallel^2 \rangle_{\mu_5} = \langle J_\parallel \rangle_{\mu_5}^2 + \chi_{\mu_5}^{\parallel}, \qquad \langle J_\perp^2 \rangle_{\mu_5} = \langle J_\perp \rangle_{\mu_5}^2 + \chi_{\mu_5}^{\perp}$$

Current squared + Current-current Susceptibility

One-Loop Computation

Diagrammatic Expression

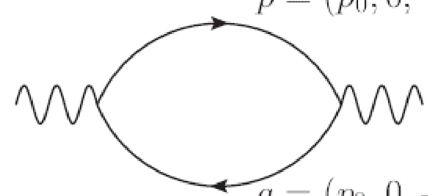
$$\chi_i = \mathrm{i}\, V T N_c \sum_f \frac{q_f^2 |q_f B|}{2\pi} \sum_{k,l} \int \frac{\mathrm{d}p_z}{2\pi} \int^T \frac{\mathrm{d}p_0}{2\pi} \int \mathrm{d}x\, \mathrm{d}y$$

$$\times\, \mathrm{tr}\Big[\gamma^i P_k(x) (\not{\tilde{p}} + \mu_5 \gamma^0 \gamma^5 - M_f)^{-1} (\Gamma_+(\tilde{p}) + \Gamma_-(\tilde{p})) P_k(y)$$

$$\times\, \gamma^i P_l(y) (\not{\tilde{q}} + \mu_5 \gamma^0 \gamma^5 - M_f)^{-1} (\Gamma_+(\tilde{q}) + \Gamma_-(\tilde{q})) P_l(x) \Big]$$

$$\Gamma_\pm(p) \equiv \frac{1}{2}(1 \pm \hat{p}\cdot\gamma\gamma^0\gamma^5)$$

$$P_k(x) = \frac{1}{2}\big[f_{k+}(x) + f_{k-}(x)\big] + \frac{\mathrm{i}}{2}\big[f_{k+}(x) - f_{k-}(x)\big]\,\gamma^1\gamma^2$$



$$f_{k+}(x) = \phi_k(x - p_y/(qB)) \qquad (k = 0, 1, 2, \dots)\,,$$
$$f_{k-}(x) = \phi_{k-1}(x - p_y/(qB)) \qquad (k = 1, 2, 3, \dots)$$

$$\phi_k(x) = \sqrt{\frac{1}{2^k k!}} \left(\frac{|qB|}{\pi}\right)^{1/4} \exp\!\Big(-\frac{1}{2}|qB|x^2\Big) H_k\big(\sqrt{|qB|}\,x\big)$$

(Ritus' method)

Linear Response Argument

Current generation rate

= Anomaly rate (Schwinger process)

KF-Kharzeev-Warringa

$$\left\langle \frac{\mathrm{d}J_\parallel}{\mathrm{d}x_0} \right\rangle = V N_c \sum_f \frac{q_f^2 |q_f B| E}{2\pi^2} + O(E^2)$$

Linear response theory

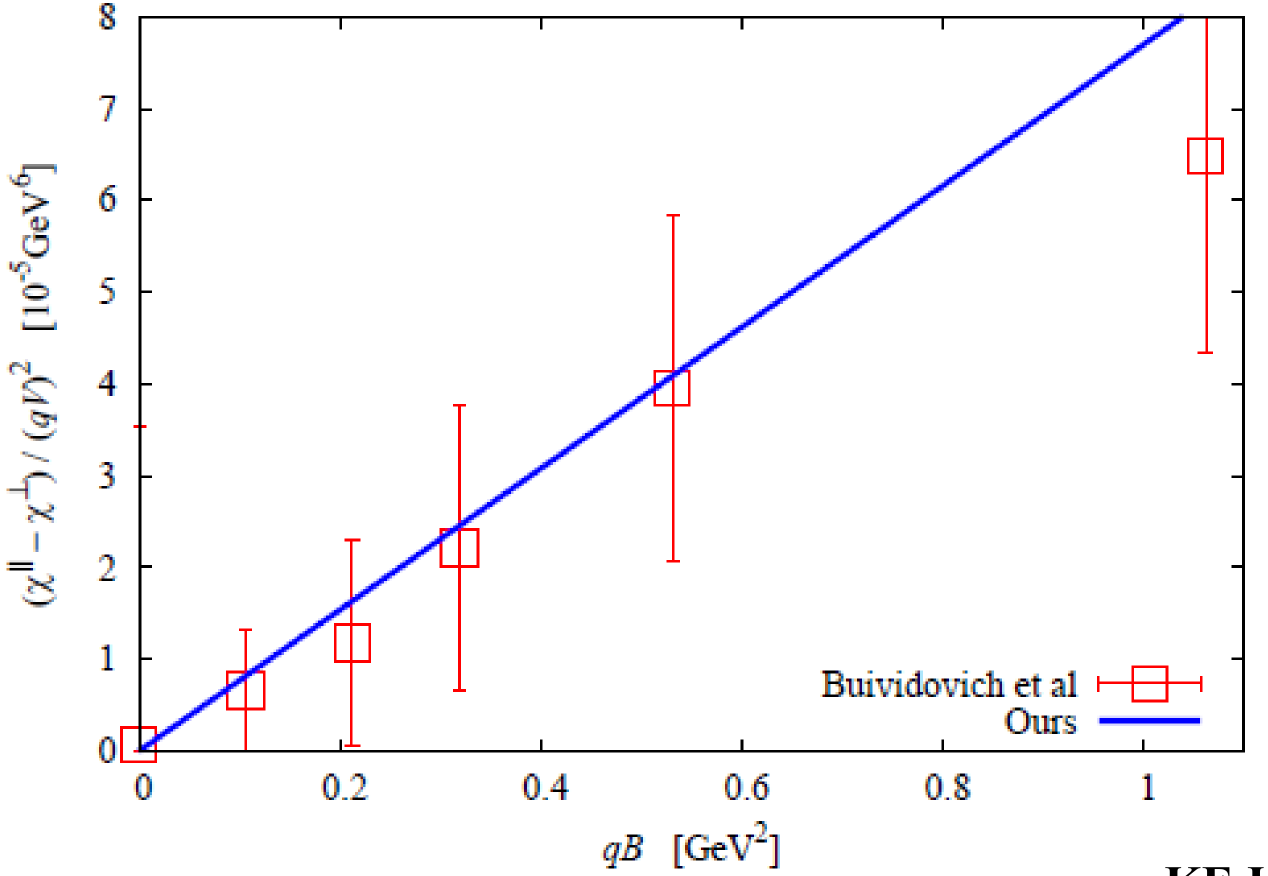
$$\left\langle \frac{\mathrm{d}J_\parallel}{\mathrm{d}x_0} \right\rangle = -\int \mathrm{d}^3x\, \mathrm{d}^4x' \left\langle \frac{\mathrm{d}j_\parallel(x)}{\mathrm{d}x_0} j_\parallel(x') \right\rangle_{\mathrm{ret}} A_z(x') + O(A_z^2)$$

Integration by parts in the gauge $(d/dx_0)A_z = E_z$

$$\left\langle \frac{\mathrm{d}J_\parallel}{\mathrm{d}x_0} \right\rangle = \int \mathrm{d}^3x\, \mathrm{d}^4x' \, \langle j_\parallel(x) j_\parallel(x') \rangle_{\mathrm{ret}} E + O(E^2)$$

(Quenched) Lattice QCD

Buividovich, Chernodub, Luschevskaya, Polikarpov (2009)



KF-Kharzeev-Warringa

Dimensional Reduction to (1+1)

Vector = Axial-Vector $(\gamma^5 = \gamma^0 \gamma^1)$

$$\gamma^\mu \gamma^5 = -\epsilon^{\mu\nu} \gamma_\nu \quad + \quad j_V^\mu = \bar{\Psi} \gamma^\mu \psi, \qquad j_A^\mu = \bar{\Psi} \gamma^\mu \gamma^5 \psi$$

$$\longrightarrow \quad j_V^1 = j_A^0, \qquad j_A^1 = j_V^0$$

Axial Anomaly in (1+1) Dimensions

$$\partial_\mu j_A^\mu = \frac{e}{2\pi} \epsilon^{\mu\nu} F_{\mu\nu} = \frac{e}{\pi} E$$

$$= -\frac{e}{2\pi} (\partial^0 A^1 - \partial^1 A^0)$$

Electric Field = Topological Charge
Vector Potential = Chern-Simons Current

Trivial Derivation of the Currents

Vector and Axial-Vector Currents

Chiral Magnetic Effect

$$j_V^1 = j_A^0 \quad \to \quad J_V^1 = N_5 = -2Q_W$$

Chiral Separation Effect

$$j_A^1 = j_V^0 \quad \to \quad J_A^1 = N$$

Ohm's law !

Trivially derived from the γ-matrix structure

More Familiar Forms using $E = F_{01}$

$$J_V^1 = -\frac{e}{\pi}\int dx \quad A^1(t,x) = \frac{1}{\pi}\int dx \quad \mu_5$$

$$J_A^1 = -\frac{e}{\pi}\int dx \quad A^0(t,x) = \frac{1}{\pi}\int dx \quad \mu$$

μ_5 is the longitudinal component of the QED vector potential

Chiral Magnetic Currents

(1+1) Dimensions → (3+1) Dimensions

$$e\,J^1_V = \frac{e}{\pi}\int dx\;\;\mu_5 \qquad \text{in (1+1) dimensions}$$

$$e\,j^1_V = \frac{e}{\pi}\mu_5 \times \frac{eB}{2\pi} = \frac{e^2\mu_5}{2\pi^2}B \qquad \text{in (3+1) dimensions}$$

Current Generation Rate

Instanton Configuration

$$A^1(t) = -Et \quad \left(= \frac{2\pi Q_W}{eL}\frac{t}{T} \right)$$

$$J^1_V = -\frac{e}{\pi}\int dx\;\; A^1(t\,,x)$$

➡

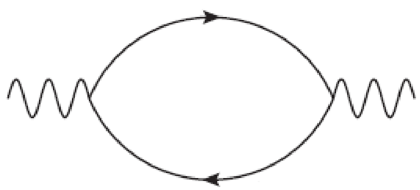
$$\frac{\partial}{\partial t} e\,j^1(t) = \frac{e^2 E}{\pi} \qquad \text{in (1+1)}$$

$$\frac{\partial}{\partial t} e\,j^1(t) = \frac{e^3 EB}{2\pi^2} \qquad \text{in (3+1)}$$

Anomaly Rate

Results in (1+1) Dimensions

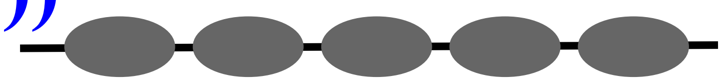
One-loop Calculation



$$\chi^{\text{one-loop}} = \frac{e^2}{\pi} \qquad \left(\frac{e^3 B}{2\pi^2} \quad \text{in } (3+1) \right)$$

known as the "mass"
in the Schwinger model

Full-order Calculation (in (1+1))



$$\chi^{\text{full-order}}(\omega, k) = \frac{e^2}{\pi} \frac{\omega^2}{\omega^2 - k^2 - e^2/\pi}$$

Screening by fermion loops

Vanishing in the zero momentum-frequency limit

Summary

- CME = Electric current induced by B (*fields*) and N_5 (*chirality from instanton/sphaleron*)

- All the known expressions related to the CME immediately derived from the (1+1)-dimensional setup.

- Electric current-current computation; one-loop result is exactly reproduced ("*Schwinger mass*").

  Full-order result is also available but it is under discussions if this can be generalized to (3+1) ?