QCD thermodynamics with improved Wilson quarks at zero and finite densities

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MOTIVATION

QCD thermodynamics on the lattice

phase structure and T_c

nature of the transition/crossover

$\mu=0$

$\mu\neq0$

EOS, v_s, **η**, ...

> properties of quark matter at heavy ion collisions

> development of early universe / genesis of matter

Numerical simulations of QCD on the lattice

* the only systematic / ab initio way to investigate the issue
* becoming quantitatively powerful

MOTIVATION

status of lattice studies

$T = 0$ $\quad N_F = 2{+}1$ (degenerate ud + s) simulation directly at the physical point (PACS-CS Collab.)

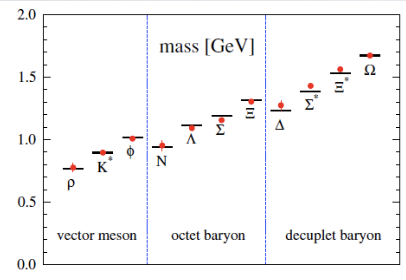
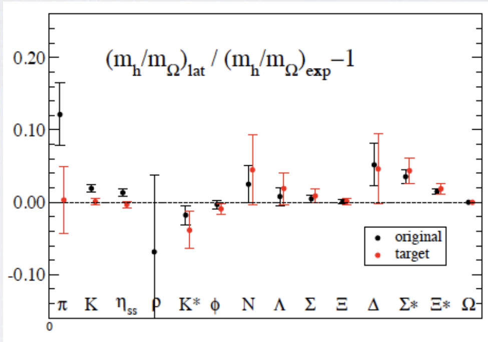


FIG. 24 (color online). Light hadron spectrum extrapolated to the physical point using m_π, m_K and m_Ω as input. Horizontal bars denote the experimental values.

[PRD79, 034503]
extrapolated
to the phys. point

→



[arXiv: 0911.2561]
directly at the phys. point
by the reweighting method

$32^3{\times}64$, a=0.09fm

planned: u-d mass difference and QED effects
yet to do: continuum extrapolation

$T > 0$, $\mu \neq 0$

$N_F = 2{+}1$ at almost physical point

e.g. EOS with improved Staggered quarks:

RBC-Bi/MILC ('07-'09) $\quad N_f = 2+1,\ N_t = 4, 6, 8,\ m_\pi \sim 220, 150\mathrm{MeV}$

Budapest-Wuppertal ('06) $\quad N_f = 2+1,\ N_t = 4, 6,\ m_\pi \sim 140\mathrm{MeV}$

MOTIVATION

lattice quarks

$T > 0$, $\mu \neq 0$

Most studies done with **staggered-type quarks** because of

less computational costs / a part of chiral sym. preserved / . . .

N_f=2+1: T>0 at (almost) physical point, $\mu \neq 0$ studies

but

4 flavors degenerate ("taste")

<= "4th root trick": replace det$D$ by its 4th root by hand

=> non-local theory

=> universality to our world not guaranteed

Need crosschecks with theoretically sound lattice quarks.

* Wilson quarks much behind: no N_f=2+1, no $\mu \neq 0$, m_q not quite light ...
* lattice chiral quarks (DW, overlap, ...): yet in its infancy / too expensive

We want to extend **Wilson** studies (thus "WHOT"-QCD)

to N_f=2+1, $\mu \neq 0$, and smaller m_q

Wilson simulations are more expensive => improvements/tricks are called for.

contents

- Motivation: Why Wilson-type quarks?
- Improved Wilson quarks at μ=0

  with an introduction to T>0 QCD on the lattice.
- Improved Wilson quarks at μ>0
- Fixed scale approach with T-integral method
- Summaries and outlooks

Wilson quarks at μ=0

PRD 63, 034502 (2001); PRD 64, 074510 (2001); PRD 75, 074501 (2007)

INTRODUCTION

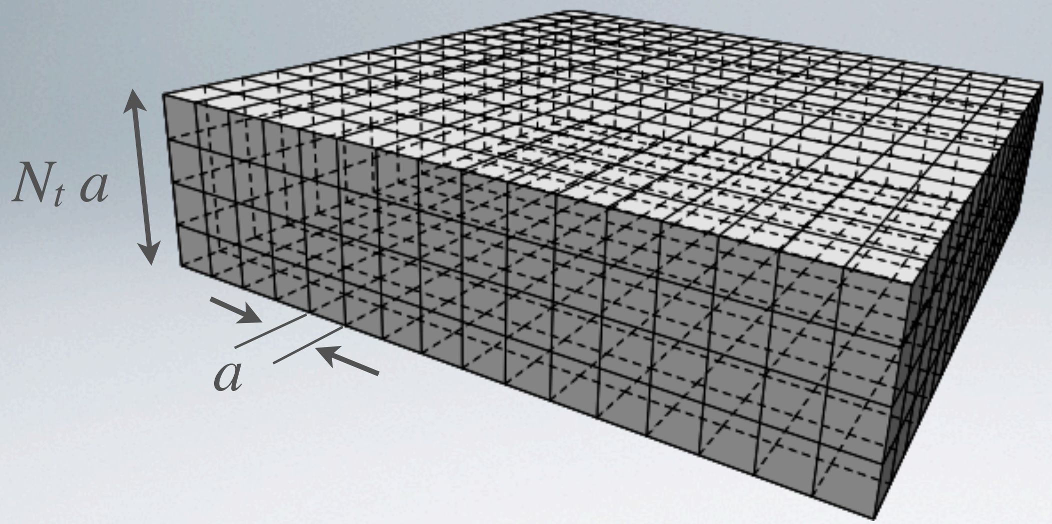
T>0 QCD on the lattice

$$Z = \int [dU][dq][d\bar{q}]\, e^{-S}$$

$$T = \frac{1}{N_t a}$$

N_t : lattice size in the euclidian time direction

a : lattice spacing



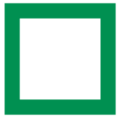
low	high	T →
large N_t	small N_t	
large a	small a	
small β	large β	

Conventionally, a is varied at a fixed N_t ("fixed Nt approach"), where a is controlled by the lattice gauge coupling $\beta = \dfrac{1}{6g^2}$.

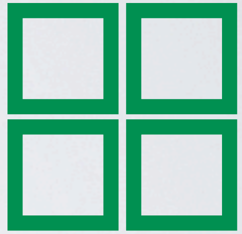
THERMODYNAMICS WITH WILSON QUARKS

$N_f = 2$ QCD, Nt=4, (6)

PRD 63, 034502 (2001); PRD 64, 074510 (2001);
PRD 75, 074501 (2007)



$$
\begin{aligned}
S &= S_g + S_q, \\
S_g &= -\beta \sum_x \left(c_0 \sum_{\mu<\nu;\mu,\nu=1}^{4} W_{\mu\nu}^{1\times1}(x) + c_1 \sum_{\mu\neq\nu;\mu,\nu=1}^{4} W_{\mu\nu}^{1\times2}(x) \right) \\
S_q &= \sum_{f=1,2} \sum_{x,y} \bar{q}_x^f D_{x,y} q_y^f,
\end{aligned}
$$



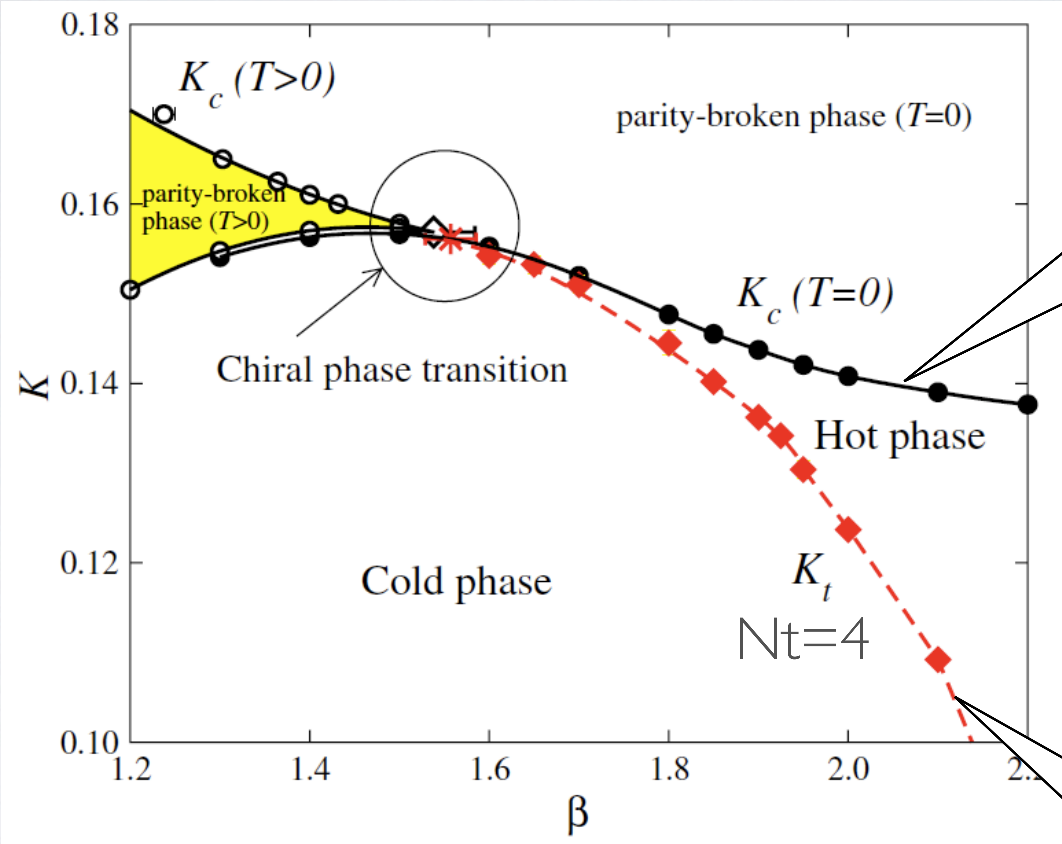
$$
D_{x,y} = \delta_{xy} - K \sum_{\mu} \{(1-\gamma_\mu)U_{x,\mu}\delta_{x+\hat{\mu},y} + (1+\gamma_\mu)U_{x,\mu}^{\dagger}\delta_{x,y+\hat{\mu}}\} - \delta_{xy} c_{SW} K \sum_{\mu<\nu} \sigma_{\mu\nu} F_{\mu\nu}
$$

$c_1 = -0.331,\ c_0 = 1 - 8c_1$ RG-improved Iwasaki glue

$c_{SW} = (W^{1\times1})^{-3/4} = (1 - 0.8412\beta^{-1})^{-3/4}$ MF-improved clover quarks

$T>0$ simulations: Nt=4, (6), V = mainly 16^3, $m_\pi >\approx 500$ MeV
$T=0$: simulations: 16^4, $12^3\times24$

PHASE STRUCTURE



chiral limit
$m_\pi(T=0) = 0$

light q

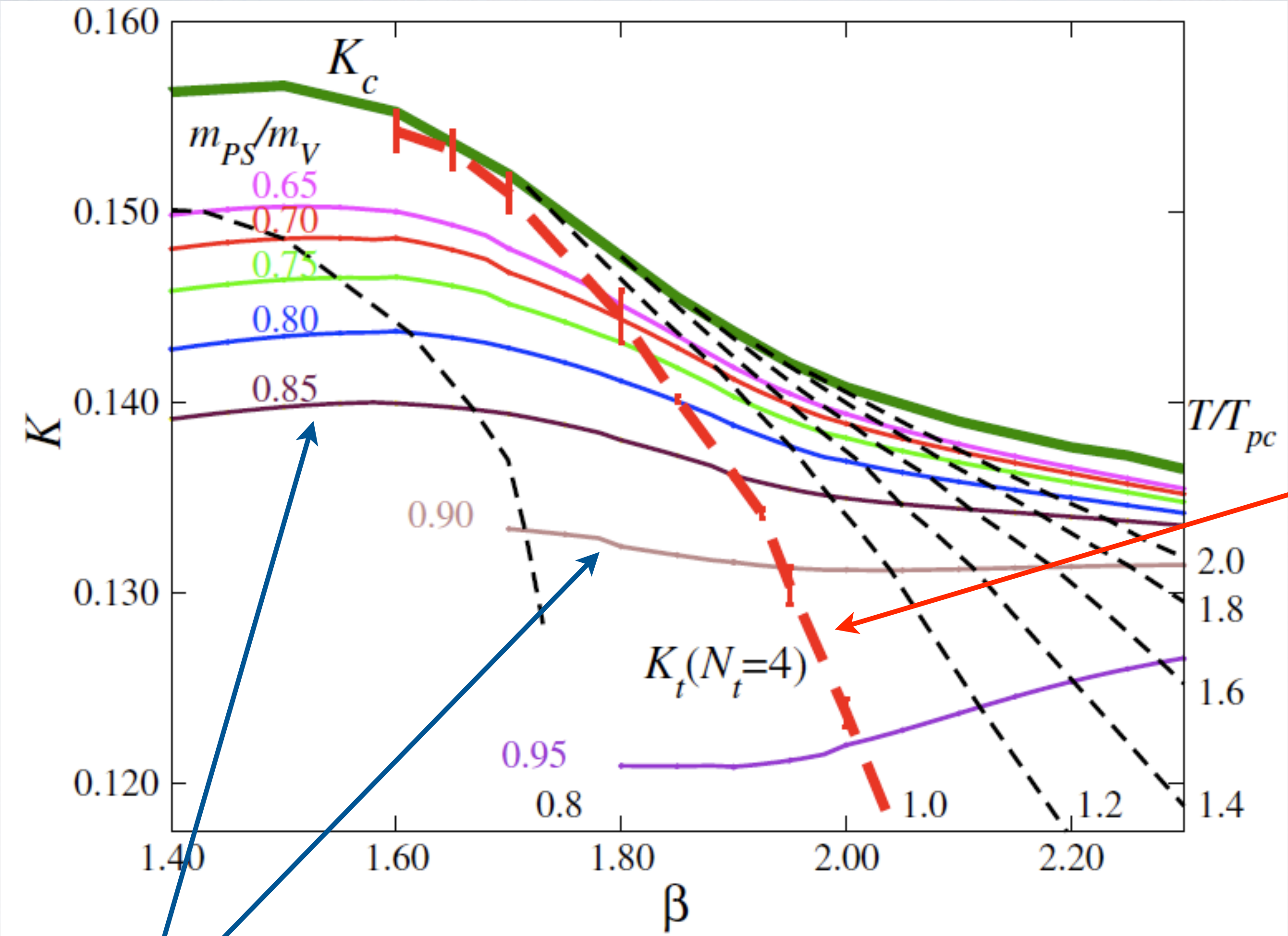
heavy q

Tc for
$Nt = 4$

low T — high T

large a — small a

PHASE STRUCTURE



Finite *T* crossover for *Nt*=4.

[arXiv:0909.2121]

LCP's (Lines of Constant Physics) determined by m_π/m_ρ

$m_\pi/m_\rho \approx 0.18$ in our world, *a* --> 0 to the left.

O(4) SCALING

- massless N_f-flavor QCD (continuum):

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times \cancel{U(1)_A}$$

SSB ↓↑ high T — anomaly

$$SU(N_f)_V \times U(1)_V$$

effective 3d σ model (GL model) — Pisarski-Wilczek ('84)

$N_f \geq 3$: 1st order

$N_f = 2$: if anomaly negligible -> 1st order

with anomaly -> 2nd order <=> O(4) Heisenberg model

O(4) scaling [crit. exponents, scaling functions, ...]

(Note: RG flow enhances the anomaly towards the IR limit.)

$$M/h^{1/\delta} = f(t/h^{1/\beta\delta})$$

$1/\beta\delta = 0.537(7)$

$1/\delta = 0.2061(9)$

- t: reduced temperature $(T-T_c)/T_c$
- f: universal scaling function
- h: external magnetic field
- M: magnetization

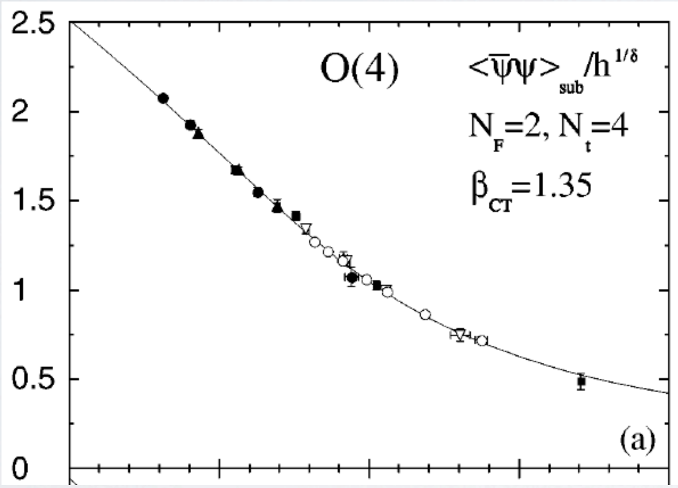
O(4) SCALING

Fit QCD data with O(4) scaling function *f*(*x*) and O(4) critical exponents.

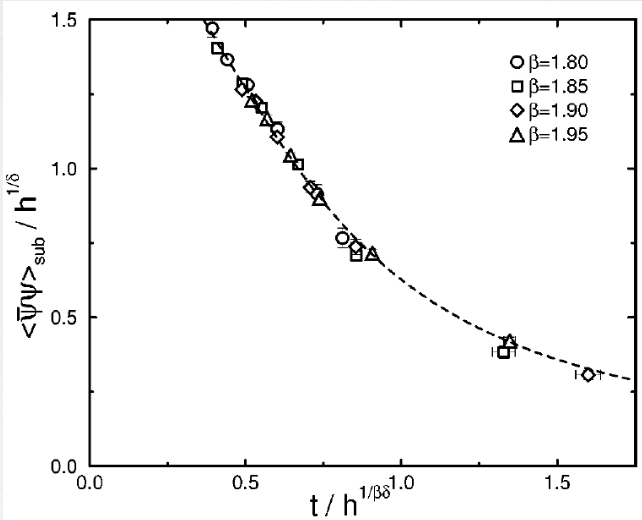
$$M/h^{1/\delta} = f(t/h^{1/\beta\delta})$$

$$t \sim \beta - \beta_{ct}\,,\ h \sim m_q a\,,\ \text{and}\ M \sim \langle\bar{\psi}\psi\rangle$$

$$\langle\bar{\psi}\psi\rangle_{\rm sub} = 2m_q a Z \sum_x \langle\pi(x)\pi(0)\rangle$$ to subtract out contributions from explicit chiral violation due to the Wilson term.



Iwasaki-KK-Kaya-Yoshie, PRL 78 ('97) 179
Iwasaki glue + unimproved Wilson



CP-PACS Collab. PRD 63 ('01) 034502
Iwasaki glue + Clover

=> can be used to precisely extract T_c in the chiral limit.

Current estimate: T_c = 171-180 (*Nt*=4), 160-184 MeV (*Nt*=6) [WHOT-QCD, arXiv:0909.2121]

(cf.) Recent study with staggered quarks: Ejiri et al. arXiv: 0909.5122

EOS

$$\epsilon = -\frac{1}{V}\frac{\partial \ln Z}{\partial T^{-1}}, \quad p = T\frac{\partial \ln Z}{\partial V}$$

Trace anomaly:

lattice action

$$\frac{\epsilon - 3p}{T^4} = N_t^4 \left\{ \left\langle a\frac{dS}{da} \right\rangle - \left\langle a\frac{dS}{da} \right\rangle_{T=0} \right\} = \frac{N_t^3}{N_s^3} \sum_i a\frac{db_i}{da} \left\{ \left\langle \frac{\partial S}{\partial b_i} \right\rangle - \left\langle \frac{\partial S}{\partial b_i} \right\rangle_{T=0} \right\}$$

$$b = (\beta, \kappa_{ud}, \kappa_s, \cdots) \equiv (b_1, b_2, \cdots)$$

measured by the simulation.
T=0 subtracted for renormalization.

lattice beta functions
scale-dep. of coupl. parameters along LCP

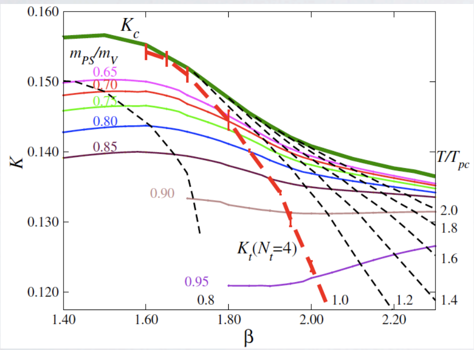
EOS

Beta functions not available for this combination of lattice actions yet.

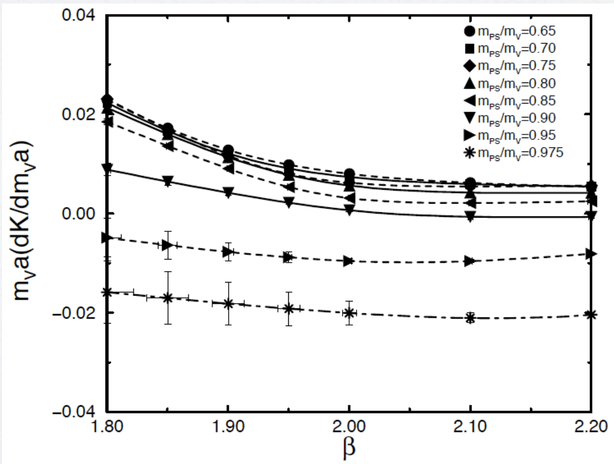
=> We have to calculate by ourselves.

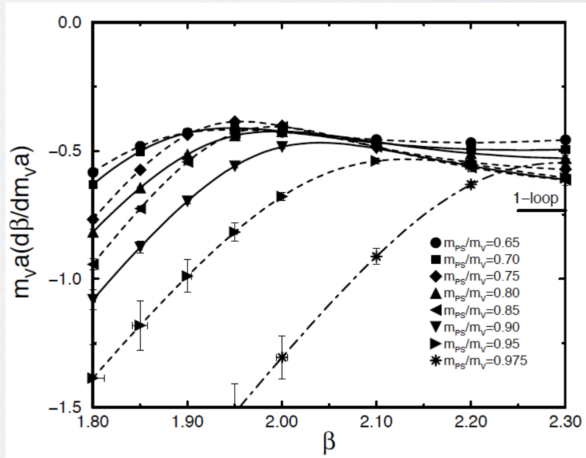
Inverse matrix method [PRD 64, 074510]

i. Collect T=0 lattice results of #param. observables (e.g. $m_\pi a$, $m_\rho a$, ...).
ii. Fit them as functions of coupling parameters
iii. Determine LCP's
iv. Invert the coupling param. dependence of observables to the observable dependence of coupling parameters along a LCP.



$$\begin{pmatrix} \dfrac{\partial \beta}{\partial (m_V a)} & \dfrac{\partial K}{\partial (m_V a)} \\ \dfrac{\partial \beta}{\partial (m_{PS}/m_V)} & \dfrac{\partial K}{\partial (m_{PS}/m_V)} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial (m_V a)}{\partial \beta} & \dfrac{\partial (m_{PS}/m_V)}{\partial \beta} \\ \dfrac{\partial (m_V a)}{\partial K} & \dfrac{\partial (m_{PS}/m_V)}{\partial K} \end{pmatrix}^{-1}$$





EOS

Integral method:

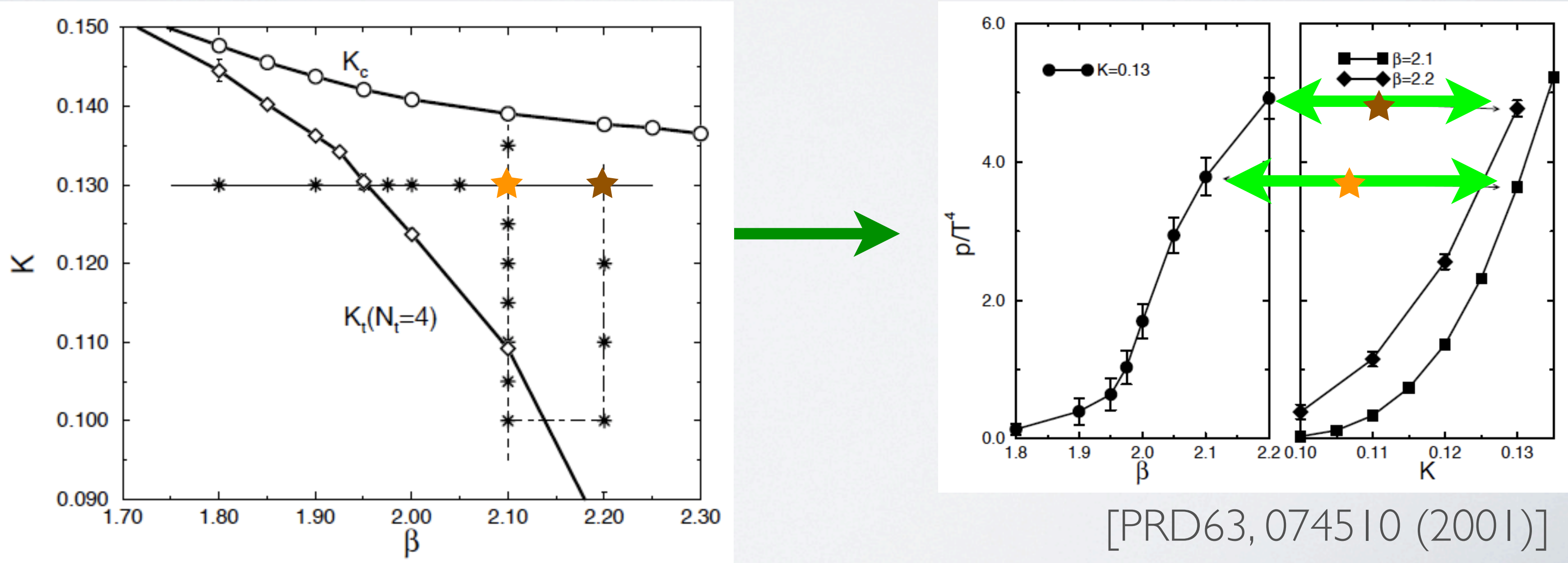
Differentiate and integrate the thermodyn. relation $p = (T/V)\ln Z$

$$p = \frac{T}{V}\int_{b_0}^{b} db\,\frac{1}{Z}\frac{\partial Z}{\partial b} = -\frac{T}{V}\int_{b_0}^{b}\sum_i db_i\left\{\left\langle\frac{\partial S}{\partial b_i}\right\rangle - \left\langle\frac{\partial S}{\partial b_i}\right\rangle_{T=0}\right\}$$

such that $p(b_0) \approx 0$

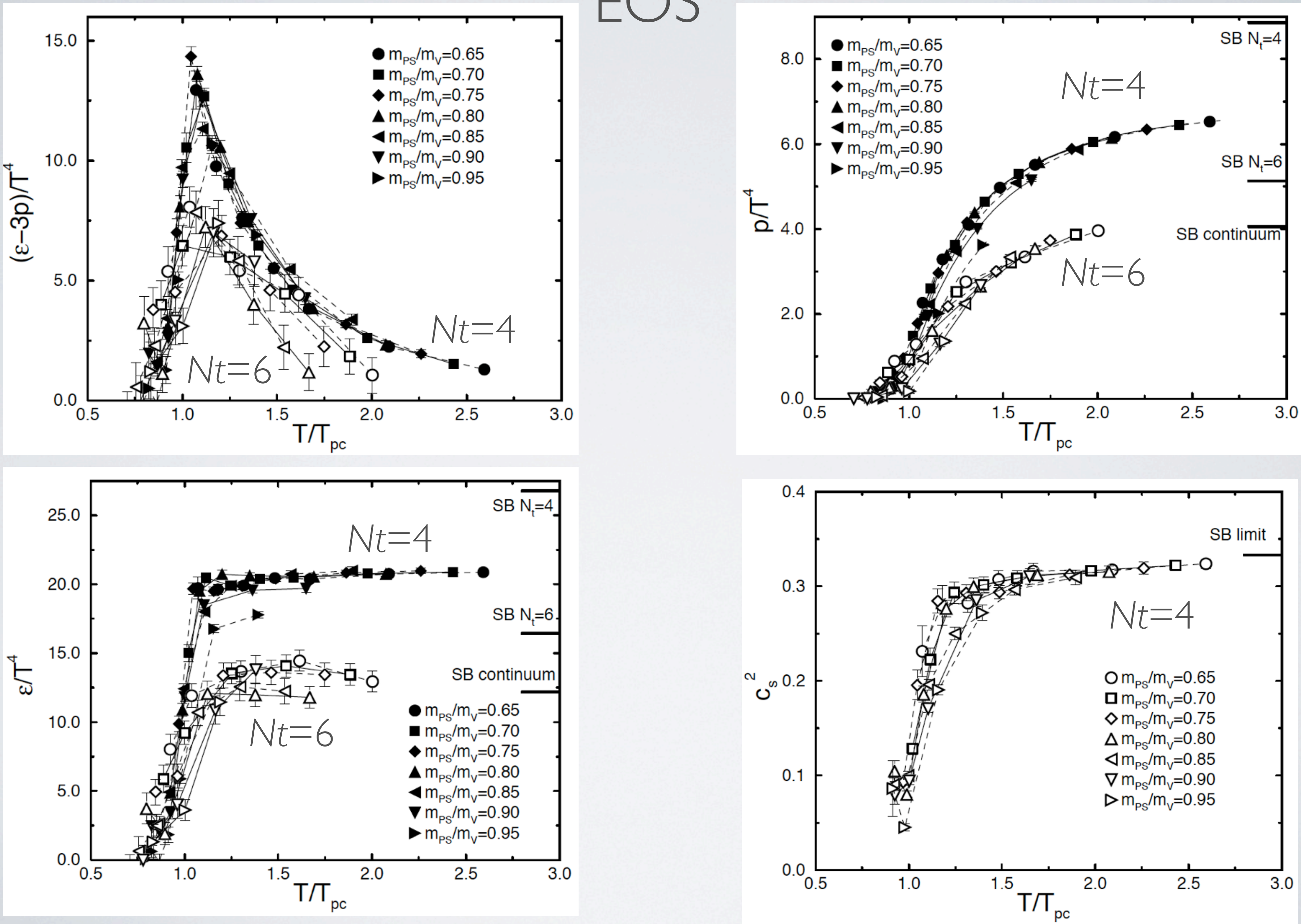
numerical integration
in the coupling param. space

The integration path is free to choose as far as $p(b_0) \approx 0$



[PRD63, 074510 (2001)]

EOS


(ε−3p)/T^4
T/T_pc
m_PS/m_V=0.65
m_PS/m_V=0.70
m_PS/m_V=0.75
m_PS/m_V=0.80
m_PS/m_V=0.85
m_PS/m_V=0.90
m_PS/m_V=0.95
Nt=4
Nt=6
p/T^4
SB N_t=4
SB N_t=6
SB continuum
ε/T^4
c_s^2
SB limit

Wilson quarks at $\mu \neq 0$

arXiv:0909.2121

FORMULATION

$\mu \neq 0$ QCD on the lattice

$$U_4 \Longrightarrow U_4 e^{ia\mu_q} \text{ (positive direction)};$$
$$U_4 e^{-ia\mu_q} \text{ (negative direction)}$$

$$Z = \mathrm{Tr} e^{-H/T} = \int Dq D\bar{q} DU e^{-S}; \quad S = S_g + \sum \bar{q} M[U]\, q$$

$$M(\mu)^\dagger = \gamma_5 M(-\mu) \gamma_5$$

$$[\det M(\mu)]^* = \det M(-\mu) \neq \det M(\mu)$$

complex Boltzmann weight $\int Dq\, D\bar{q} e^{-\sum \bar{q} M[U]\, q} = \det M(\mu)$

=> sign problem (complex phase problem)

Large cancellations due to phase fluctuations: $\left\langle e^{i\theta} \right\rangle \propto e^{-V}$

Exponentially large statistics needed to keep the accuracy.

STATUS

Approaches to study at small μ:

- Reweighting from μ=0 — Fodor-Katz ('02-), ...
- Taylor expansion from μ=0 — Swansea, Bielefeld, BNL ('02-), ...
- Analytic continuation from imag. μ — deForcrand-Philipsen ('02-), ...
- Canonical ensemble — BNL-Bielefeld ('07-), ...

Taylor expansion method with improved Staggered quarks:

Bielefeld-Swansea ('02-05) Nf=2, Nt=4, heavy

MILC, RBC-Bielefeld ('08-) Nf=2+1, Nt=4,6, $m_\pi \approx 220$MeV

No Wilson until ~'07.

STUDY WITH WILSON QUARKS AT $\mu \neq 0$

N_f=2 QCD, Nt=4 (based on the μ=0 study) [arXiv:0909.2121]

- Observables

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z} \equiv \omega, \qquad \frac{n_f}{T^3} = \frac{1}{VT^3} \frac{\partial \ln \mathcal{Z}}{\partial(\mu_f/T)} = \frac{\partial (p/T^4)}{\partial(\mu_f/T)}, \qquad (f = u,\, d)$$

$$\frac{\chi_q}{T^2} = \left(\frac{\partial}{\partial(\mu_u/T)} + \frac{\partial}{\partial(\mu_d/T)} \right) \frac{n_u + n_d}{T^3} \qquad \frac{\chi_I}{T^2} = \left(\frac{\partial}{\partial(\mu_u/T)} - \frac{\partial}{\partial(\mu_d/T)} \right) \frac{n_u - n_d}{T^3}$$

- Taylor expansion in μu=μd(=μq) at μu=μd=0

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu_q}{T} \right)^n, \qquad c_n(T) = \frac{1}{n!} \frac{N_t^3}{N_s^3} \left. \frac{\partial^n \ln \mathcal{Z}}{\partial(\mu_q/T)^n} \right|_{\mu_q=0}$$

$$\frac{\chi_q(T,\mu_q)}{T^2} = 2c_2 + 12 c_4 \left(\frac{\mu_q}{T} \right)^2 + \cdots \qquad c_2 = \frac{N_t}{2N_s^3} \mathcal{A}_2, \quad c_4 = \frac{1}{4! N_s^3 N_t} (\mathcal{A}_4 - 3\mathcal{A}_2^2)$$

$$\mathcal{A}_2 = \langle \mathcal{D}_2 \rangle + \langle \mathcal{D}_1^2 \rangle, \qquad \mathcal{A}_4 = \langle \mathcal{D}_4 \rangle + 4 \langle \mathcal{D}_3 \mathcal{D}_1 \rangle + 3 \langle \mathcal{D}_2^2 \rangle + 6 \langle \mathcal{D}_2 \mathcal{D}_1^2 \rangle + \langle \mathcal{D}_1^4 \rangle$$

$$\mathcal{D}_n = N_f \frac{\partial^n \ln \det M}{\partial \mu^n}$$

$$\mu \equiv \mu_q a,$$

$$\mathcal{D}_1 = N_f \mathrm{tr} \left(M^{-1} \frac{\partial M}{\partial \mu} \right)$$

$$\mathcal{D}_2 = N_f \left[\mathrm{tr} \left(M^{-1} \frac{\partial^2 M}{\partial \mu^2} \right) - \mathrm{tr} \left(M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} \right) \right]$$

NUMERICAL METHODS

Wilson quarks are more expensive => require improvements!

- Noise method for traces

Its too expensive to calculate all elements of M^{-1}.

$$\frac{1}{N_{\rm noise}}\sum_i \eta_{i,\alpha}\eta^*_{i,\beta} \approx \delta_{\alpha,\beta} \qquad {\rm tr}A = \sum_\alpha A_{\alpha,\alpha} \approx \frac{1}{N_{\rm noise}}\sum_i \eta_i^\dagger A \eta_i$$

$${\rm tr}\left(\frac{\partial^n M}{\partial^n \mu} M^{-1}\right) \approx \frac{1}{N_{\rm noise}}\sum_i \eta_i^\dagger \frac{\partial^n M}{\partial^n \mu} X_i; \quad X_i = M^{-1}\eta_i$$

Noise method efficient when off-diagonal elements are small.

Because M^{-1} is a propagator, distant contributions are naturally suppressed.

For Wilson quarks, 3(color)x4(spin) competitors at the same point.

=> We generate independent η for each color and spin.

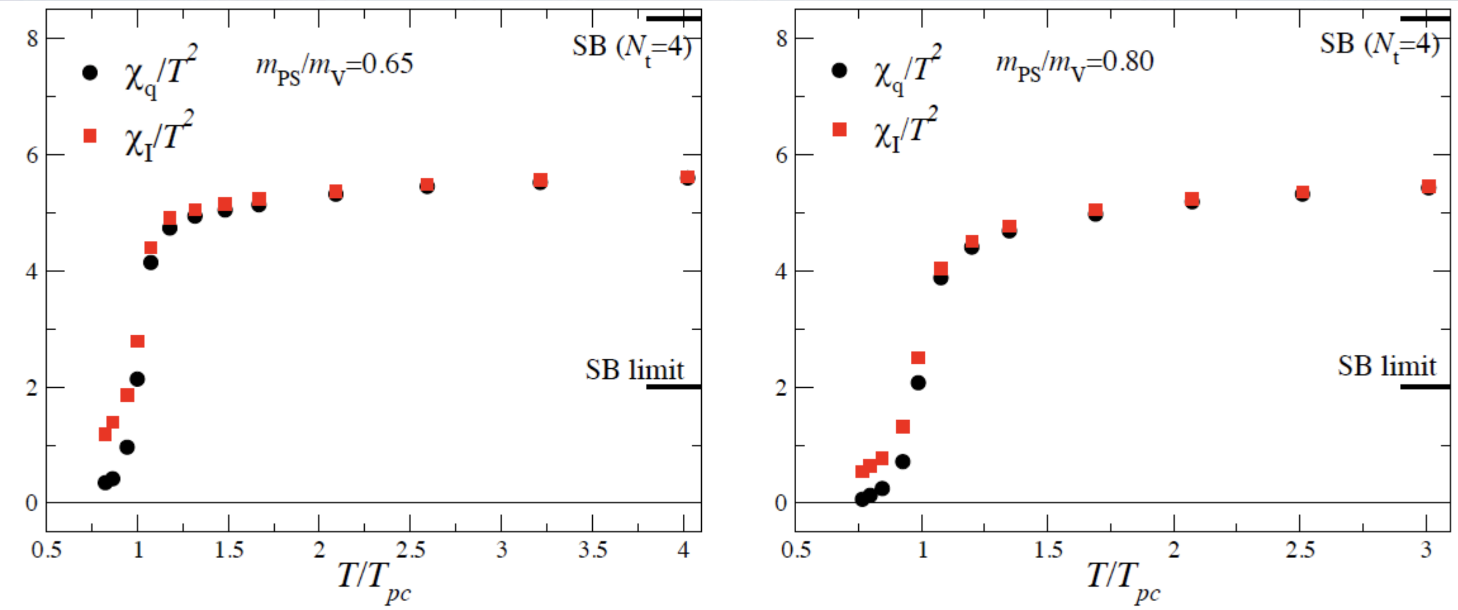
(cf.) staggered quark: spins are staggered on a cube.

Error from D_1 turned out to be dominating in the results.

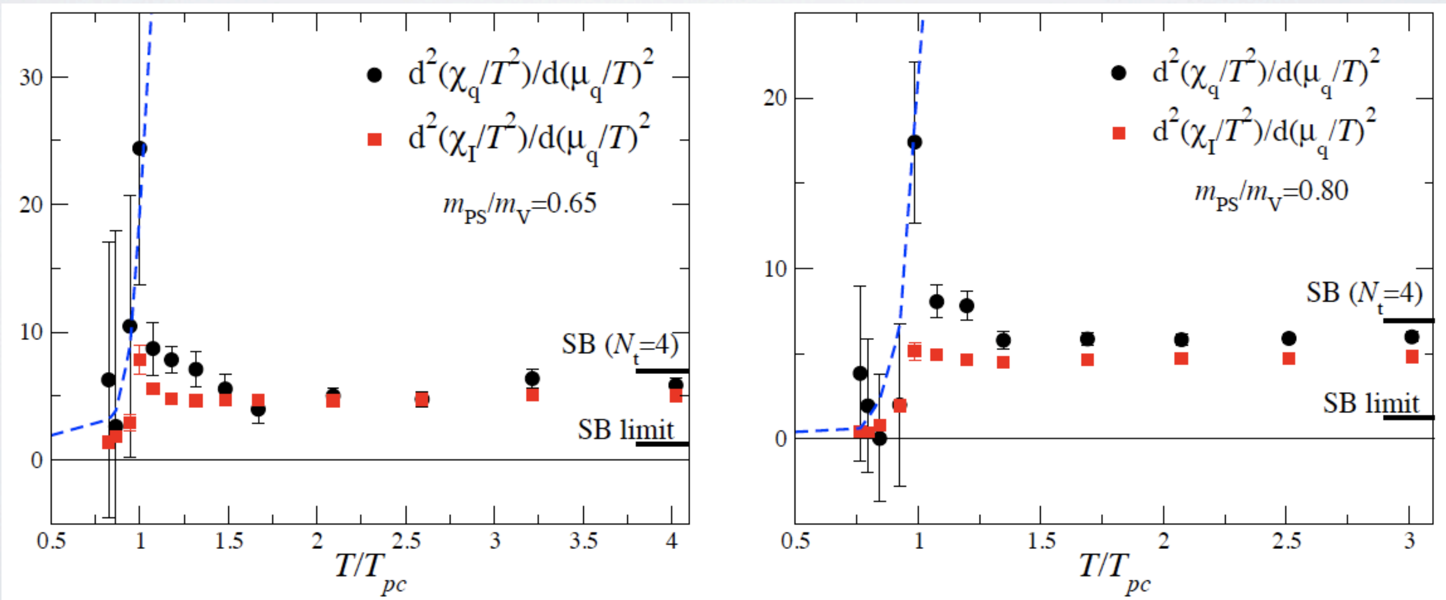
=> We adopt about 100 times larger $N_{\rm noise}$ for D_1.

RESULTS (I)

- μ=0



- c_2



RESULTS (I)

- χ_q

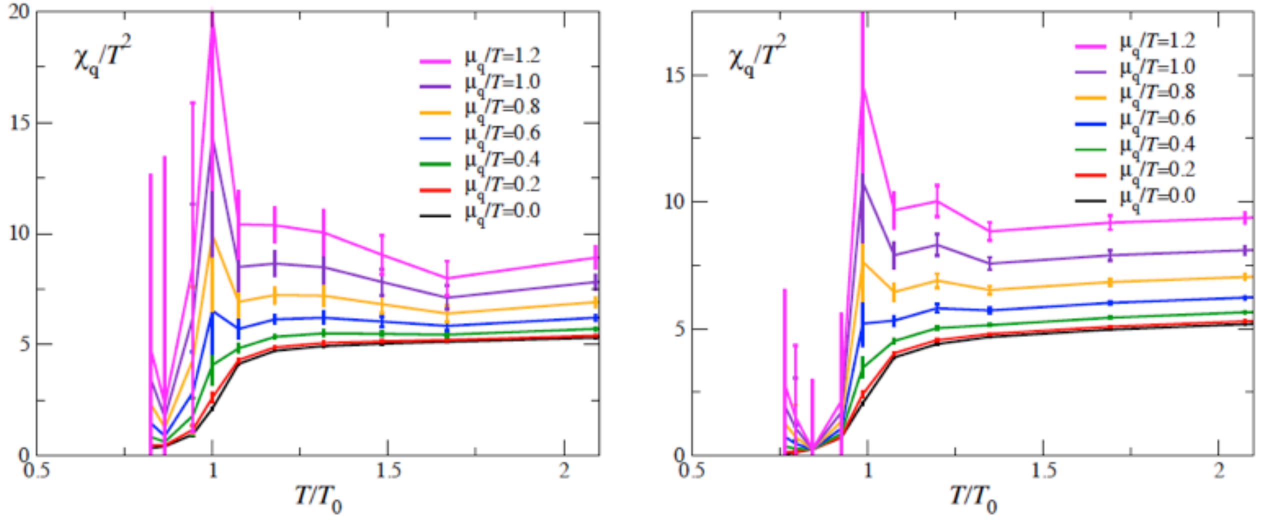


FIG. 14: Quark number susceptibility at finite μ_q for $m_{\rm PS}/m_{\rm V} = 0.65$ (left) and 0.80 (right).

- χ_I

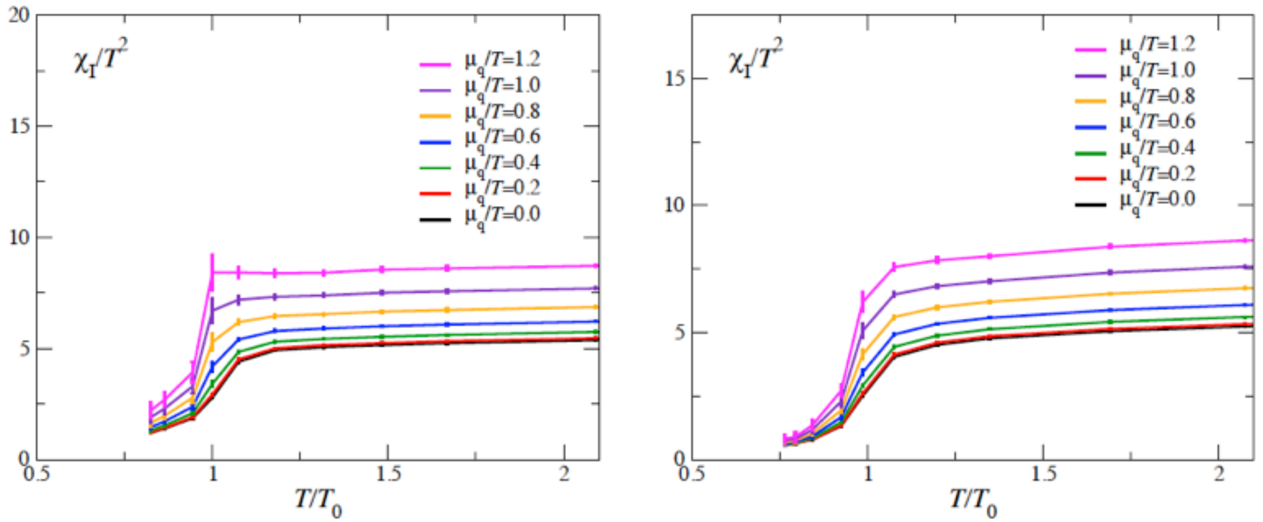


FIG. 15: Isospin susceptibility at finite μ_q for $m_{\rm PS}/m_{\rm V} = 0.65$ (left) and 0.80 (right).

Suggest critical pt. at finite μ, which is insensitive to the iso-spin number.

GAUSSIAN METHOD

To further improve the calculation

- A hybrid Taylor+reweighting method

Allton et al., PRD 66, 074507 ('02)
Ejiri, PRD 77, 014508 ('08)

Reweight the grand canonical partition function from μ=0:

$$\mathcal{Z}(T,\mu_q) = \mathcal{Z}(T,0)\left\langle \left(\frac{\det M(\mu)}{\det M(0)}\right)^{N_f}\right\rangle_{(\mu_q=0)} \equiv \mathcal{Z}(T,0)\left\langle e^{F(\mu)}e^{i\theta(\mu)}\right\rangle_{(\mu_q=0)}$$

and Taylor-expand the terms in exp:

$$\begin{aligned} F(\mu) &\equiv N_f \mathrm{Re}\left[\ln\left(\frac{\det M(\mu)}{\det M(0)}\right)\right] \\ &= N_f \sum_{n=1}^{\infty}\frac{1}{(2n)!}\mathrm{Re}\left[\frac{\partial^{2n}(\ln\det M)}{\partial\mu^{2n}}\right]_{(\mu=0)}\mu^{2n} = \sum_{n=1}^{\infty}\frac{1}{(2n)!}\mathrm{Re}\mathcal{D}_{2n}\mu^{2n}. \end{aligned}$$

$$\begin{aligned} \theta(\mu) &= N_f \mathrm{Im}\left[\ln\det M(\mu)\right] \\ &= N_f \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}\mathrm{Im}\left[\frac{\partial^{2n+1}(\ln\det M(\mu))}{\partial\mu^{2n+1}}\right]_{(\mu=0)}\mu^{2n+1} = \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}\mathrm{Im}\mathcal{D}_{2n+1}\mu^{2n+1}. \end{aligned}$$

Truncate the expansions up to D_4

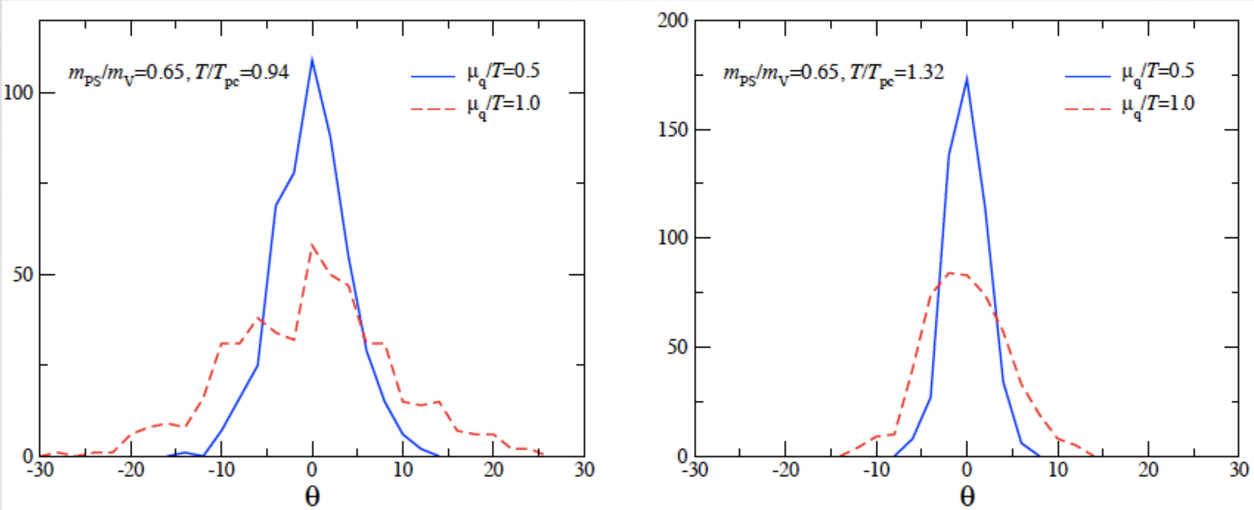
* Identical to the truncated Taylor expansion up to the 4th order,
but contain a part of higher orders through the exponential function.

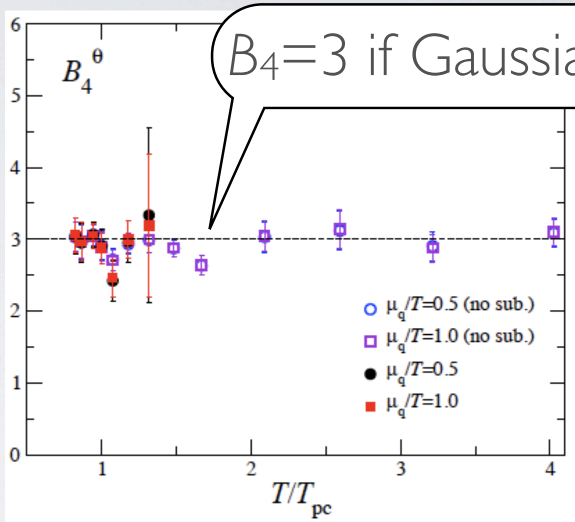
* Exact for free QGP, in which D_n=0 for n>4, => The truncation will be OK at high T.

GAUSSIAN METHOD

- Gaussian approximation for θ

Ejiri, PRD 77, 014508 ('08)





B_4=3 if Gaussian

[arXiv:0909.2121]

=> θ-integration tractable by measuring $\frac{1}{2a_2(\bar{F})} = \langle \theta^2 \rangle_{\bar{F}}$

This mildens the sign problem (fluctuation due to $e^{i\theta}$).

(Note) θ as defined by the expansion is not restricted between -π and +π.
Conventional periodic θ is recovered by mapping the result into (-π,+π].

RESULTS (2)

- χ_q

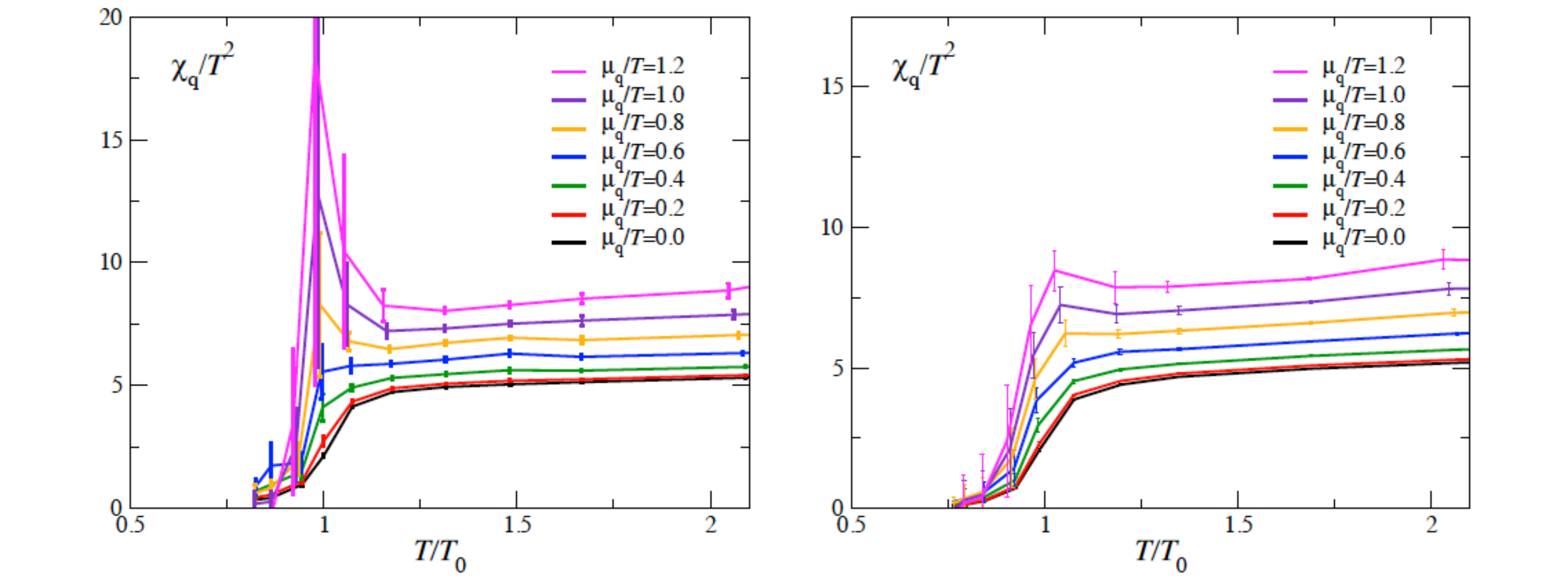
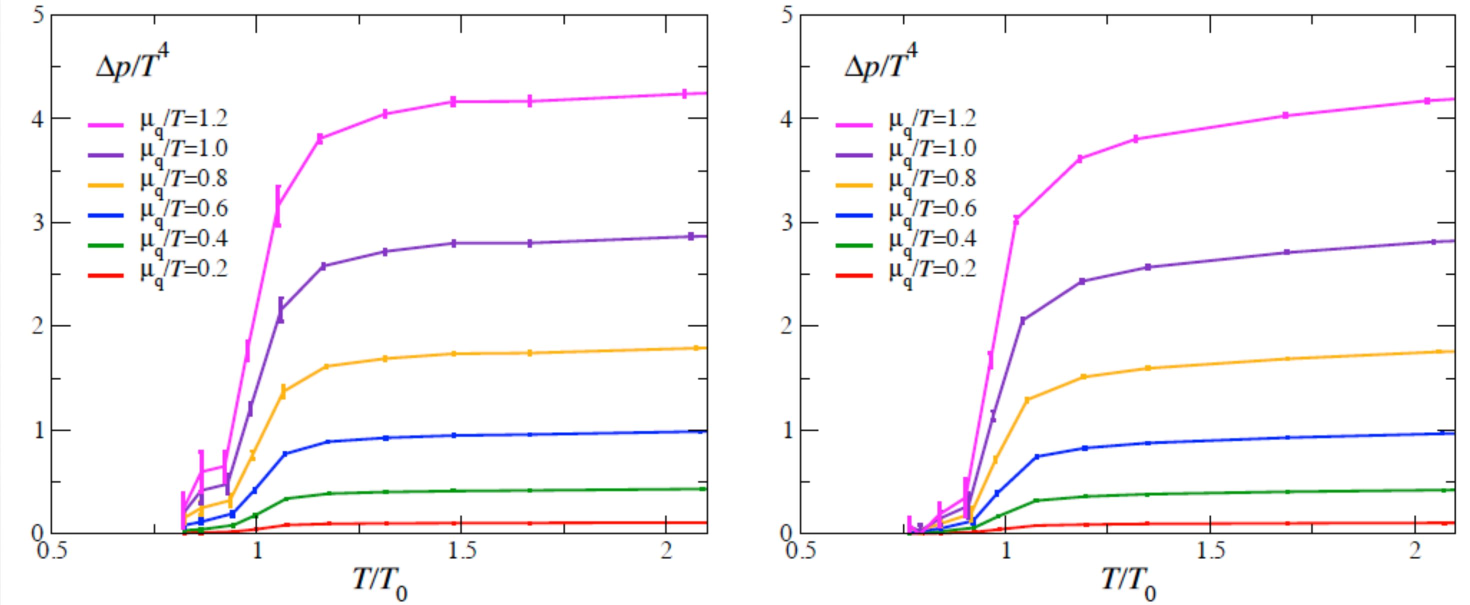


FIG. 24: Quark number susceptibility for each μ_q/T at $m_{\rm PS}/m_{\rm V} = 0.65$ (left) and 0.80 (right).

- $p(\mu)$-$p(0)$



RESULTS (3)

- heavy quark free energies

$$V^R(r,T,\mu_q) = v_0^R + v_1^R\left(\frac{\mu_q}{T}\right) + v_2^R\left(\frac{\mu_q}{T}\right)^2 + O(\mu^3)$$

$Q\bar{Q}$ QQ

$R = \mathbf{1},\ \mathbf{8},\ \mathbf{6},\ \mathbf{3}^*$

$$\begin{aligned}
\Omega^{\mathbf{1}}(r) &= \frac{1}{3}\mathrm{tr}\Omega^\dagger(\mathbf{x})\Omega(\mathbf{y}),\\
\Omega^{\mathbf{8}}(r) &= \frac{1}{8}\mathrm{tr}\Omega^\dagger(\mathbf{x})\mathrm{tr}\Omega(\mathbf{y}) - \frac{1}{24}\mathrm{tr}\Omega^\dagger(\mathbf{x})\Omega(\mathbf{y}),\\
\Omega^{\mathbf{6}}(r) &= \frac{1}{12}\mathrm{tr}\Omega(\mathbf{x})\mathrm{tr}\Omega(\mathbf{y}) + \frac{1}{12}\mathrm{tr}\Omega(\mathbf{x})\Omega(\mathbf{y}),\\
\Omega^{\mathbf{3}^*}(r) &= \frac{1}{6}\mathrm{tr}\Omega(\mathbf{x})\mathrm{tr}\Omega(\mathbf{y}) - \frac{1}{6}\mathrm{tr}\Omega(\mathbf{x})\Omega(\mathbf{y}),
\end{aligned}$$

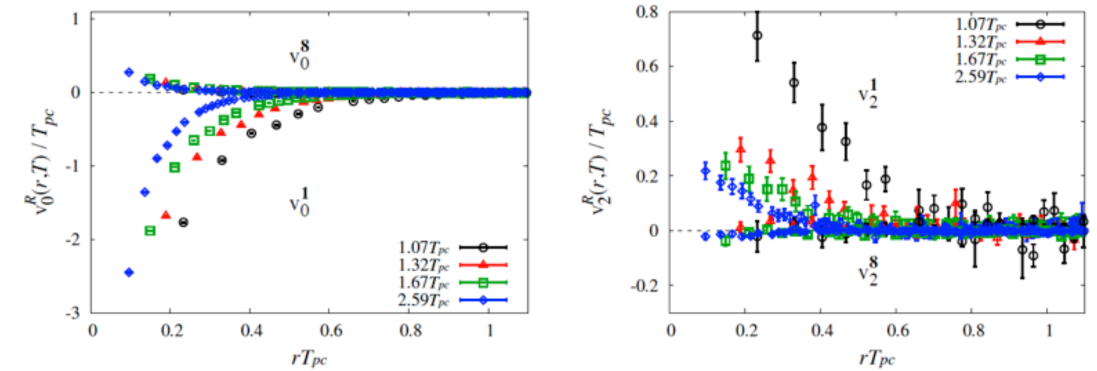


FIG. 25: v_0^R (left) and v_2^R (right) for color-singlet and octet $Q\bar{Q}$ channels above T_{pc} at $m_{PS}/m_V = 0.65$.

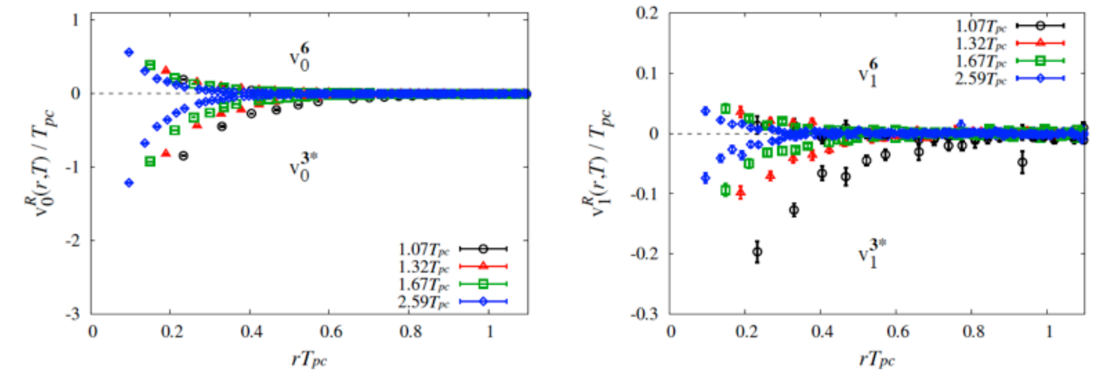


FIG. 26: v_0^R (left) and v_1^R (right) for color-sextet and antitriplet QQ channels above T_{pc} at $m_{PS}/m_V = 0.65$.

→ $Q\bar{Q}$ interaction: weaker at μ>0

QQ interaction: stronger at μ>0

(leading order in μ)

SUMMARY

- ✦ Study of finite density QCD with Wilson-type quarks possible by improvements.

- ✦ So far, results are consistent with previous studies with staggered quarks.
  - suggest a critical point at finite μ
    - sensitive to quark number,
    - but insensitive to iso-spin number

- ✦ Some new results for heavy quark free energies, etc.

Fixed scale approach with T-integral method

Phys. Rev. D79, 051501 (2009) [arXiv:0809.2842]
arXiv:0910.5284, arXiv: 0911.0254

MOTIVATION

We want to extend Wilson studies (thus "WHOT"-QCD)

to N_F=2+1, larger N_t, and smaller m_q

to avoid theoretical uncertainties of stag. quarks (universality, locality, etc.).

More improvements of the method are longed for.

A large fraction of the cost: $T = 0$ simulations for both stag. and Wilson

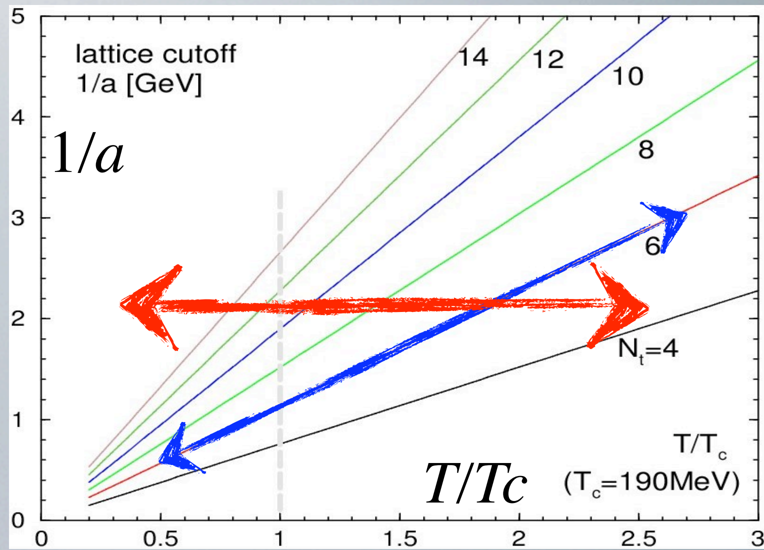
- Determination of basic info about the simulation point:
  scale, non-perturbative beta function, etc.
- $T = 0$ subtractions
  needed at all simulation points !!
- Determination of the Lines of Constant Physics (LCP)

→ To (partially) overcome the problem,
we propose a **fixed scale approach**
armed with a **T-integral method**.

FIXED SCALE APPROACH

Vary $T = \dfrac{1}{N_t a}$ by varying N_t

with all coupling params. fixed.



Conventional integral method not applicable.

↓

T-integral method

A thermodynamic relation at μ=0:

$$T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right) = \frac{\epsilon - 3p}{T^4}$$

↓

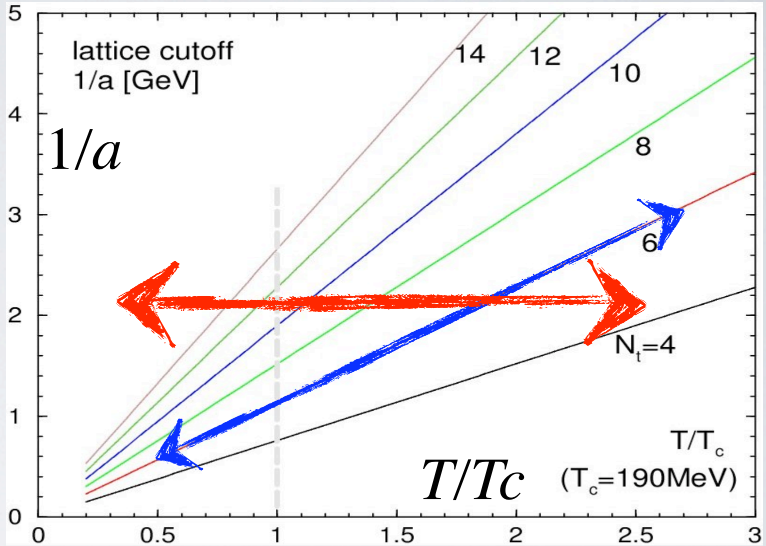
$$\frac{p}{T^4} = \int_{T_0}^{T} dT\, \frac{\epsilon - 3p}{T^5}$$

↖ such that $p(T_0) \approx 0$

numerical integration in *T*

Note: *T* can be precisely determined up to the common overall scale *a*.

FIXED SCALE APPROACH + T-INTEGRAL METHOD



Pros and cons:

- A common T=0 simulation enough for all T=0 subtractions.
  - We can even borrow publicly available high statistic configurations on ILDG
- Automatically on a LCP w/o fine tuning.

→ Most of the T=0 simulation costs removable.

- The resolution in *T* is limited because *Nt* is discrete. => No problem for EOS (see below)

- At high T: (T>2-3Tc)
  - *Nt* too small
  - Keep the lattice volume large.
- Around Tc :
  - More costs due to larger *Nt*.
  - Keep the lattice spacing small.

↔ fixed *Nt* approach

powerful at high T
coarse at low T

complementary!

Note: Finite volume effects when *Ns*/*Nt* is small (physical effects).

TEST OF THE METHOD

Phys. Rev. D79, 051501 (2009) [arXiv:0809.2842]

quenched QCD (w/o dynamical quarks)

Set	β	ξ	N_s	N_t	r_0/a_s	a_s(fm)	L(fm)	$a(dg^{-2}/da)$
i1	6.0	1	16	3–10	$5.35(^{+2}_{-3})$	0.093	1.5	−0.098 172
i2	6.0	1	24	3–10	$5.35(^{+2}_{-3})$	0.093	2.2	−0.098 172
i3	6.2	1	22	4–13	7.37(3)	0.068	1.5	−0.112 127
a2	6.1	4	20	8–34	5.140(32)	0.097	1.9	−0.107 04

$Tc \approx 290$ MeV

i1, i2: $N_t \approx 7$–8

i3: $N_t \approx 10$

a2: $N_t \approx 10$

borrowing the results of previous $T = 0$ simulations

$\xi = a_s/a_t$

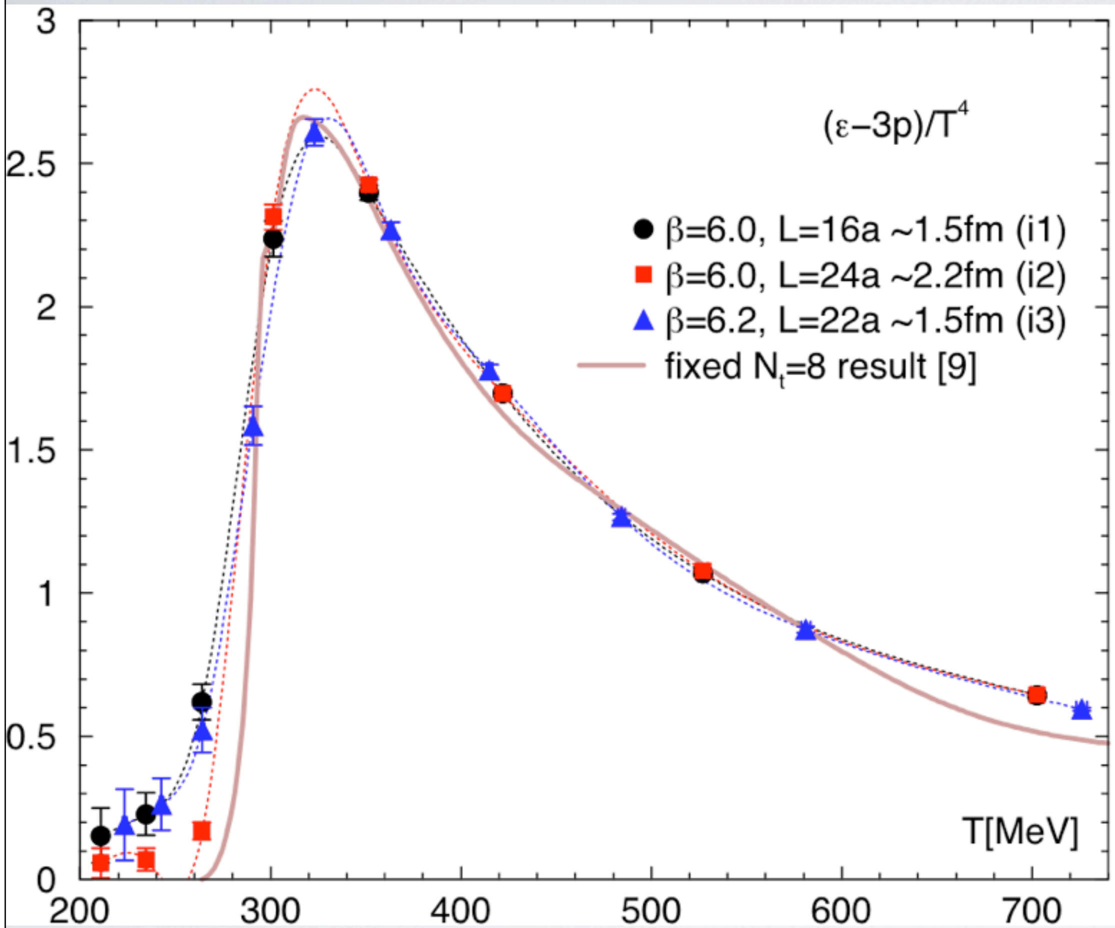
$T \sim 200$-700 MeV

(240-1010 MeV for a2)

- Lattice cutoff effects <= i1 vs. i3
- Spatial volume effects at low T <= i1 vs. i2
- Small Nt artifacts at high T <= i2 vs. a2 / conventional method
- Error from interpolation of T <= i2 vs. a2

TEST(1): ISOTROPIC LATTICES

Trace anomaly



☑ i1 vs. i3

⇒ Lattice cutoff effs. negligible at all T.

(i1 sufficiently fine.)

☑ i1 vs. i2

⇒ Spatial vol. effects near T_c.

(Physical effects)

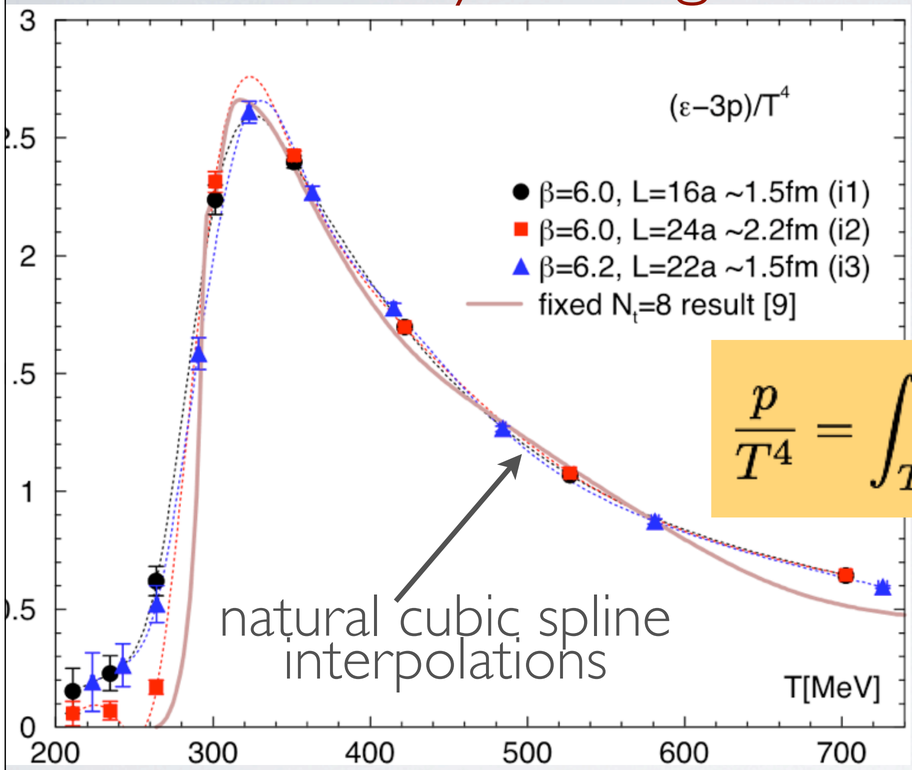
For higher T, spatial size of 1.5fm sufficient.

fixed Nt method
Nt=8, Ns = 32
L ~ 2.7 fm at Tc
Boyd et al. NP B469 ('96)

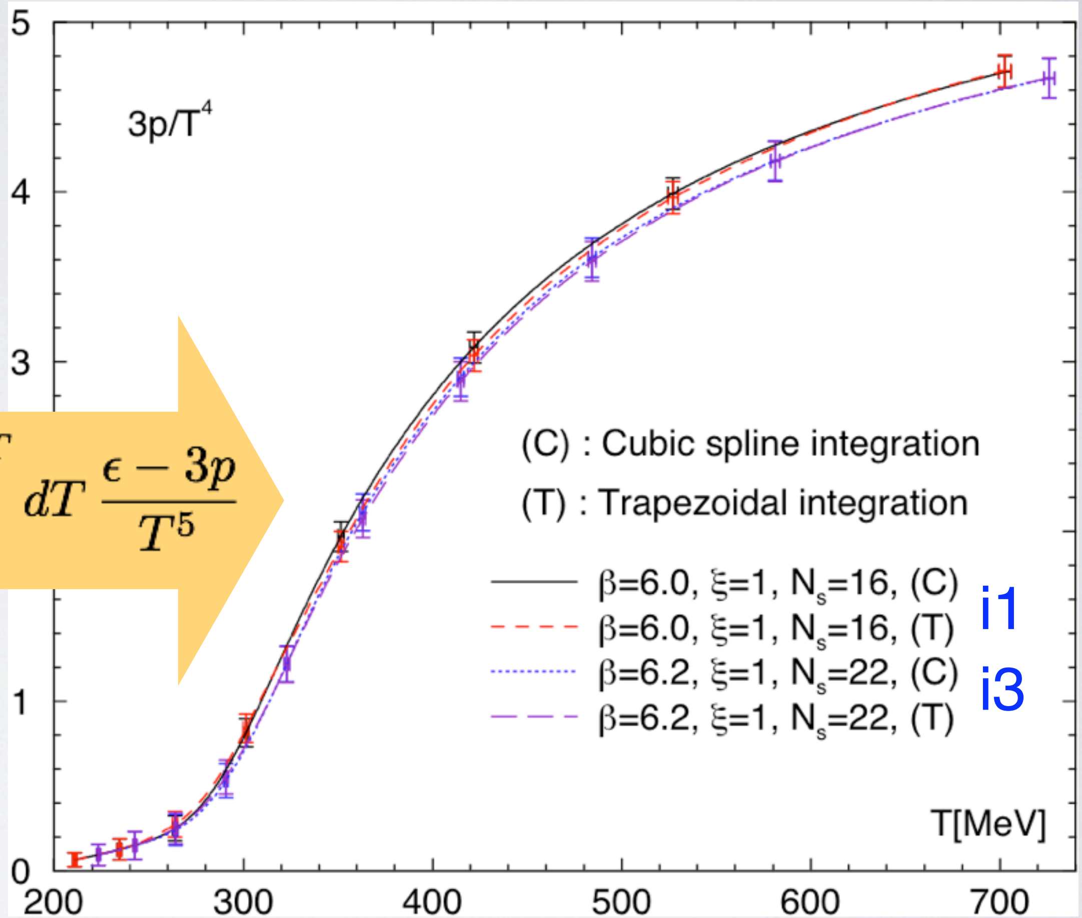
☑ i2 vs. fixed Nt=8 result ⇒ Consistent for $T \lesssim 600$ MeV

TEST(I): ISOTROPIC LATTICES

Pressure by T-integration



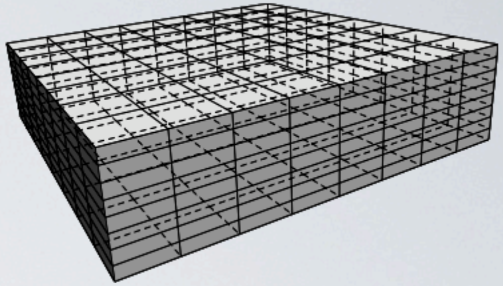
$$\frac{p}{T^4} = \int_{T_0}^{T} dT\, \frac{\epsilon - 3p}{T^5}$$



- ☑ Sufficient points to interpolate.
- ☑ Trapezoidal interpolation leads to almost identical values.
  ⇒ Systematic errors due to the limited resolution in *T* are small.

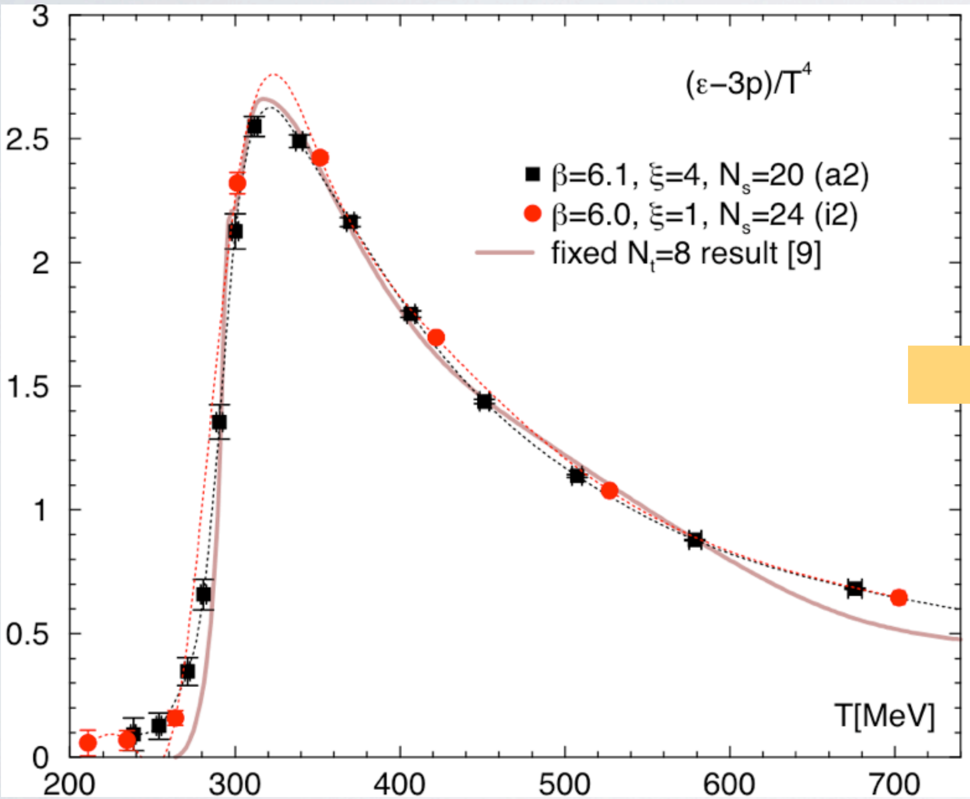
TEST(2): ANISOTROPIC LATTICE

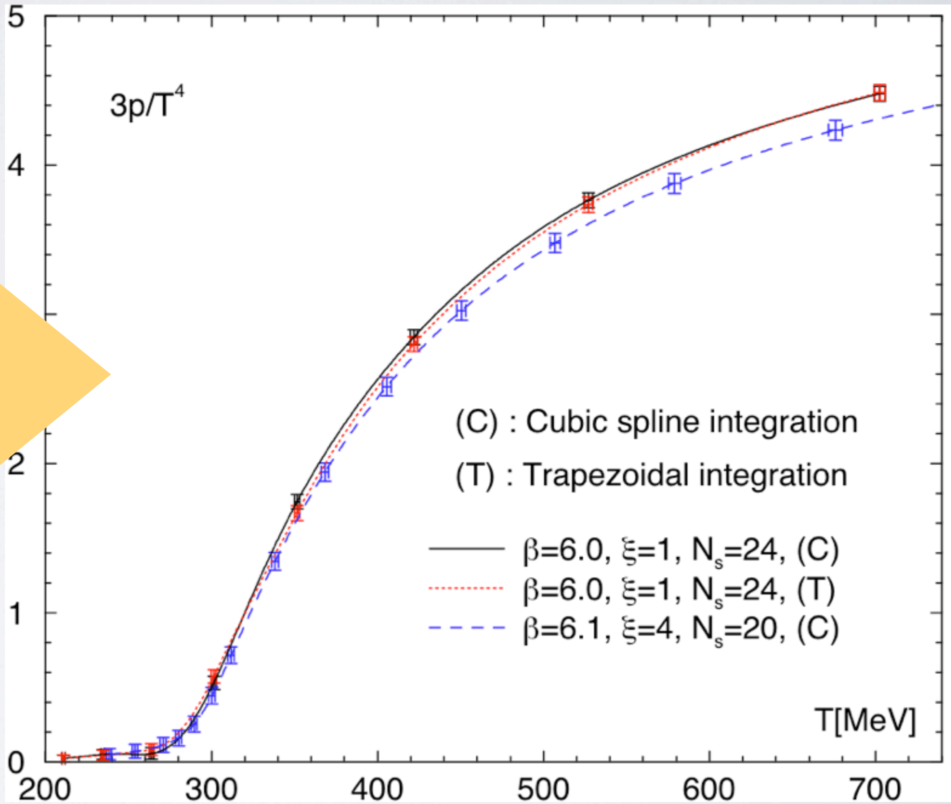
Another test by anisotropic lattice: **i2** vs. **a2**



$$\xi = a_s/a_t = 4$$

⇒ about 4 times finer resolution in $T = 1/N_t a_t$ at similar a_s.

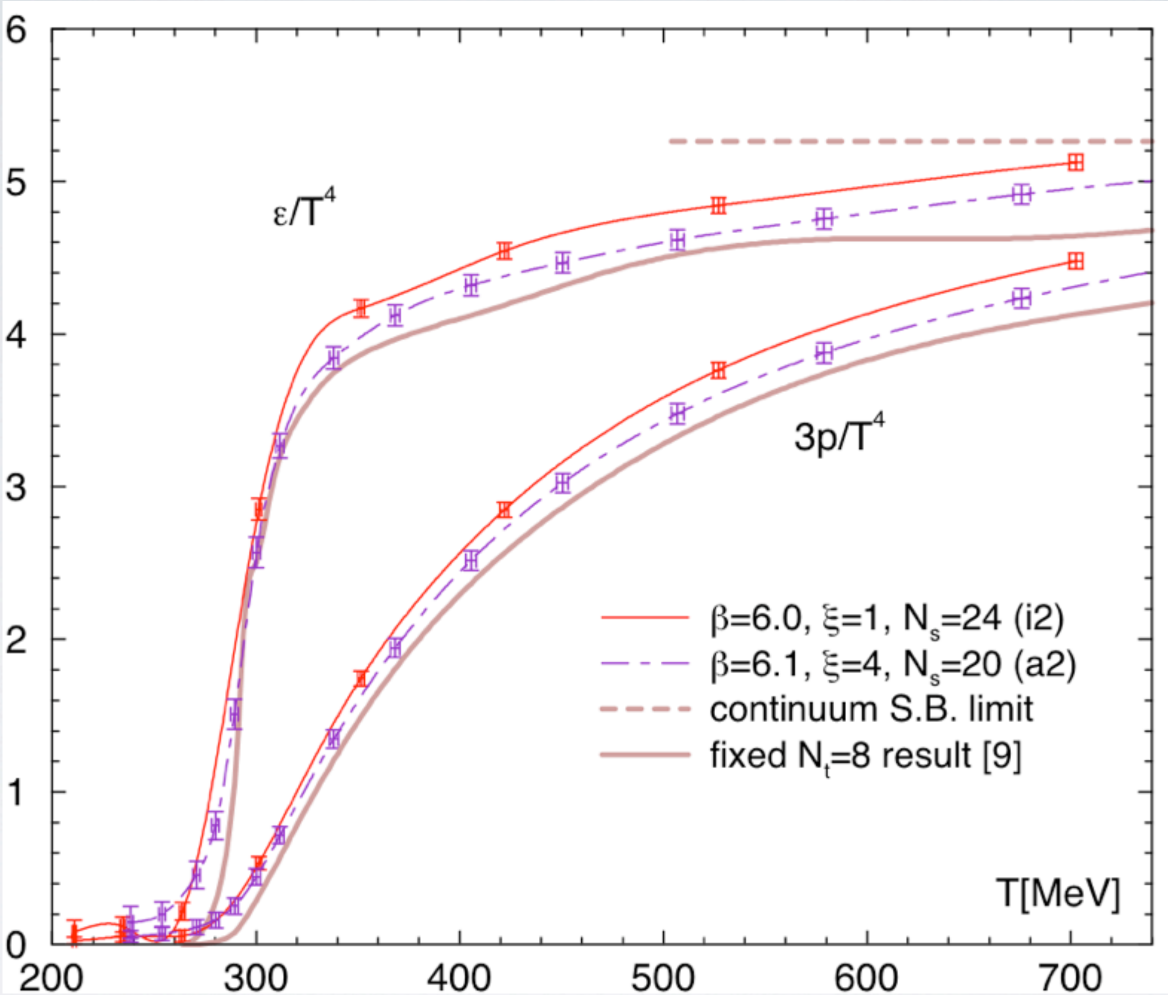




☑ a2 points on the i2 interpolation line. ☑ Well consistent up to ~600MeV.

⇒ Systematic errors due to the limited resolution in T are under control.

EOS



- ☑ EOS well consistent with each other.
- ☑ The fixed scale approach is capable to calculate EOS precisely.

SUMMARY

Fixed scale approach + T-integral method

- Vary T by varying N_t with all coupling parameters fixed.
- Evaluate the pressure non-perturbatively by $\frac{p}{T^4} = \int_{T_0}^{T} dT \, \frac{\epsilon - 3p}{T^5}$

- Can largely reduce the cost for T=0 simulations.
  - We can even borrow public configurations on ILDG
- Automatically on a LCP
- Our approach is complementary to the fixed *Nt* approach, and has advantages around Tc.

Tests in quenched QCD: promising

- Lattice cutoff effects small.
- Errors from the limited resolution in T controlable and small.
- Results ≈consistent with previous large scale study at fixed Nt=8.

EOS IN 2+1 FLAVOR QCD

N_f=2+1QCD simulation

based on a T=0 study by the CP-PACS+JLQCD Collaboration

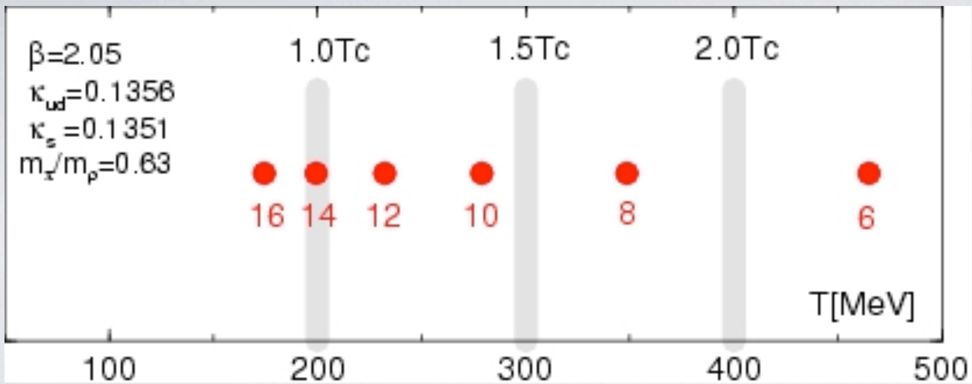
- RG-improved Iwasaki glue + NP clover-improved Wilson quarks
- $(2\ \text{fm})^3$ lattice, a=0.07, 0.1, 0.12 fm, exact PHMC algorithm for s, etc.
- configurations available on ILDG

PRD78,011502(08)

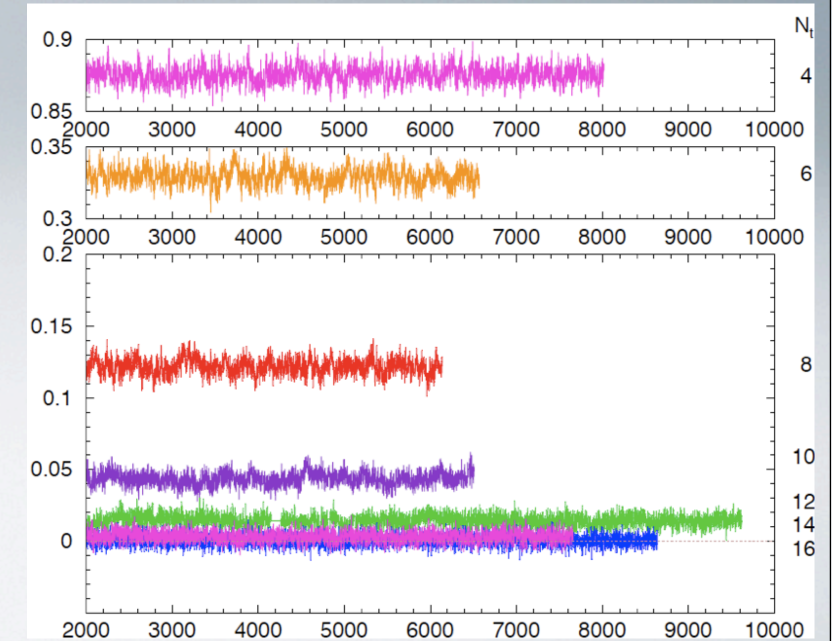
$$\beta = 2.05,\ a = 0.07\text{fm},\ m_\pi/m_\rho = 0.63,\ 28^3 \times 56,\ 6000\text{traj.}$$

T > 0 simulations

$$32^3 \times N_t, \quad N_t = 4,\ 6, \cdots,\ 16$$



Scales set by r_0=0.5 fm.

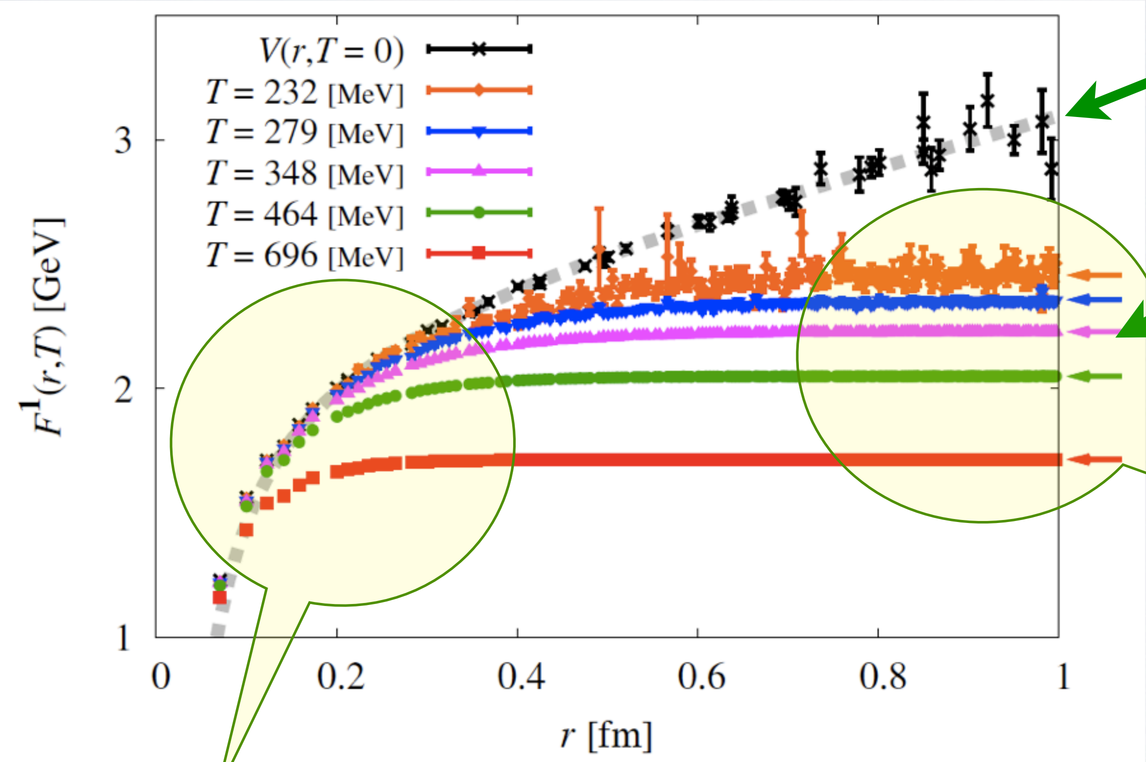


Polyakov loop history after thermalization
(simulation on-going)

[arXiv:0911.0254]

Heavy quark free energy at $T>T_c$

$$F^1(r,T) = -T \ln\langle \mathrm{Tr}\,\Omega^\dagger(\mathbf{x})\Omega(\mathbf{y})\rangle$$ in the Coulomb gauge



T=0 by CP-PACS+JLQCD

$$V(r) = -\frac{\alpha_0}{r} + \sigma r + V_0$$

$$2F_Q = -2T \ln\langle \mathrm{Tr}\,\Omega\rangle$$

2 x single quark free energy

F^1 --> $2F_Q$ at long distances.

i.e. deconfinement

F^1 --> $V(r) = F^1(T{=}0)$ at short distances.

- No vertical adjustment needed for F^1 in the fixed scale approach.

(cf.) F^1 -->$V(r)$ is used as an input to adjust the constant term of F^1 in the fixed Nt approach.

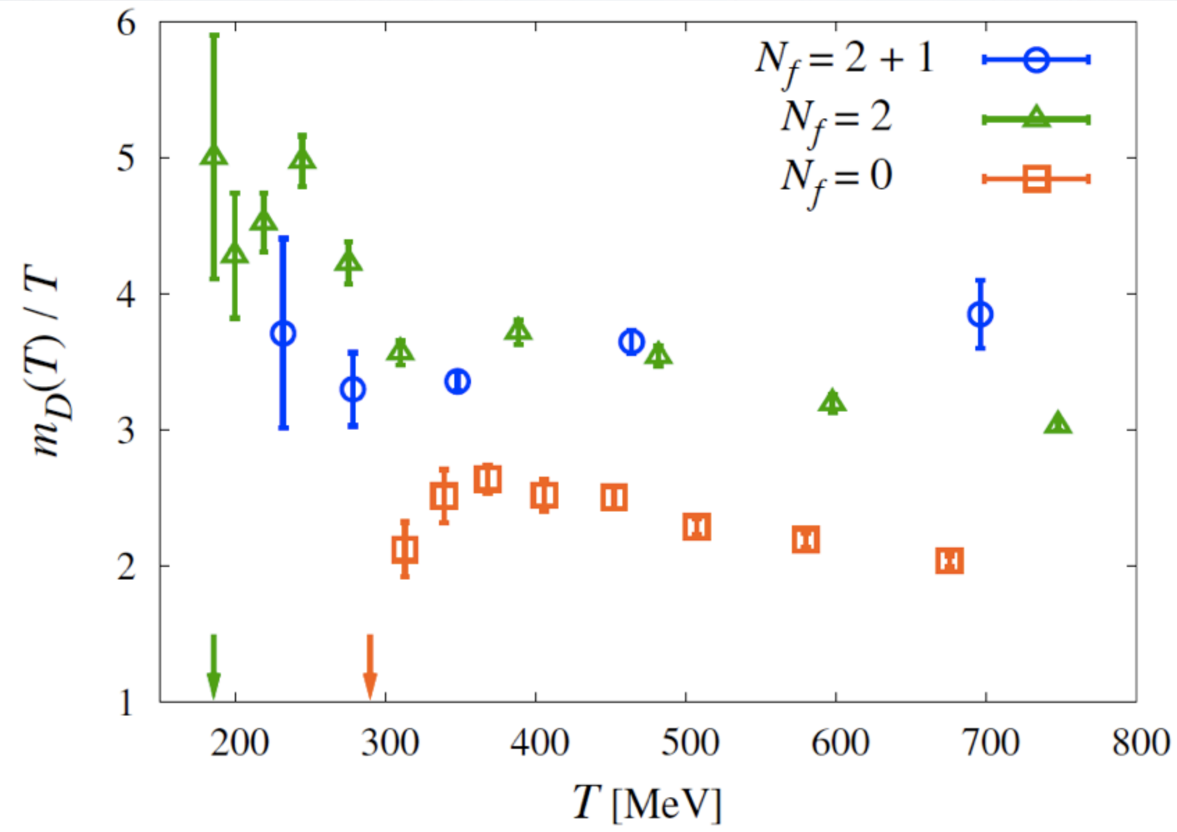
We have thus proved the T-insensitivity of F^1 at short distances.

Temperature effects down to shorter distances at higher T.

[arXiv:0911.0254]

- Debye screening mass at $T>T_c$

$$F^1(r,T) = -\frac{\alpha(T)}{r} e^{-m_D(T)\,r}$$



N_f=0: Umeda et al., PRD 79, 051501 (2009) fixed scale
N_f=2: Maezawa et al., PRD 75, 074501 (2007) fixed *Nt*

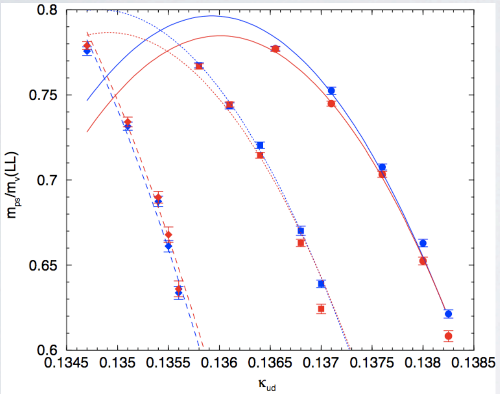
[arXiv:0910.5284]

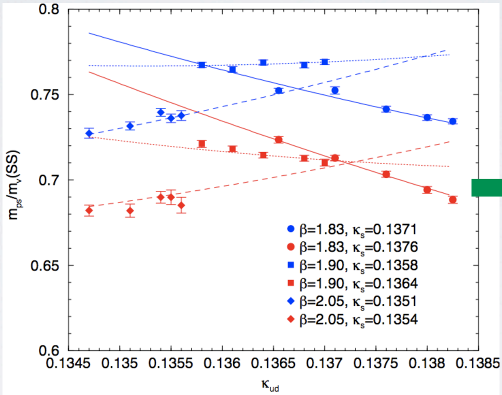
EOS (status report)

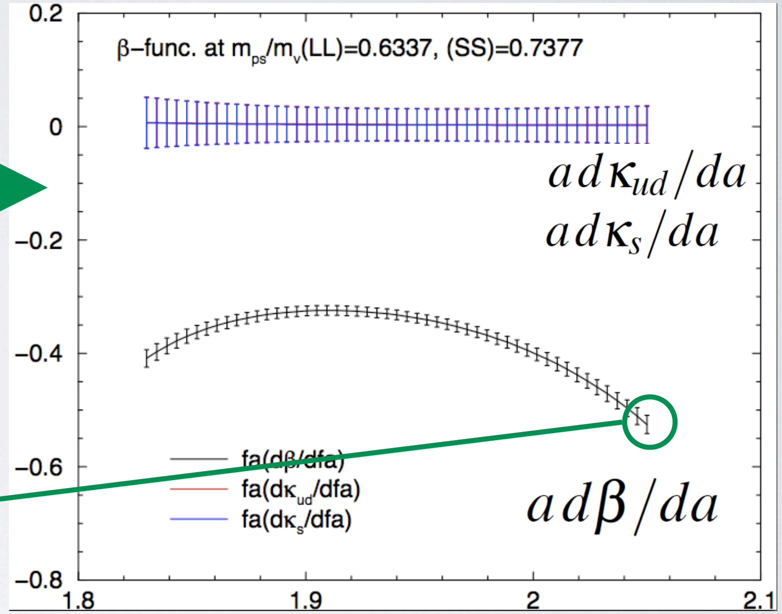
Beta functions not available for this lattice action yet.

=> We have to calculate by ourselves.

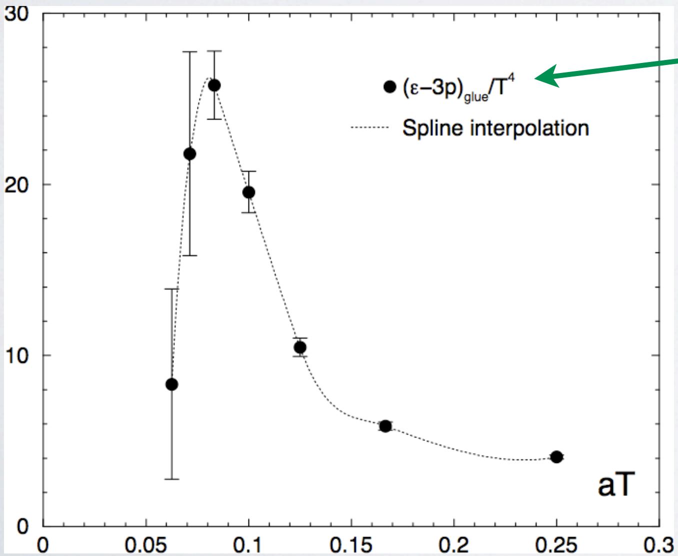
Inverse matrix method using the CP-PACS+JLQCD data for m_{PS}/m_V(ll), m_{PS}/m_V(ss)







Gluon contribution to the trace anomaly.



(cf.) N_f=2 case (Nt=4):

peak height of the trace anomaly ~ 13

gluon ~ 45, quark ~ –32

peak lower at larger Nt

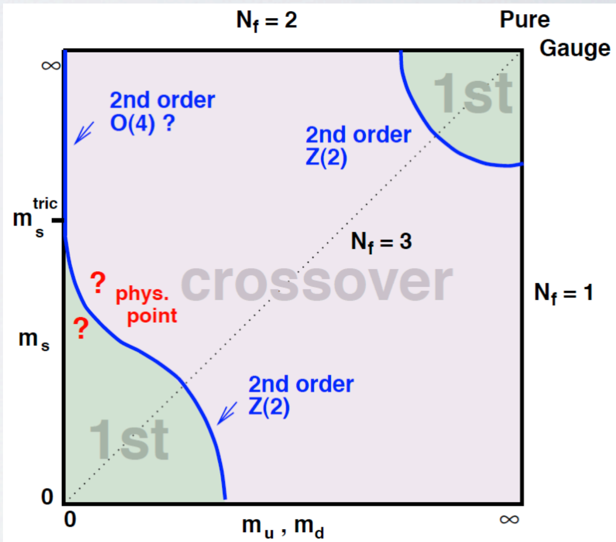
PERSPECTIVES

Beta functions

More precise data needed.
Reweighting method to directly calculate beta functions at the simulation point.

N_F=2+1QCD just at the physical point

T=0 configurations by the PACS-CS Collab. under accumulation.
T>0 simulations just at the reweighted point.



Finite density

We can combine our approach with the Taylor expansion method, to explore $\mu > 0$.

thank you!