# Freeze-out dynamics of expanding quantum fields

Nov 18, 2009

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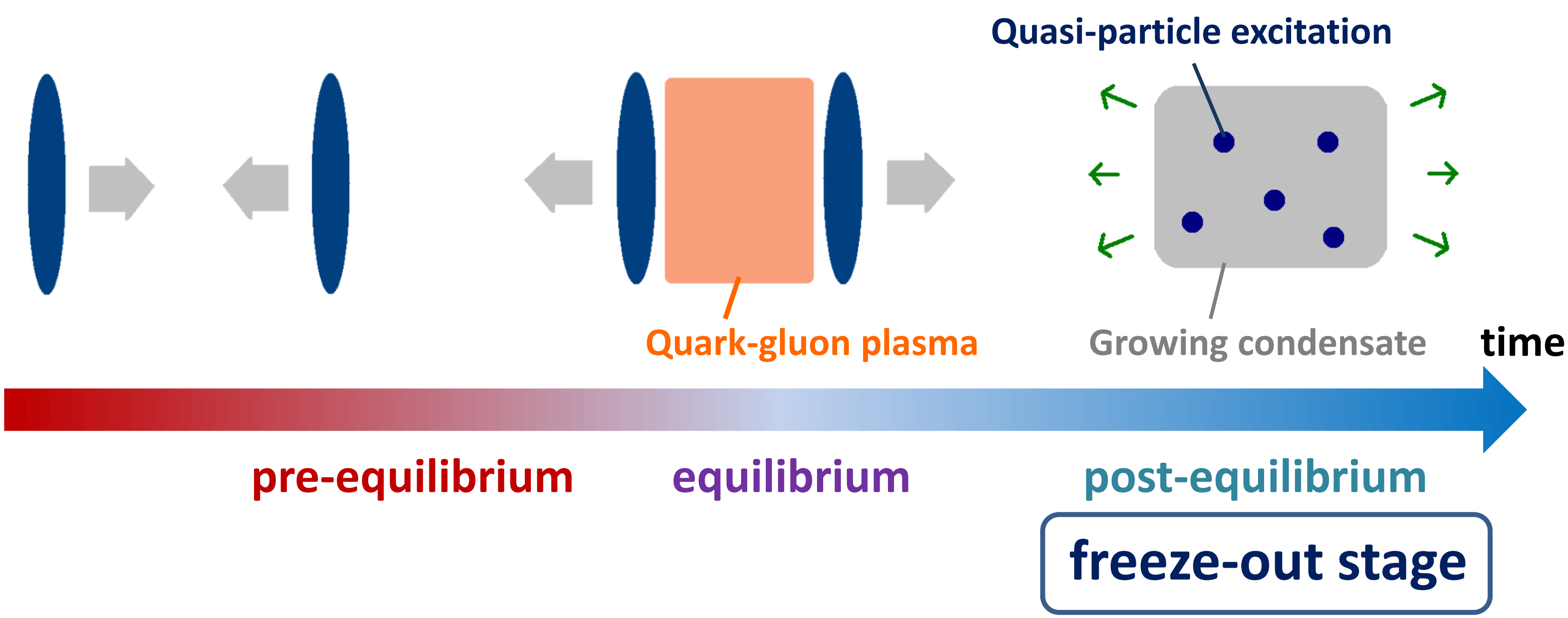
**Ultra-relativistic heavy ion collisions (RHIC, LHC, etc)**



# Role of chiral phase transition

O(4) linear sigma model has been used to describe chiral transition in the classical filed approximation.

***Disoriented chiral condensate***(DCC) is described in terms of the fluctuating classical mean field, but **no particle excitation**.

**Rajagopal-Wilczek(1993), etc**

↓

***Quasi-particle excitation*** has been introduced in terms of the Wigner functions which describe **quantum fluctuation** of meson fields.

**Matsui-Matsuo(2008)**

We apply this theory to **freeze-out dynamics** taking into account boost-invariant expansion.

# Kinetic theory of meson *condensate* and *quasi-particle excitations* (*quantum fluctuation*)

Hamiltonian

$$H = \int d\mathbf{r} \left[ \frac{1}{2}\hat{\pi}^2 + \frac{1}{2}(\nabla \hat{\phi})^2 + \frac{1}{2}m^2\hat{\phi}^2 + \frac{\lambda}{4!}\hat{\phi}^4 \right]$$

$$\hat{\phi}(\mathbf{r},t) = \boxed{\phi_c(\mathbf{r},t)} + \boxed{\tilde{\phi}(\mathbf{r},t)}$$

quantization

$$\tilde{\phi}(\mathbf{r},t) = \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{1}{\sqrt{2\omega_{\mathbf{p}}}} \left[ a_{\mathbf{p}}(t) + a^{\dagger}_{-\mathbf{p}}(t) \right]$$

$\omega_{\mathbf{p}} = \sqrt{p^2 + \mu^2}$ $\mu$ : mass parameter

***classical mean field***

$$\phi_c(\mathbf{r},t) = \langle \hat{\phi}(\mathbf{r},t) \rangle$$

describes **meson condensates**.

**coupled**

***quantum fluctuation***

describes **quasi-particle meson excitations** in terms of the Wigner functions.

Statistical average

$$\langle \hat{O}(t) \rangle = \mathrm{tr}\left[ \hat{O}(t)\hat{\rho} \right]$$

$\hat{\rho}$ : density matrix

The Wigner functions
(the quantum distribution functions)

$$\begin{pmatrix} F & \bar{G} \\ G & \bar{F} \end{pmatrix} = \begin{pmatrix} \langle a^{\dagger}_{\mathbf{p}+\mathbf{k}/2} a_{\mathbf{p}-\mathbf{k}/2} \rangle & \langle a^{\dagger}_{\mathbf{p}+\mathbf{k}/2} a^{\dagger}_{-\mathbf{p}+\mathbf{k}/2} \rangle \\ \langle a_{-\mathbf{p}-\mathbf{k}/2} a_{\mathbf{p}-\mathbf{k}/2} \rangle & \langle a_{-\mathbf{p}-\mathbf{k}/2} a^{\dagger}_{-\mathbf{p}+\mathbf{k}/2} \rangle \end{pmatrix}$$

# Problem in the previous formalism

Lorentz covariance in the previous formalism is not manifest.

e.g. The distribution function $f(\mathbf{p},\mathbf{r},t)$ don't have explicit dependence on energy paired with $\mathbf{p}$ to form a Lorentz vector.

This leads to difficulty in constructing boost-invariant solution.

$\downarrow$

**We introduce the two-time Wigner functions.**

| | one-time | two-time |
|---|---|---|
| Wigner function | $F(\mathbf{p},\mathbf{k},t) = \langle a^\dagger_{\mathbf{p}+\mathbf{k}/2}(t) a_{\mathbf{p}-\mathbf{k}/2}(t)\rangle$ | $F(\mathbf{p},\mathbf{k},t,t') = \langle a^\dagger_{\mathbf{p}+\mathbf{k}/2}(t+t'/2) a_{\mathbf{p}-\mathbf{k}/2}(t-t'/2)\rangle$ |
| distribution function (Fourier transforms) | $f(\mathbf{p},\mathbf{r},t)$ | $f(\mathbf{p},\omega;\mathbf{r},t)$ |
| the equations of motion | 4 equations for t | 8 equations for t and t' |

$t$ : time paired with $\mathbf{r}$ to form a Lorentz vector $(\mathbf{r},t)$ $\quad \mathbf{k} \xrightarrow[\text{F.T.}]{} \mathbf{r}$

$t' \xrightarrow[\text{F.T.}]{} \omega$ : quasi-particle energy paired with $\mathbf{p}$ to form another Lorentz vector $(\mathbf{p},\omega)$

# The equations of motion for the one-time Wigner functions

- The Landau-Vlasov equation

$$\frac{\partial}{\partial t} f(\mathbf{p},\mathbf{r},t) + \nabla_{\mathbf{p}}\epsilon\cdot\nabla_{\mathbf{r}} f - \nabla_{\mathbf{r}}\epsilon\cdot\nabla_{\mathbf{p}} f = iU_{\mathbf{p}} g_{-}(\mathbf{p},\mathbf{r},t) - \frac{1}{2}\nabla_{\mathbf{p}}U_{\mathbf{p}}\cdot\nabla_{\mathbf{r}} g_{+} + \frac{1}{2}\nabla_{\mathbf{r}}U_{\mathbf{p}}\cdot\nabla_{\mathbf{p}} g_{+}$$

**drift term** **Vlasov term**

- The equation of motion for g

$$\frac{\partial}{\partial t} g(\mathbf{p},\mathbf{r},t) + 2i\epsilon g = -iU_{\mathbf{p}} f_{+}(\mathbf{p},\mathbf{r},t) - \frac{1}{2}\nabla_{\mathbf{p}}U_{\mathbf{p}}\cdot\nabla_{\mathbf{r}} f_{-} + \frac{1}{2}\nabla_{\mathbf{r}}U_{\mathbf{p}}\cdot\nabla_{\mathbf{p}} f_{-}$$

$$\epsilon = \sqrt{p^2+\mu^2} + \frac{\Delta\Pi(\mathbf{r},t)}{2\sqrt{p^2+\mu^2}} \equiv \omega_{\mathbf{p}} + U_{\mathbf{p}}$$

$\Delta\Pi(\mathbf{r},t)$ : self energy

$$f_{\pm}(\mathbf{p},\mathbf{r},t) = f \pm \bar{f} \qquad g_{\pm}(\mathbf{p},\mathbf{r},t) = g \pm \bar{g}$$

Off-diagonal component of the Wigner function

$$\begin{pmatrix} F & \bar{G} \\ G & \bar{F} \end{pmatrix} = \begin{pmatrix} \langle a^{\dagger}_{\mathbf{p}+\mathbf{k}/2} a_{\mathbf{p}-\mathbf{k}/2}\rangle & \langle a^{\dagger}_{\mathbf{p}+\mathbf{k}/2} a^{\dagger}_{-\mathbf{p}+\mathbf{k}/2}\rangle \\ \langle a_{-\mathbf{p}-\mathbf{k}/2} a_{\mathbf{p}-\mathbf{k}/2}\rangle & \langle a^{\dagger}_{\mathbf{p}+\mathbf{k}/2} a^{\dagger}_{-\mathbf{p}+\mathbf{k}/2}\rangle \end{pmatrix} \xrightarrow[\text{F.T.}]{} \begin{pmatrix} f & \bar{g} \\ g & \bar{f} \end{pmatrix}$$

It is difficult to impose the boost-invariance in the Landau-Vlasov equation.

The reason is that the Lorentz covariance in the equations is not manifest.

## The equations of motion for the two-time Wigner functions

We can eliminate g by redefinition of the distribution function.

- The Landau-Vlasov equation

$$\frac{\partial}{\partial t} f'(\mathbf{p},\omega;\mathbf{r},t) + \nabla_{\mathbf{p}}\epsilon' \cdot \nabla_{\mathbf{r}} f' - \nabla_{\mathbf{r}}\epsilon' \cdot \nabla_{\mathbf{p}} f' + \frac{\partial}{\partial t}\epsilon' \frac{\partial}{\partial \omega} f' = 0$$

- **Energy spectrum of quasi-particle** (Fourier transform of equation for t')

$$\boxed{\omega = \sqrt{p^2 + \mu^2 + \Delta\Pi(\mathbf{r},t)} \equiv \epsilon'}$$

$$f'(\mathbf{p},\omega;\mathbf{r},t) = \frac{1}{2}\left(\frac{\epsilon'}{\epsilon}+1\right) f + \frac{1}{2}\left(\frac{\epsilon'}{\epsilon}-1\right)\bar{f}$$

The Bogoliubov transformation

quasi-particle energy in the one-time formalism $\epsilon = \sqrt{p^2+\mu^2} + \frac{\Delta\Pi(\mathbf{r},t)}{2\sqrt{p^2+\mu^2}}$

- **The Landau-Vlasov equation in a manifestly Lorentz covariant form**

$$\boxed{p_\mu \partial^\mu f'(\mathbf{p},\omega;\mathbf{r},t) + \frac{1}{2}\partial_\mu \Delta\Pi(\mathbf{r},t)\partial_p^\mu f'(\mathbf{p},\omega;\mathbf{r},t) = 0 \qquad p^\mu = (\omega,\mathbf{p})}$$

From these equations, we can obtain the same result to the collective excitation near the equilibrium as obtained in the one-time formalism. Matsui, Matsuo(2008)

# The Bogoliubov transformation

We choose a mass parameter in definition of creation/annihilation operators.

Creation/annihilation operators and the Wigner functions depend on this choice.

$$\tilde{\phi} = \sum_{\mathbf{p}} \frac{e^{i\mathbf{p}\cdot\mathbf{r}}}{\sqrt{2\omega_{\mathbf{p}}}} \left( a_{\mathbf{p}} + a^{\dagger}_{-\mathbf{p}} \right) = \sum_{\mathbf{p}} \frac{e^{i\mathbf{p}\cdot\mathbf{r}}}{\sqrt{2\omega'_{\mathbf{p}}}} \left( b_{\mathbf{p}} + b^{\dagger}_{-\mathbf{p}} \right)$$

$$\omega_{\mathbf{p}} = \sqrt{p^2 + \mu^2}$$
$$\omega'_{\mathbf{p}} = \sqrt{p^2 + \mu'^2}$$

The Bogoliubov transformation

$$\begin{pmatrix} b_{\mathbf{p}} \\ b^{\dagger}_{-\mathbf{p}} \end{pmatrix} = \begin{pmatrix} \cosh\alpha_{\mathbf{p}} & \sinh\alpha_{\mathbf{p}} \\ \sinh\alpha_{\mathbf{p}} & \cosh\alpha_{\mathbf{p}} \end{pmatrix} \begin{pmatrix} a_{\mathbf{p}} \\ a^{\dagger}_{-\mathbf{p}} \end{pmatrix} \equiv \mathbf{M}_{\mathbf{p}} \begin{pmatrix} a_{\mathbf{p}} \\ a^{\dagger}_{-\mathbf{p}} \end{pmatrix}$$

$$e^{\alpha_{\mathbf{p}}} = \sqrt{\frac{\omega'_{\mathbf{p}}}{\omega_{\mathbf{p}}}}$$

The relation between the Wigner functions

$$\begin{pmatrix} \langle b^{\dagger} b \rangle & \langle b^{\dagger} b^{\dagger} \rangle \\ \langle b b \rangle & \langle b b^{\dagger} \rangle \end{pmatrix} = \mathbf{M} \begin{pmatrix} \langle a^{\dagger} a \rangle & \langle a^{\dagger} a^{\dagger} \rangle \\ \langle a a \rangle & \langle a a^{\dagger} \rangle \end{pmatrix} \mathbf{M}$$

If we take $\mu'^2$ as $\mu'^2 = \mu^2 + \Delta\Pi(\mathbf{r}, t)$ , this relation corresponds to the redefinition of the distribution function as

$$f'(\mathbf{p}, \omega; \mathbf{r}, t) = \frac{1}{2}\left(\frac{\epsilon'}{\epsilon} + 1\right) f + \frac{1}{2}\left(\frac{\epsilon'}{\epsilon} - 1\right) \bar{f}$$

# The boost-invariant equation in 1+1 dimension

Impose $f'(\mathbf{p},\omega;\mathbf{r},t) = f'(\tau, \xi = y - \eta, m_T, p_T)$

$$\Delta\Pi(\mathbf{r},T) = \Delta\Pi(\tau)$$

$$\tau = \sqrt{t^2 - z^2} \qquad m_T = \sqrt{\omega^2 - p_z^2}$$

$$\eta = \frac{1}{2}\log\frac{t+z}{t-z} \qquad y = \frac{1}{2}\log\frac{\omega + p_z}{\omega - p_z}$$

$$\xi = y - \eta$$

A boost transformation for z-direction

$$\Delta v = \tanh \Delta y$$

$$\tau' = \tau \qquad m_T' = m_T$$

$$\eta' = \eta - \Delta y \qquad y' = y - \Delta y$$

$$\xi = y - \eta \longrightarrow \xi' = \xi$$

$\tau$ , $m_T$, and $\xi$ is boost-invariant

**The Landau-Vlasov equation in a boost-invariant form**

$$\frac{\partial}{\partial\tau}f' - \tanh\xi\left[\frac{1}{\tau} + \frac{1}{2m_T^2}\frac{\partial}{\partial\tau}\Delta\Pi\right]\frac{\partial f'}{\partial\xi} + \frac{1}{2m_T}\frac{\partial}{\partial\tau}\Delta\Pi\frac{\partial f'}{\partial m_T} = 0$$

equation of **quasi-particle excitations**

coupled equations

**The equation of motion of $\phi_c(\tau)$ in a boost-invariant form**

**Meson condensate**

# Summary

We construct **a kinetic theory for interacting quantum meson fields in a manifestly Lorentz covariant form** in order to describe freeze-out stage of expanding meson fields.

We have introduced **the *two-time* Wigner functions** in order to ensure manifest Lorentz covariance.

We have shown that the two-time formalism leads to the same result to the collective excitation near the equilibrium as obtained using the one-time formalism.

We have derived the equations of motion **in a boost-invariant form**.

We are now constructing **boost-invariant solutions** numerically.

**Work in progress**