Space-time picture of Schwinger mechanism

- Non-perturbative pair creation under a color electric field
- Time-evolution with back reaction
- Effects of a magnetic field

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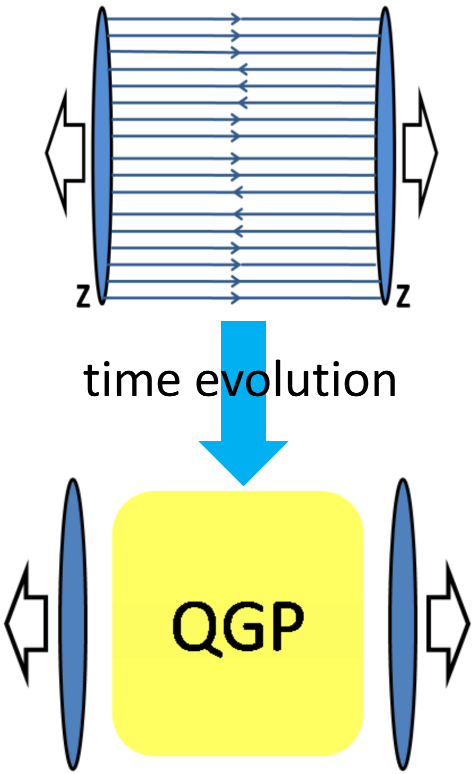
discussion session with Prof. Blaizot, 09.11.18

Initial stage of heavy-ion collision



• the Low-Nussinov model
• the color glass condensate

classical color electric fields
in the longitudinal direction

How classical fields evolve?

1. classical time-evolution
   • expansion of the system • instability • back reaction
2. quantum effects
   • non-perturbative pair creation (Schwinger mechanism)
   • interaction between quanta . . .

Need to describe the time-evolution of a system where classical fields and quantum fields interact with each other

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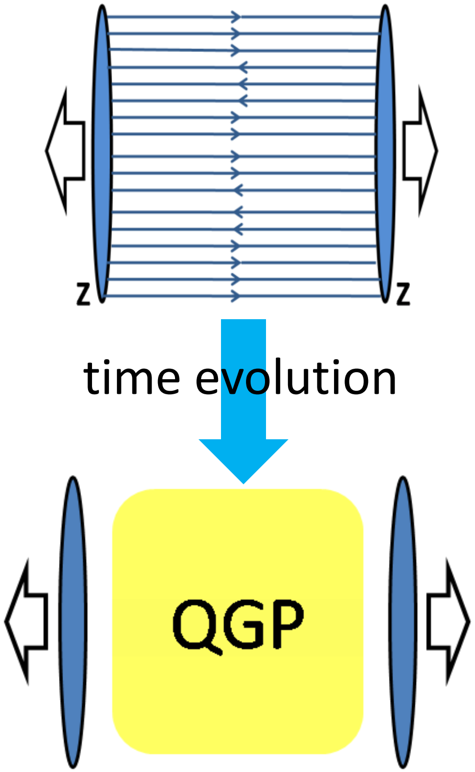

- the Low-Nussinov model
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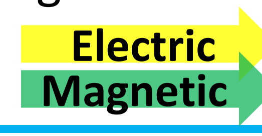
Also, longitudinal magnetic field



classical color electric fields in the longitudinal direction

How classical fields evolve?

1. classical time-evolution
   - expansion of the system
   - instability
   - **back reaction**
2. quantum effects
   - **non-perturbative pair creation (Schwinger mechanism)**
   - interaction between quanta . . .

Need to describe the time-evolution of a system where classical fields and quantum fields interact with each other

- Dynamical view of quark pair creation in uniform color electric fields
- Effects of color magnetic fields on pair creation

Formalism

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{\psi} \left(i\gamma^\mu D_\mu - m \right) \psi + \mathcal{L}_{\mathrm{GF+FP}}$$

Separate the gauge field $A^a_\mu = \bar{A}^a_\mu + \mathcal{A}^a_\mu$

classical field

quantum field

Eq. of motion for the quantum gauge field

$$\bar{D}^\nu \bar{D}_\nu \mathcal{A}^{a\mu} - 2g f^{abc} \bar{F}^{b\mu\nu} \mathcal{A}^c_\nu = 0$$

spin-electromagnetic field interaction

Eq. of motion for the quark field

$$\left(i\gamma^\mu \bar{D}_\mu - m \right) \psi = 0$$

$$\bar{D}_\mu = \partial_\mu + igT^a \bar{A}^a_\mu$$

Eq. of motion for the classical gauge field (“Maxwell” eq.)

$$\bar{D}_\nu \bar{F}^{a\mu\nu} + \langle J^{a\mu} \rangle = 0$$

current generated by quarks and gluons

Abelianization

Assume a spatially uniform color electric field

$$\mathbf{E}^a = \mathbf{E} n^a \Longleftarrow \bar{A}^a_\mu = \bar{A}_\mu n^a$$

a constant vector indicating a color direction of electromagnetic fields

diagonalize $T^a n^a$

$$\bar{D}_\mu \psi = (\partial_\mu + igT^a n^a \bar{A}_\mu)\psi = \left[\partial_\mu + ig\begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix}\bar{A}_\mu\right]\begin{pmatrix}\psi_1 \\ \psi_2 \\ \psi_3\end{pmatrix}$$

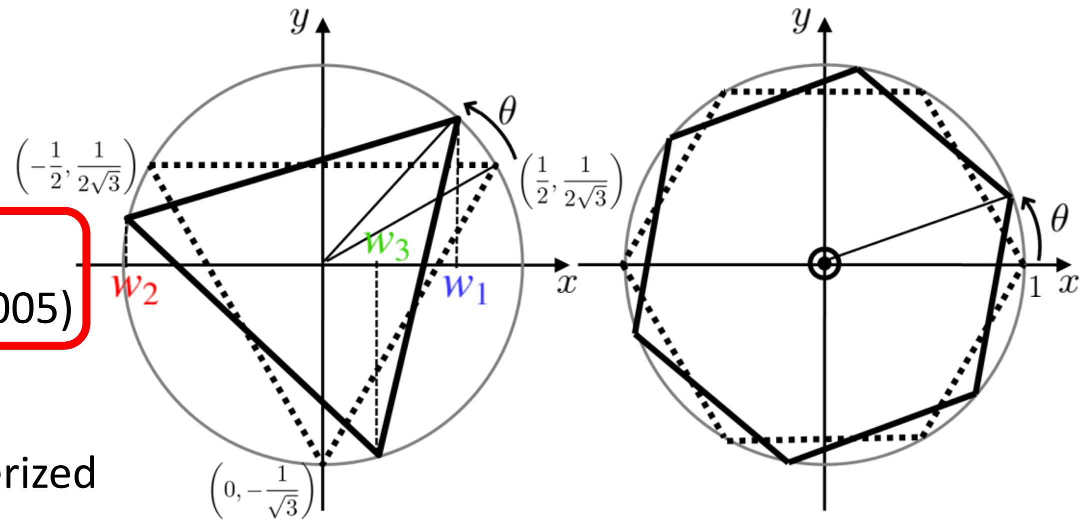
Each quark field ψ_i couples to the Abelianized gauge field $\bar{A}_\mu$ via coupling $w_i g$.

Relation between w_i and n^a

$$\sin^2 3\theta = 3(d^{abc} n^a n^b n^c)^2$$

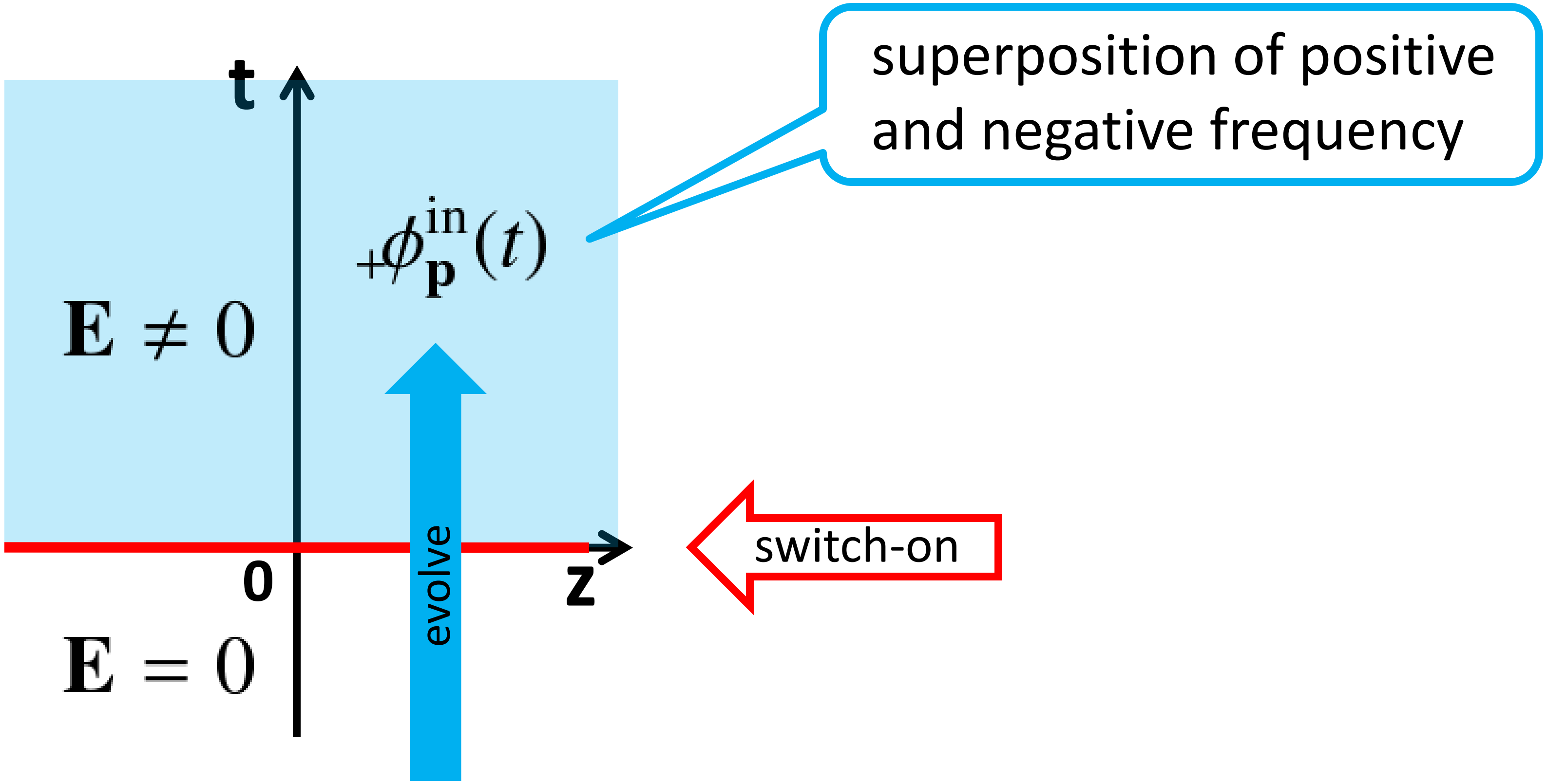
gauge (Casimir) invariant quantity characterizing a color direction (Nayak 2005)

There are two independent directions in SU(3) color space, which are parameterized by $E^a E^a$ and θ in a gauge-invariant way.



rotating weight/root diagram $(0^\circ \le \theta \le 30^\circ)$

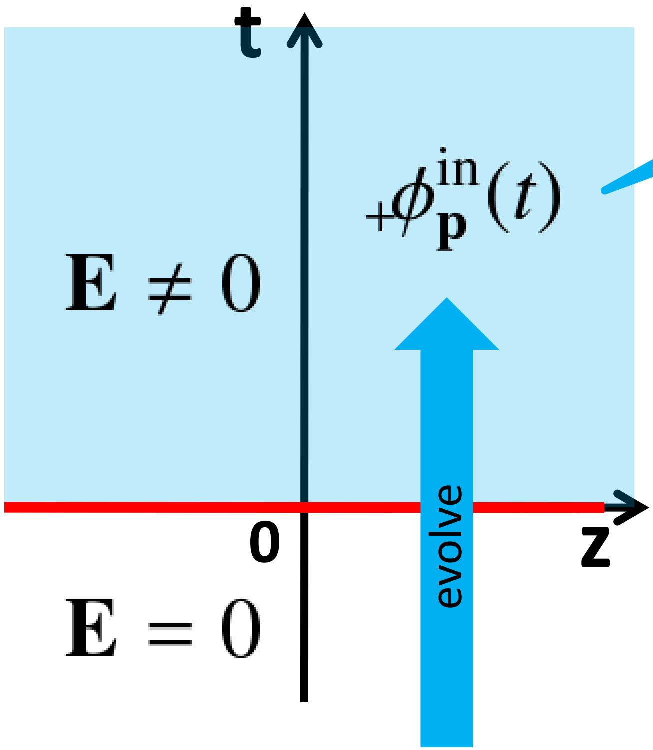
Quantization in background fields



$$_{+}\phi^{\text{in}}_{\mathbf{p}}(t) = \frac{1}{\sqrt{2\omega_p}} e^{-i\omega_p t}$$

positive frequency

Quantization in background fields



$$_{+}\phi^{\rm in}_{\mathbf{p}}(t) = \frac{1}{\sqrt{2\omega_p}} e^{-i\omega_p t}$$

superposition of positive and negative frequency

time-dependent Bogoliubov transformation

$$a_{\mathbf{p}}(t) = \alpha_{\mathbf{p}}(t) a^{\rm in}_{\mathbf{p}} + \beta_{\mathbf{p}}(t) b^{\rm in \dagger}_{-\mathbf{p}}$$

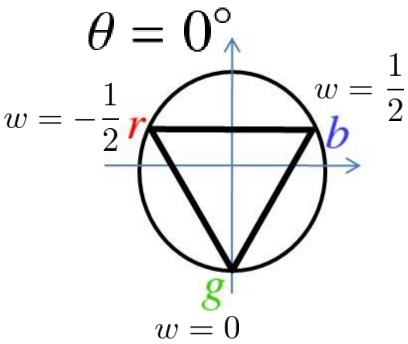
$$b^{\dagger}_{-\mathbf{p}}(t) = \alpha^{*}_{\mathbf{p}}(t) b^{\rm in \dagger}_{-\mathbf{p}} + \beta^{*}_{\mathbf{p}}(t) a^{\rm in}_{\mathbf{p}}$$

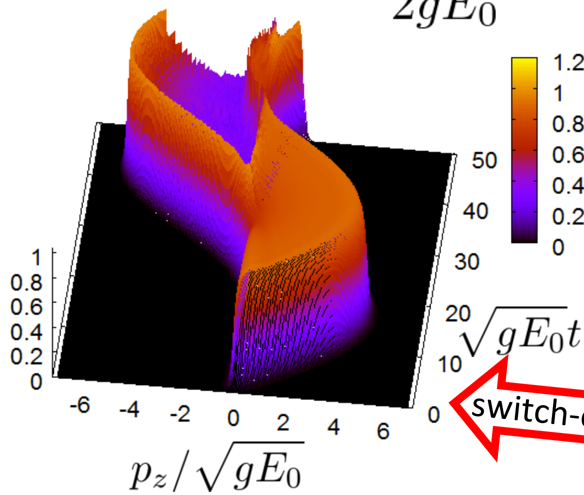
$$f_{\mathbf{p}}(t) = \langle 0, {\rm in} | a^{\dagger}_{\mathbf{p}}(t) a_{\mathbf{p}}(t) | 0, {\rm in} \rangle \frac{(2\pi)^3}{V}$$

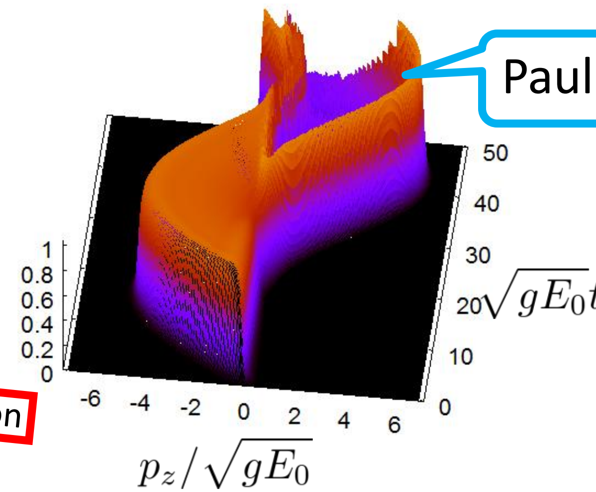
is non-zero.

Numerical result (quark pair creation) $g = 1$

$$a = \frac{m^2}{2gE_0} = 0.01 \quad (E_0 \text{ : initial field strength})$$





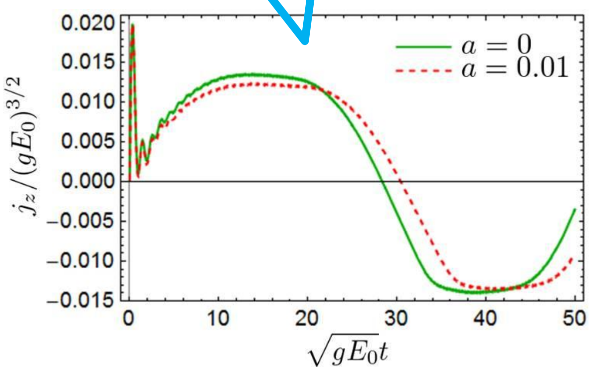


0

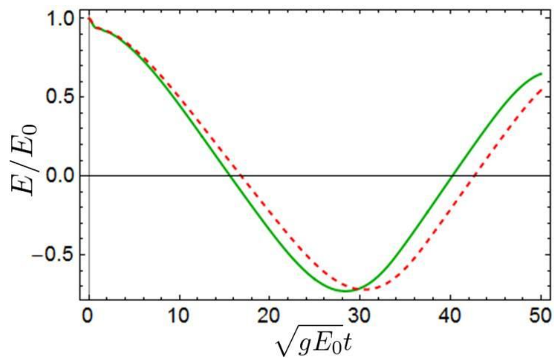
g

longitudinal momentum distribution $(p_T = 0)$

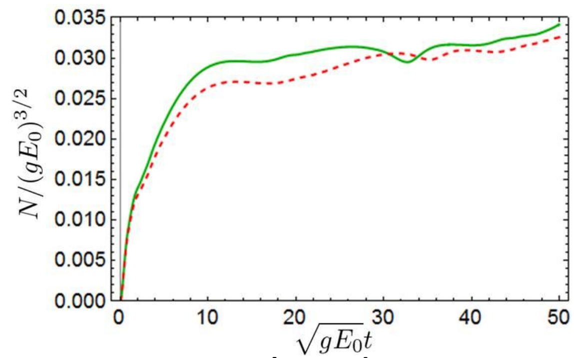
plasma oscillation



current density



electric field

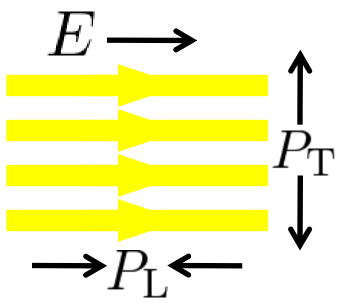


number density

Pressure

The initial state with the longitudinal electric field is quite anisotropic.

$$P_{\rm L} = -E^2/2 \qquad P_{\rm T} = E^2/2$$



In local thermalized QGP, pressure must be isotropic.
How does the system become isotropic?

It is worthwhile to investigate how pressure evolves due to pair creation, although full isotropization cannot be achieved in our treatment.

energy-momentum tensor

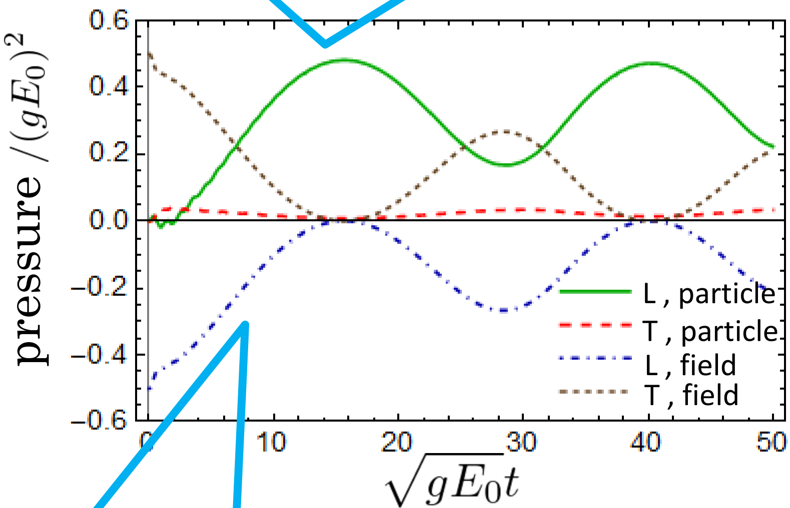
$$\begin{aligned}\langle \Theta^{\mu\nu} \rangle &= -\frac{i}{4} \langle 0, \mathrm{in} | \bar{\psi} \left(\gamma^{\mu} \overleftrightarrow{\partial}^{\nu} + \gamma^{\nu} \overleftrightarrow{\partial}^{\mu} \right) \psi | 0, \mathrm{in} \rangle \\ &= \mathrm{diag}\left(\mathcal{E}, P_{\rm T}, P_{\rm T}, P_{\rm L} \right)\end{aligned}$$

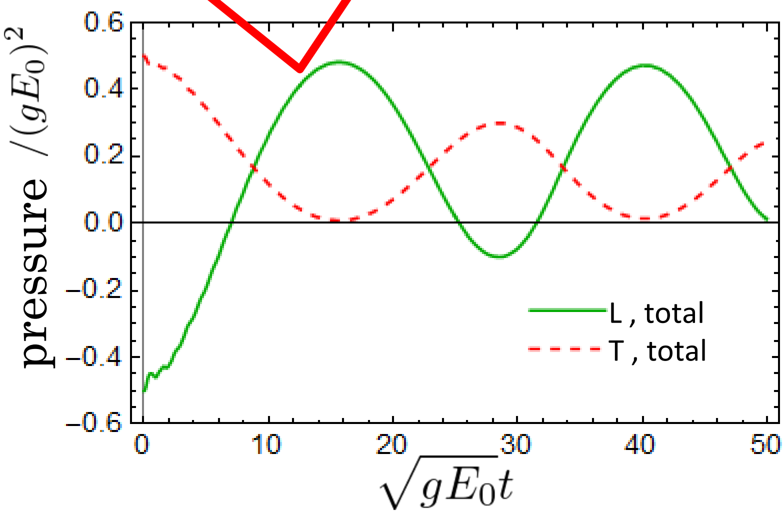
$$\langle \Theta^{\mu}{}_{\mu} \rangle = \mathcal{E} - 2P_{\rm T} - P_{\rm L} = m \langle \bar{\psi} \psi \rangle$$

Pressure

Created particles generate positive longitudinal pressure.

Negative longitudinal pressure is compensated by pressure generated by particles.





Due to back reaction,
the field strength is weakened.

total pressure

Degree of anisotropy is moderated by pair creation.

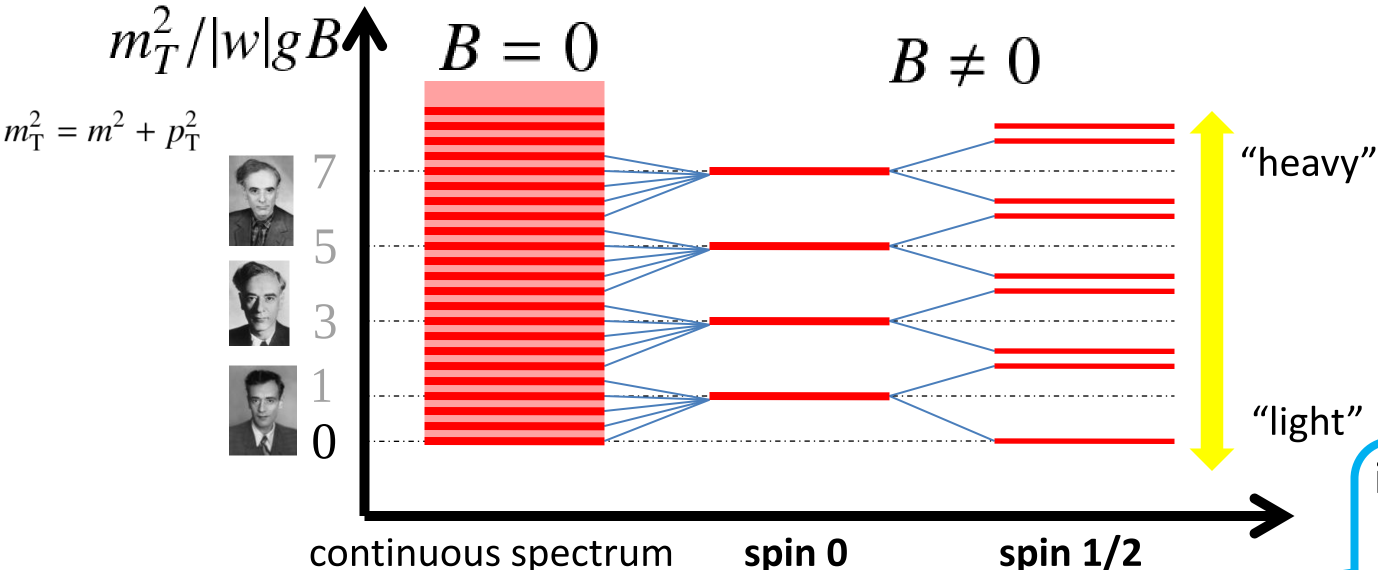
The effects of a magnetic field

Electric $E^a = \mathbf{E} n^a$

Magnetic $B^a = \mathbf{B} n^a$

Longitudinal magnetic field

- Landau level $\quad p_T^2 \to (2n+1)|w|gB \ (n = 0, 1, 2, \cdots)$
- Zeeman splitting $\quad \pm 2s|w|gB \quad (s : \text{spin})$



lowest level: $|w|gB$ (spin 0), 0 (spin 1/2)

The strong magnetic field makes particles "heavy" and suppresses pair creation.

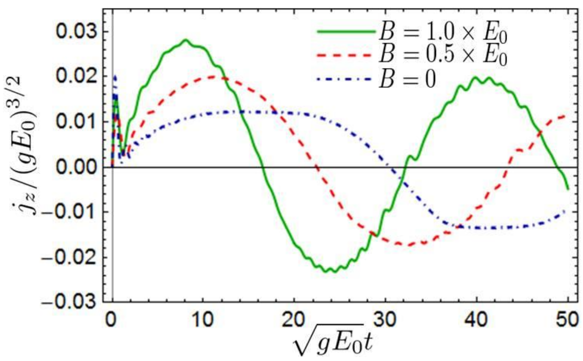
independent of B → Pair creation is not suppressed.

+

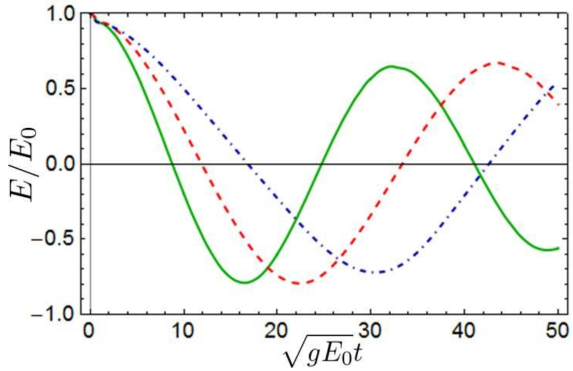
The number of modes degenerating in one Landau level is proportional to B

The effects of a magnetic field

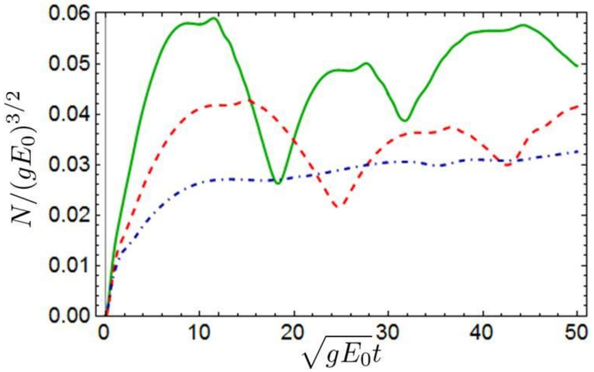
1. Enhancement of pair creation



current density



electric field



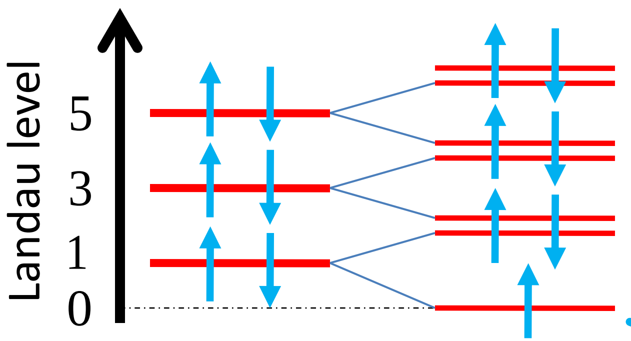
number density

Pair creation is enhanced ➡ Time evolution gets faster

The effects of a magnetic field

2. Induction of chiral charge

$$Q_5 = \langle 0, \mathrm{in}|\bar{\psi}\gamma_0\gamma_5\psi|0, \mathrm{in}\rangle = N_\mathrm{R} - N_\mathrm{L}$$



Due to spin-magnetic field interaction, energy levels are split.

The lowest Landau level induces chiral charge.

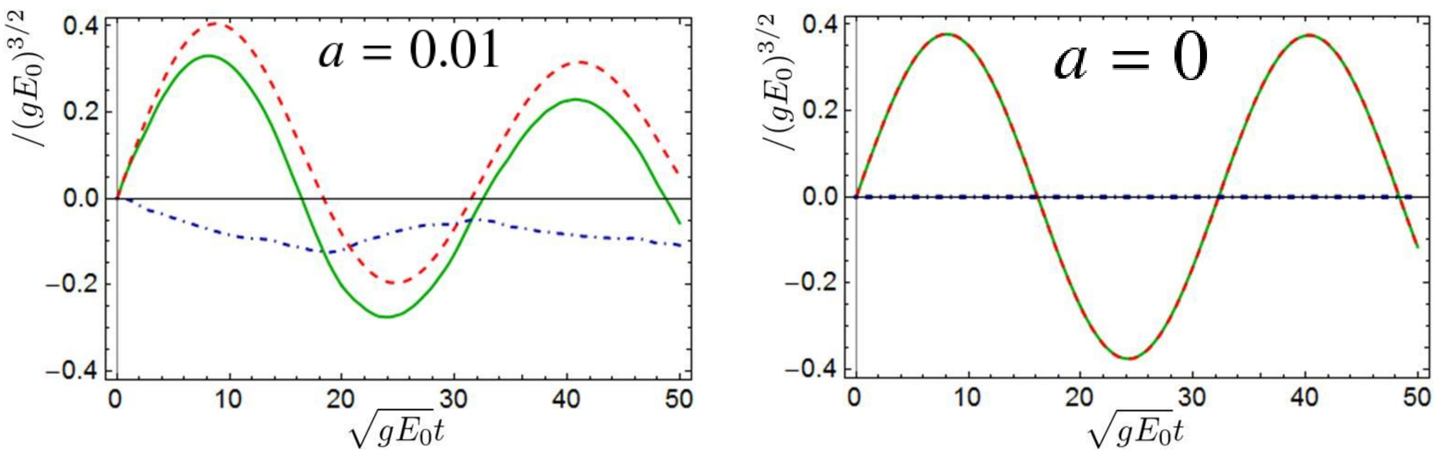
The Adler-Bell-Jackiw anomaly equation

$$\frac{d}{dt}Q_5 = \frac{N_f g^2}{4\pi^2}EB + 2m\langle\bar{\psi}i\gamma_5\psi\rangle$$

explicit breaking of chiral symmetry by nonzero mass

The effects of a magnetic field

2. Induction of chiral charge



nonzero mass | massless

The time evolution of chiral charge

$$Q_5(t) = \frac{N_f g^2}{4\pi^2}\int_0^t dt' EB + 2m\int_0^t dt' \langle \bar{\psi} i\gamma_5 \psi \rangle$$

Summary

- The momentum distributions of created quarks and their time-evolution in uniform color electric fields have been revealed.

- Negative longitudinal pressure of the electric field is compensated by pressure of created particles.

- The magnetic field speeds up the time-evolution of the quark system and induces chiral charge.