Discussion with Prof. Blaizot

2009/11/25

Stability of Superfluid

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Abstract

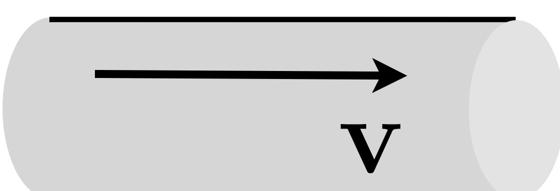
local density spectral function

$$\mathcal{I}_n(x,\varepsilon) = \sum_l |\langle l|\delta\hat{n}(x)|g\rangle|^2 \delta(\varepsilon - E_l + E_g)$$

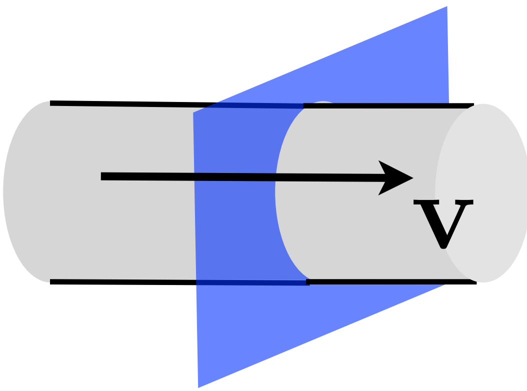
characterize the stability of superfluid by an exponent β

$$\mathcal{I}_n(\mathbf{r},\varepsilon) \propto \varepsilon^{\beta} \quad \text{for} \quad \varepsilon \longrightarrow 0$$

Landau instability



soliton instability

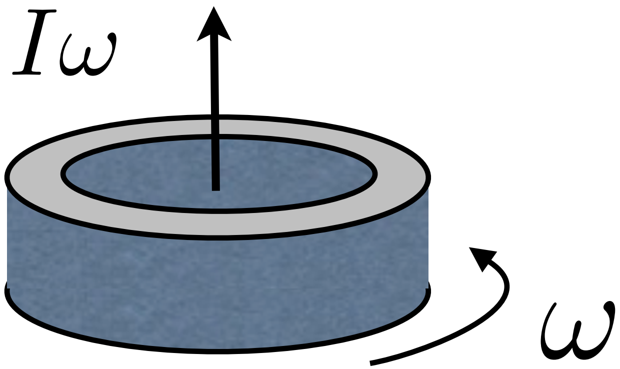


Superfluidity

non-classical
rotational inertia

(Leggett '71)

$$L = I\omega$$



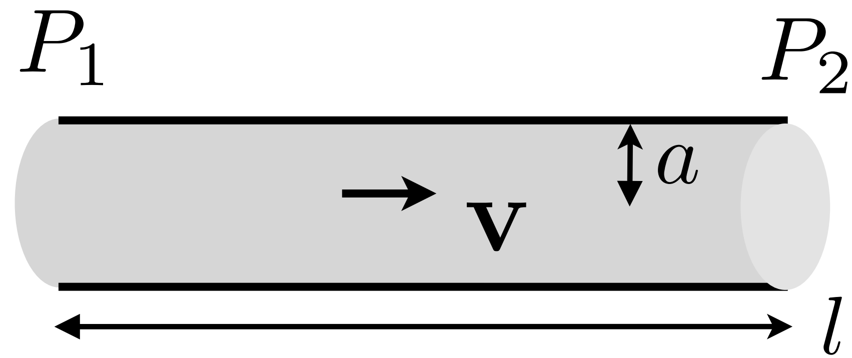
normal fluid

$$I = I_\mathrm{c}$$

superfluid

$$I = \frac{\rho_\mathrm{n}}{\rho} I_\mathrm{c} < I_\mathrm{c}$$

dissipationless flow



normal fluid

Hagen-Poiseuille flow

$$\mathbf{v} = \frac{a^2}{8l\eta}(P_1 - P_2)$$

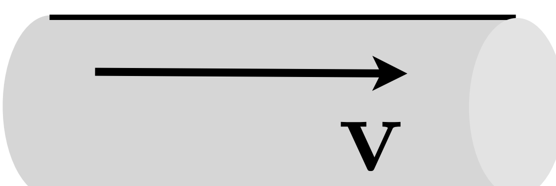
Landau's criterion

Landau (1941)

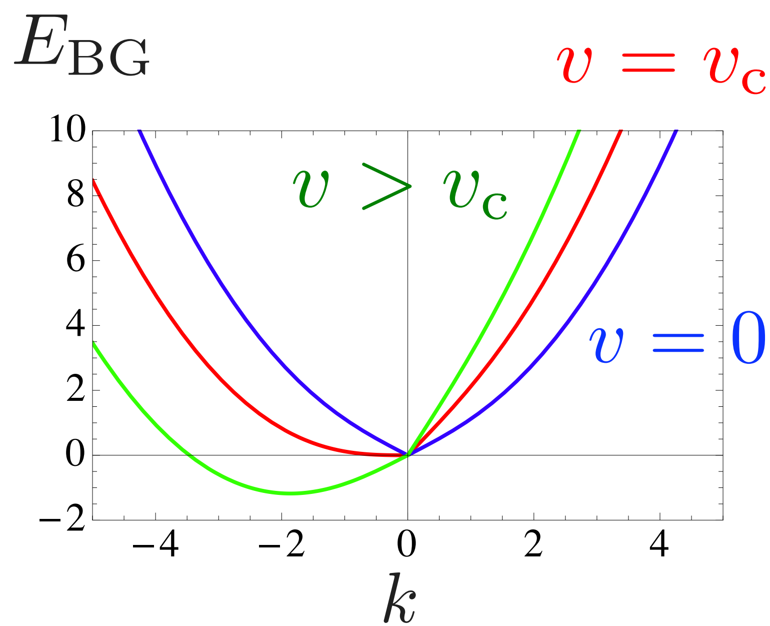
idea

Galilean transformation

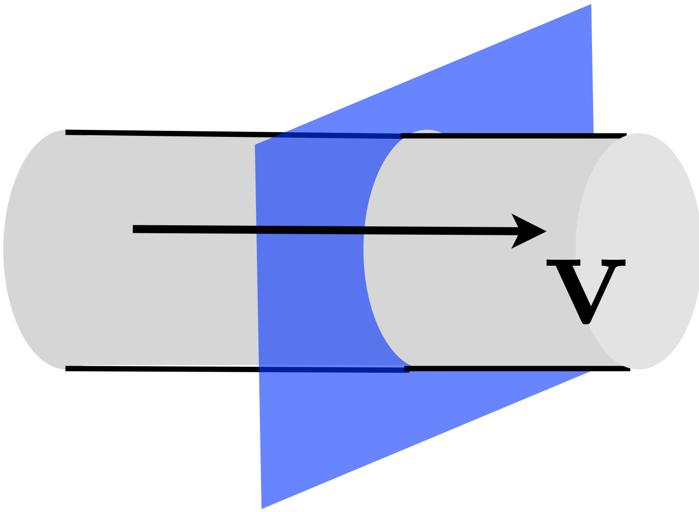
$$v_{\mathbf{c}} = \min\left(\frac{\varepsilon_{\mathbf{p}}}{p}\right)$$



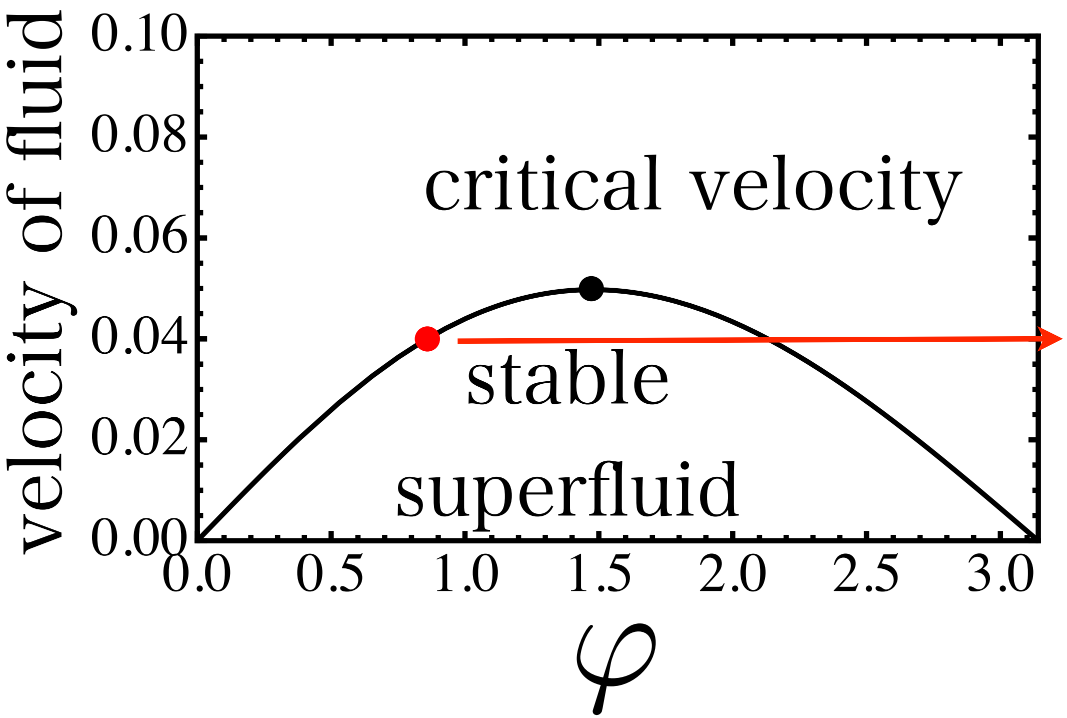
$$\varepsilon_p + \mathbf{p}\cdot\mathbf{v} < 0$$

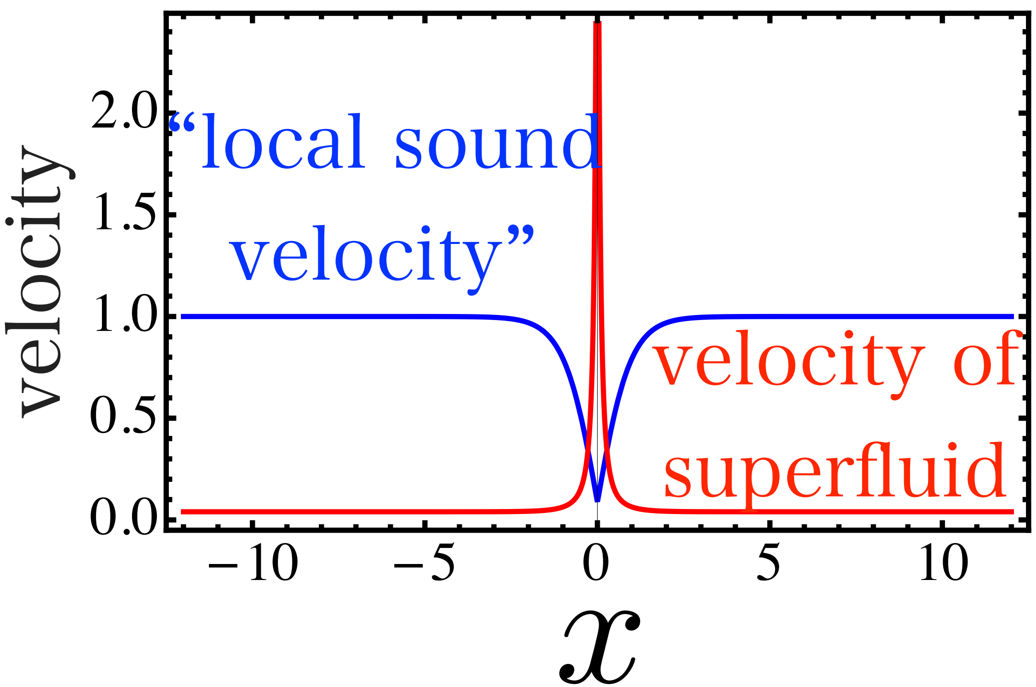


Local Landau's Criterion



Is the Landau's criterion applicable to inhomogeneous system?



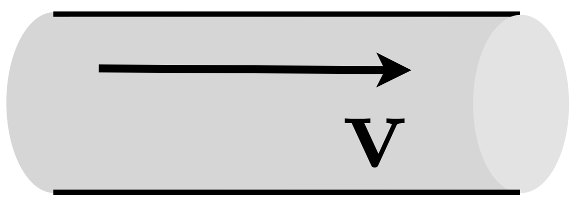


The answer is "NO".

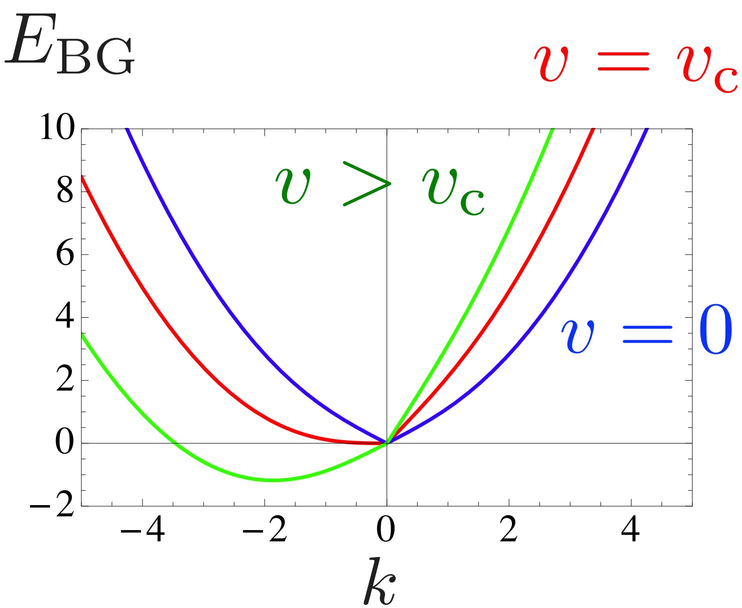
Emission of Excitations

Landau's criterion

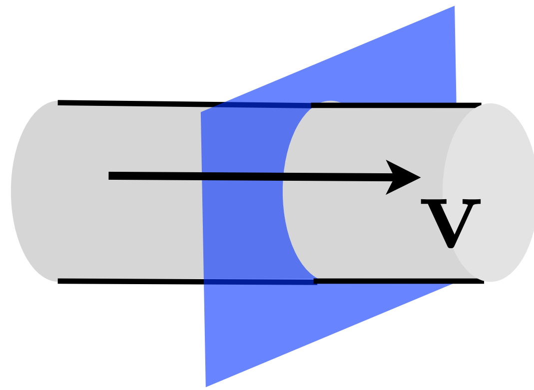
Landau (1941)

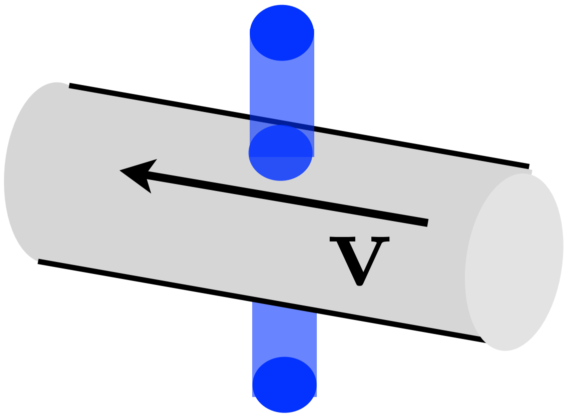


$$\varepsilon_p + \mathbf{p} \cdot \mathbf{v} < 0$$

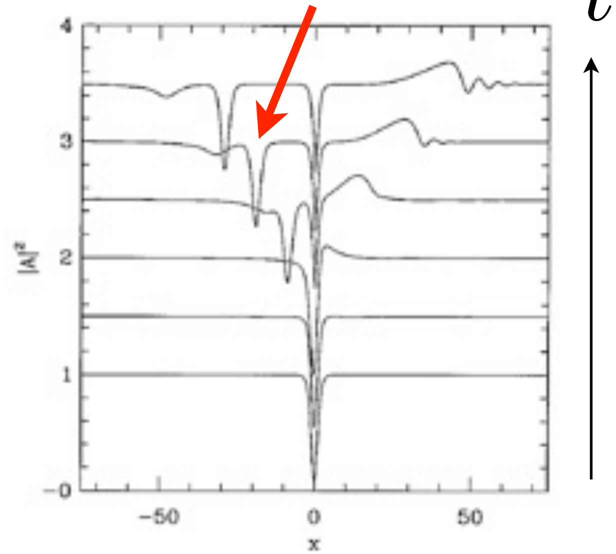


soliton and vortex emissions



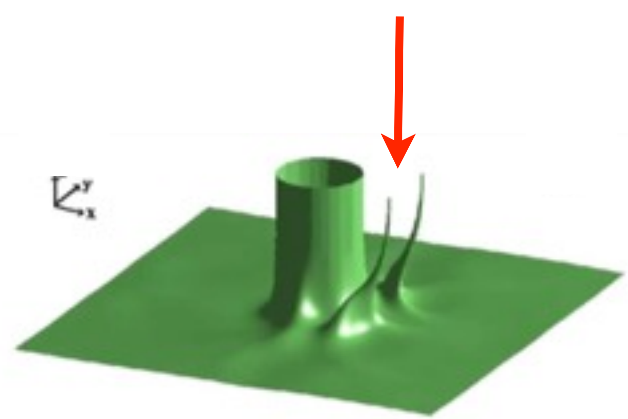


soliton



Hakim (1997)

vortex pairs



Aftalion (2003)

Critical Velocities in Cold Atoms

Raman (1999)

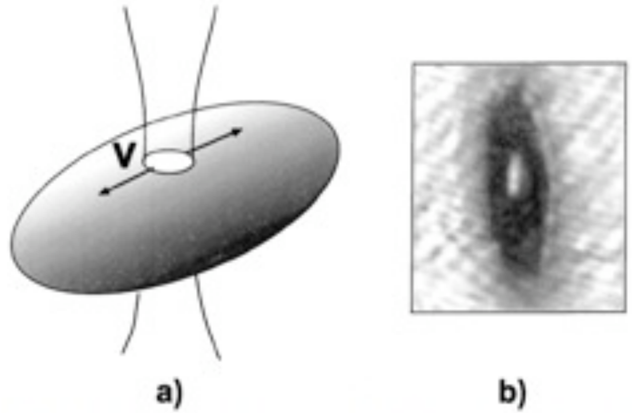


FIG. 1. Stirring a condensate with a blue detuned laser beam. (a) The laser beam diameter is 13 μm, while the radial width of the condensate is 45 μm. The aspect ratio of the cloud is 3.3. (b) *In situ* absorption image of a condensate with the scanning hole. A 10 kHz scan rate was used for this image to create the time-averaged outline of the laser trajectory through the condensate.

Onofrio (2000)

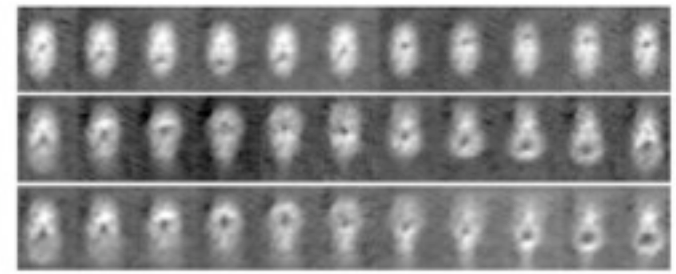
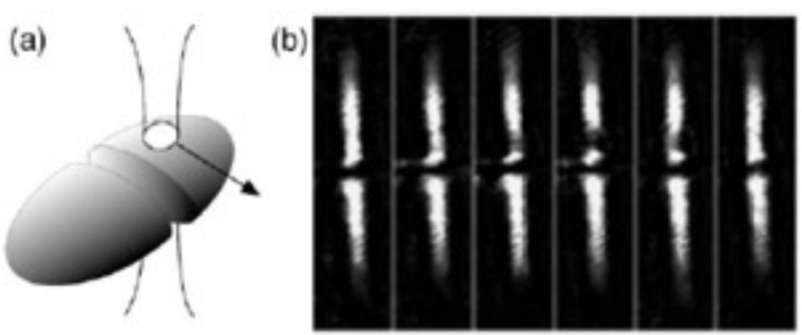
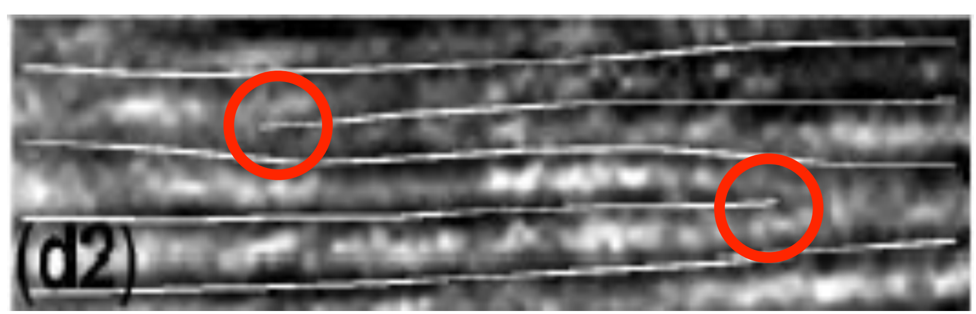


FIG. 1. Hydrodynamic flow in a Bose-Einstein condensate. Eleven phase contrast images of a condensate stirred for one cycle are taken *in situ* for various velocities of the laser beam. From top to bottom: 0.35, 5.6, and 9.0 mm/s. The exposure time is 300 μs per frame, and the frame rates are 33, 526, and 909 Hz, respectively. The maximum density and the corresponding sound velocity without the laser are 2×10^{14} cm^{-3} and 7 mm/s, respectively. The position of the laser beam is seen as a depletion of the condensate around its center. The vertical size of each image is ≈200 μm.

Inouye (2001)

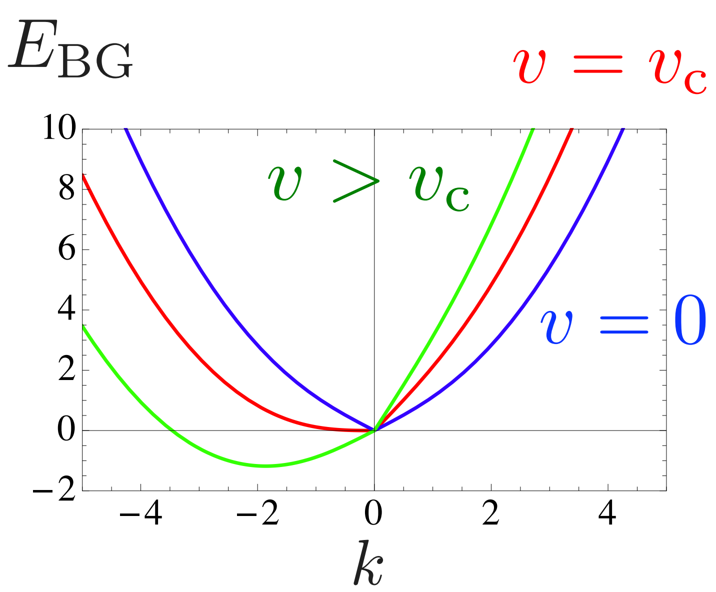


vortex observed by interference fringes



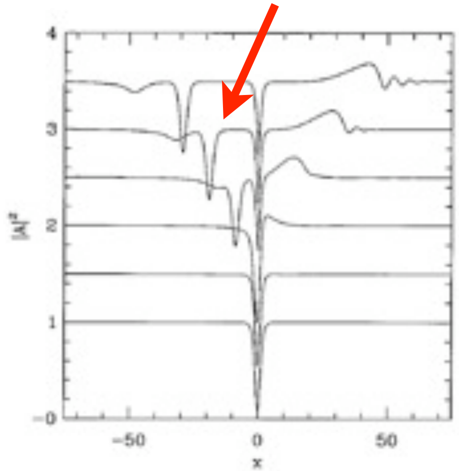
Aim

Landau's criterion



soliton emission

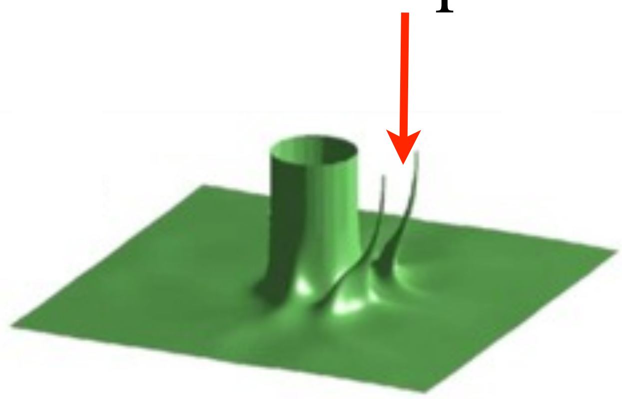
soliton



Hakim (1997)

vortex emission

vortex pair



Aftalion et al. (2003)

Is it possible to characterize these instability in a unified way?

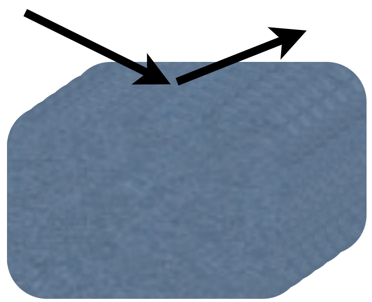
saddle node bifurcation
Rica (2001)

Proposal

Local Density Spectral Function

How to characterize the stability?

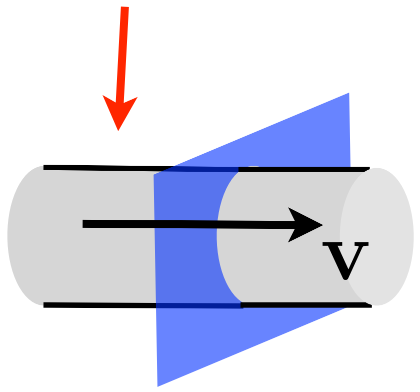
dynamical structure factor $S(\mathbf{q}, \omega)$



function of momentum

applicable to uniform system

local density spectral function $\mathcal{I}_n(\mathbf{r}, \varepsilon)$



function of position

allow to describe inhomogeneity

Local Density Spectral Function

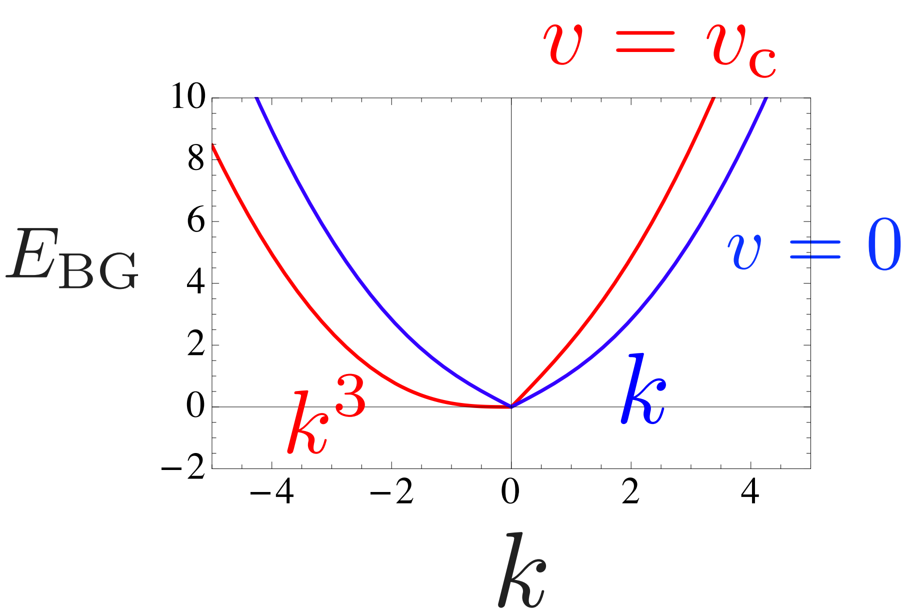
$$\mathcal{I}_n(x,\varepsilon) = \sum_l |\langle l|\delta\hat{n}(x)|g\rangle|^2 \delta(\varepsilon - E_l + E_g)$$

$$\delta\hat{n}(x) = \hat{n}(x) - \langle g|\hat{n}(x)|g\rangle$$

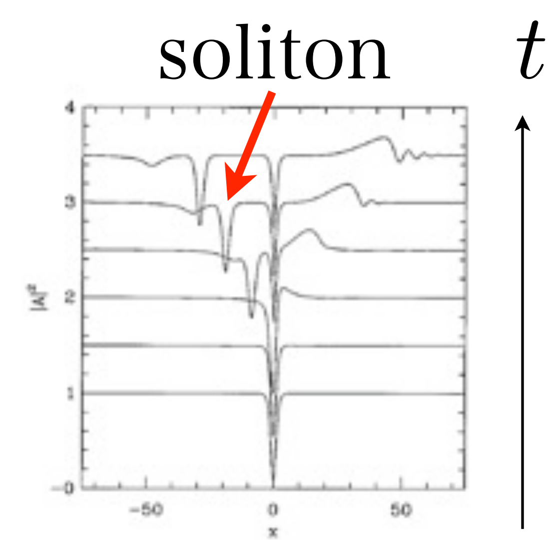
density of state

|

excitation spectrum



soliton emission (wavefunction)



cf. anomalous tunneling

Takahashi and Kato (2009)

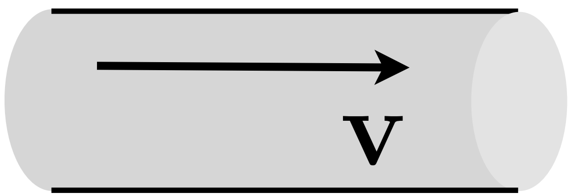
Results

weakly interacting Bose system
(Gross-Pitaevskii equation + Bogoliubov equation)

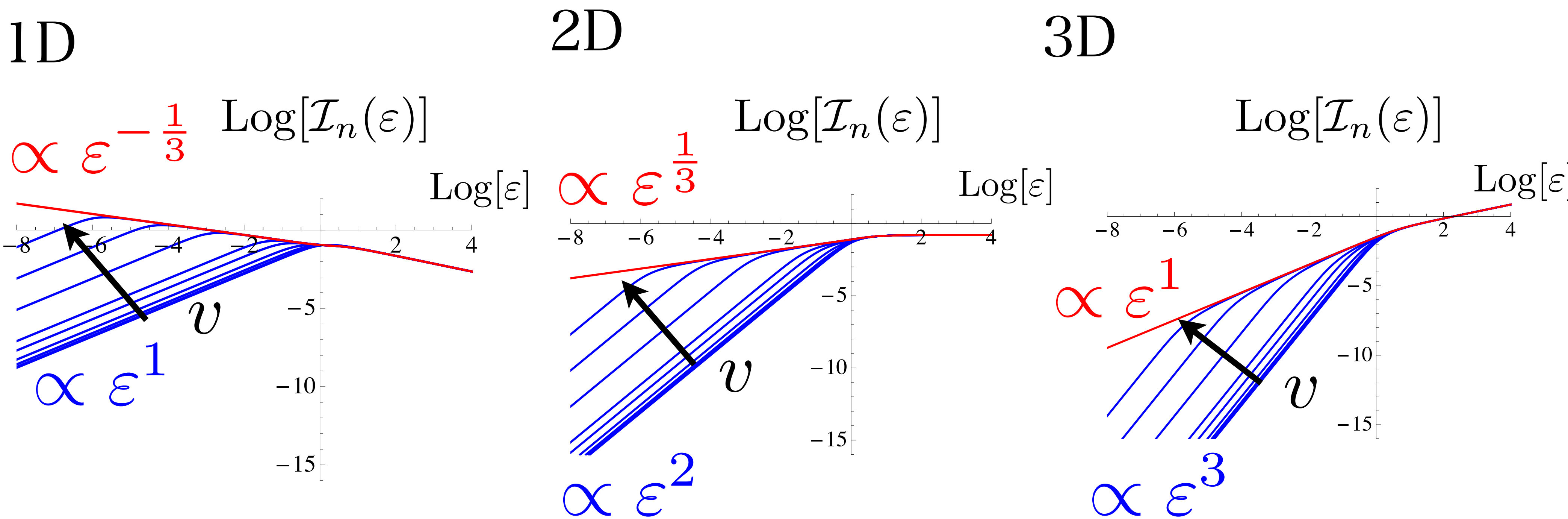
Landau instability

soliton instability

Landau instability

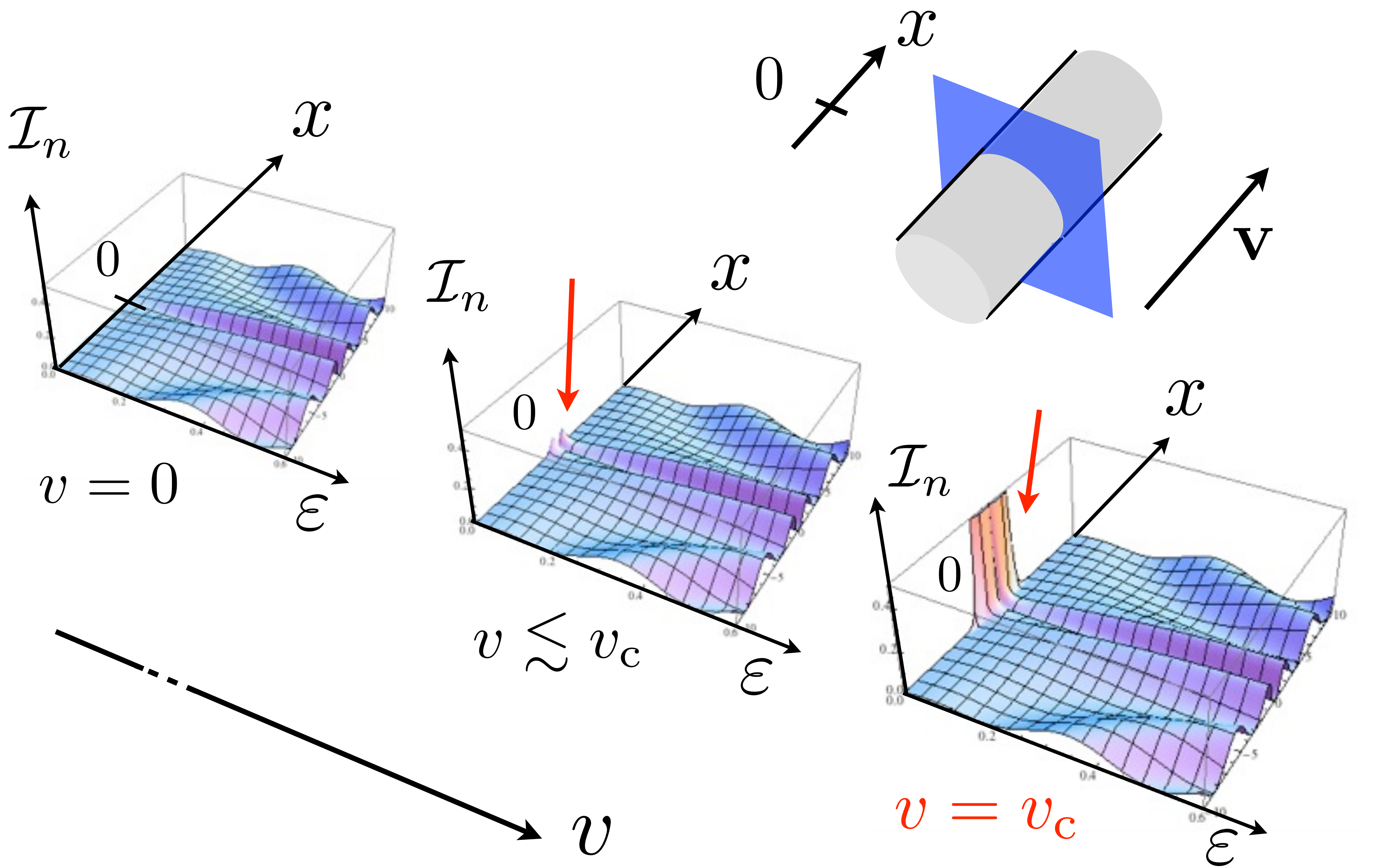


$$\mathcal{I}_n(\varepsilon) = \int \frac{d\mathbf{q}}{(2\pi)^3} S(\mathbf{q}, \varepsilon = \omega)$$



- increase of density fluctuation
- jump of exponent at the critical velocity

Soliton Instability



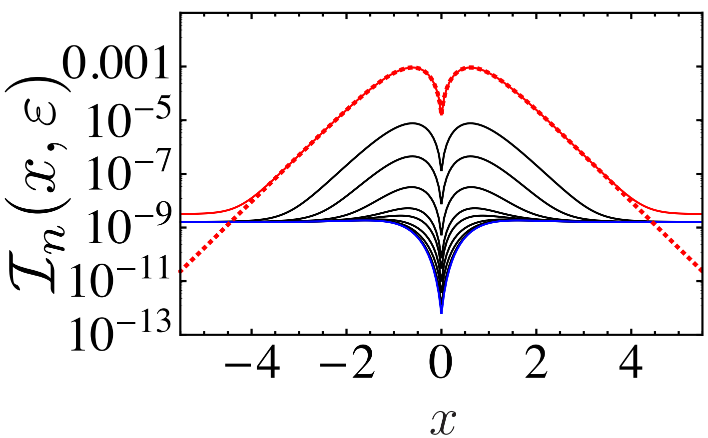
Profile of Spectral Function

$$\mathcal{I}_n(x,\varepsilon) \propto A^2(x) \left[\frac{\partial A(x)}{\partial \varphi} \right]^2$$

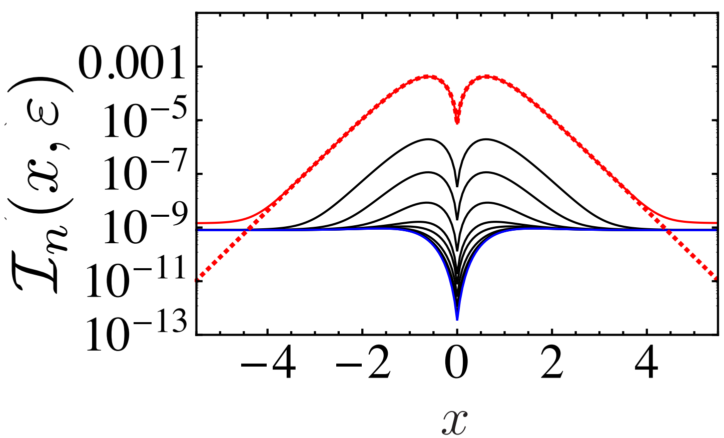
$A(x)$: amplitude of condensate wavefunction

φ : phase difference

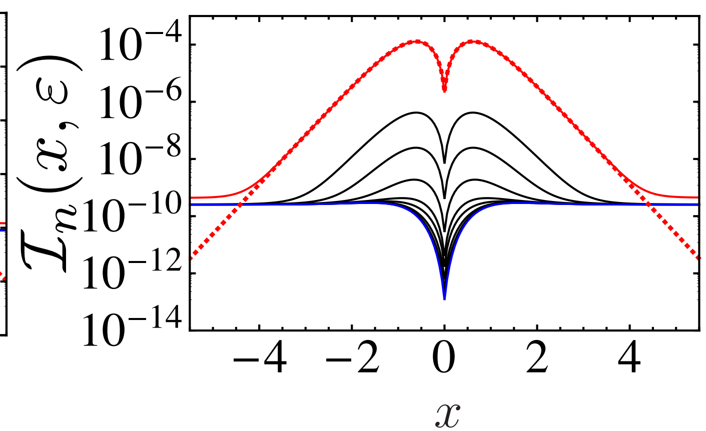
1D



2D

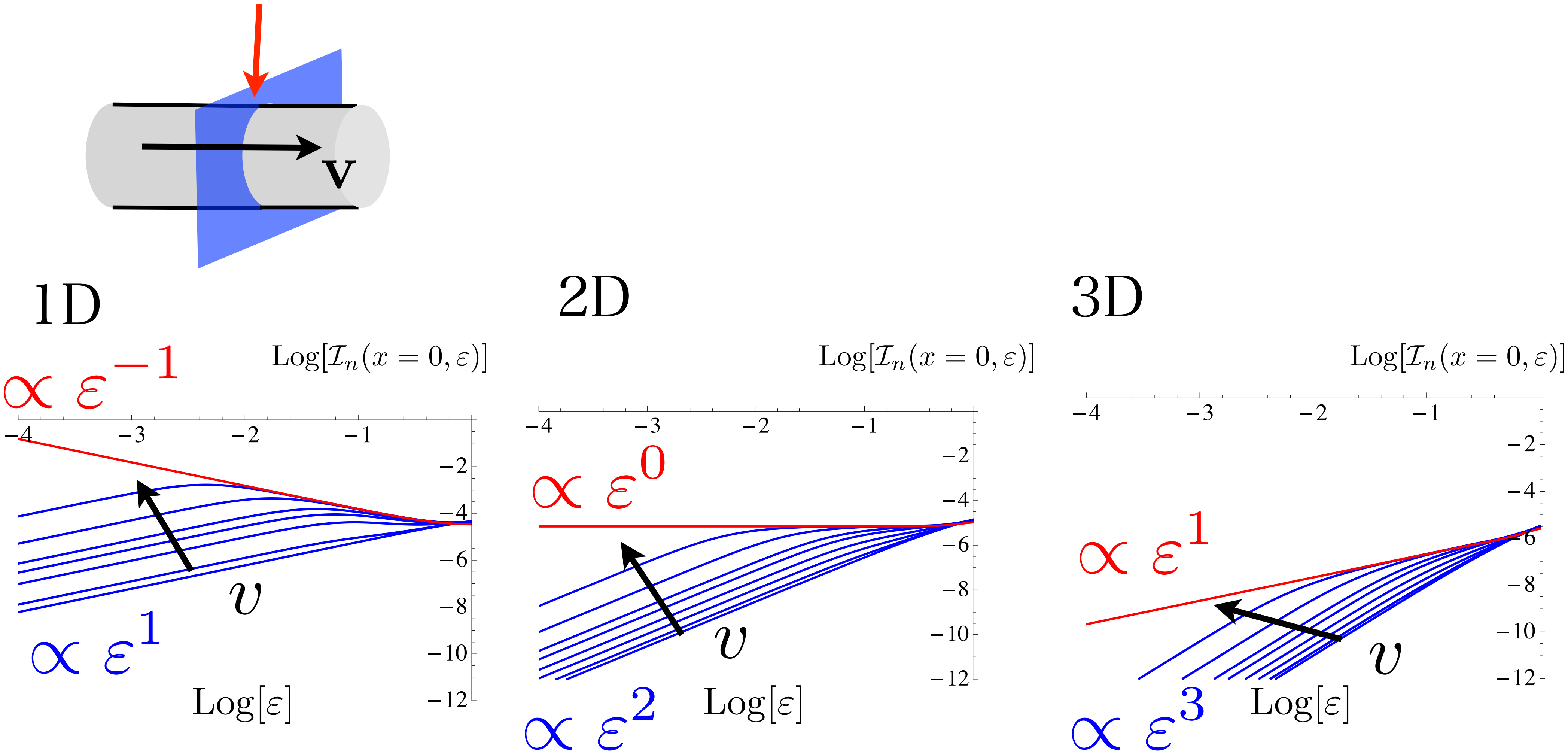


3D



- increase of density fluctuation
- prediction of profile at the critical current

Soliton Instability

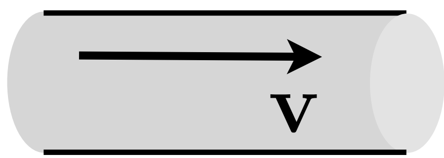
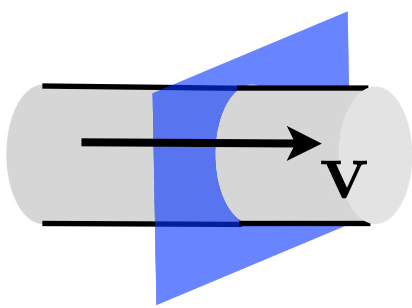


- increase of density fluctuation
- jump of exponent at the critical velocity

Exponent of Spectral Function

d dimension

When $\varepsilon \to 0$, $\mathcal{I}_n(x,\varepsilon) \propto \varepsilon^{\beta}$

	$v < v_c$	$v = v_c$
	$\beta = d$	Landau instability $\beta = -\frac{1}{3}, \frac{1}{3}, 1$
	$\beta = d$	soliton instability $\beta = -1, 0, 1$

$(d = 1, 2, 3)$

propose a new criterion

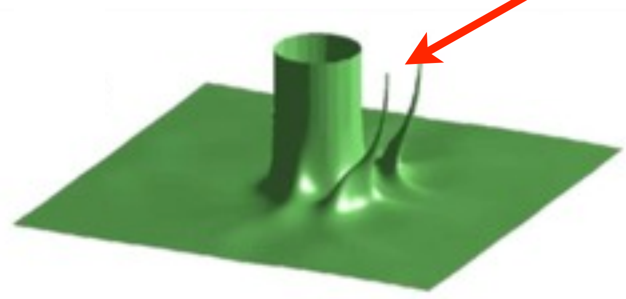
When $\varepsilon \to 0$, $\mathcal{I}_n(x,\varepsilon) \propto \varepsilon^{\beta}$ ⟹ $\beta = d$ $v < v_c$; $\beta < d$ $v = v_c$

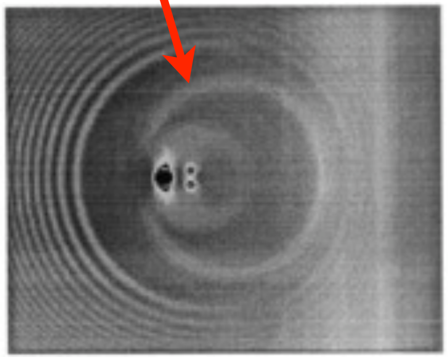
Outlook and Conclusion

Outlook

vortex pair

vortex emission





Conclusion

Unified characterization of Landau and soliton instability

<u>local density spectral function</u>

propose a new criterion for superfluid

d dimension When $\varepsilon \to 0$,

$\mathcal{I}_n(x, \varepsilon) \propto \varepsilon^{\beta}$ ⇒

$\beta = d$ $\quad v < v_{\mathrm{c}}$

$\beta < d$ $\quad v = v_{\mathrm{c}}$