session iii

Wilson loop in AdS/CFT

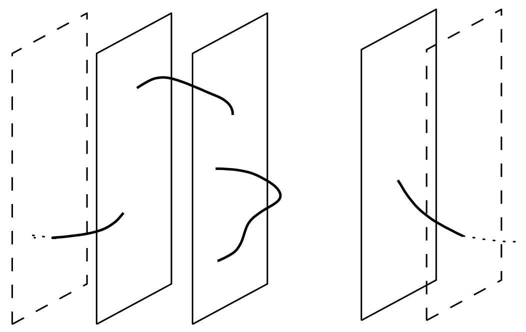
December 9th 2009

Junichiro Noguchi

string theory $\cdots$ a candidate of an unified theory for 4 interactions

$$\begin{cases} \text{open strings} \;\cdots\; \text{need a boundary condtion} \\ \text{closed strings} \end{cases}$$

put Dirichlet b.c. on open strings $\cdots$ Dp-brane



Dp-branes in d-dimensions

now think of a N D3-brane system:

$$\text{N D3}-\text{brane system} \xrightarrow{\text{low energy limit}} \begin{cases} \text{4d. } \mathcal{N}=4 \text{ SYM } \cdots \text{ CFT} \\ \qquad\qquad\qquad \text{(strong coupling region)} \\ \text{string theory on AdS} \cdots \text{ AdS} \\ \qquad\qquad\qquad\qquad \text{(classical solution)} \end{cases}$$

AdS/CFT correspondence:

they are equivalent theories,
since we are looking at a same system from different point of view

if it holds, for some observable, you can say:

$$\underset{\text{gauge theory side}}{\text{(quantum calculation in s.c.r.)}} = \underset{\text{string theory side}}{\text{(classical calculation)}}$$

to check the correspondence, hang down a string from a probe D3-brane to this system and close the contour, then you will get a corresponding Wilson loop on both theory side.

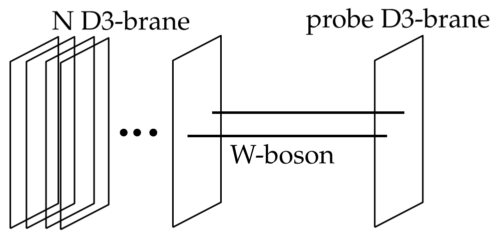


図 1 gauge theory side

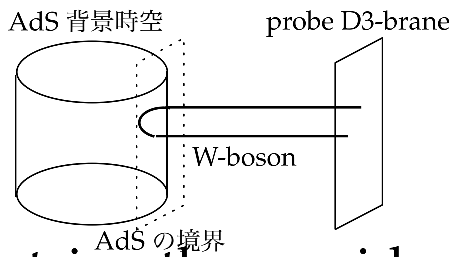


図 2 string theory side

so $\langle W(C) \rangle$ is conjectured to be a minimal surface in string theory side:

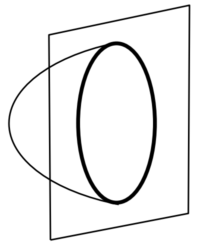


図 3 an image of a minimal surface

notes:

4d. SYM $\cdots$ QCD
Wilson loop $\cdots$ confinement

an investigation of Wilson loop in 4d. SYM in great detail is interesting and meaningful.

def. of Wilson loop is

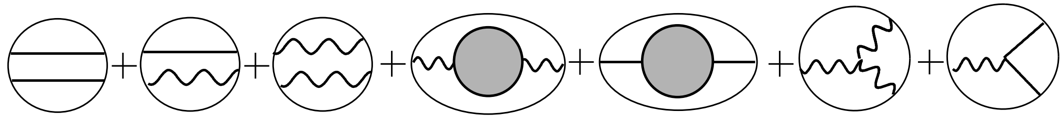
$$W(x^\mu, \theta_i)|_C = \frac{1}{N} Tr\ P\ exp[\int_C ds(iA_\mu \dot{x}^\mu + |\dot{x}|\phi_i \theta^i)]$$

$$\mu = 1, 2, 3, 4, \quad i = 5, 6, 7, 8, 9, 10$$

think of supersymmetric(BPS) Wilson loop which satisfy BPS eq.:

$$\delta\ W \sim (i\not{\dot{x}} + |\dot{x}|\ \not{\theta}\ )(\ \varepsilon\ _0 + \not{x}\ \varepsilon\ _1) = 0$$

some BPS solutions are known and for some of them, it is shown that 1 loop corrections cancel out exactly:



and believed that they do for all order. $\cdots$ not yet confirmed what I have studied:

1. relationship between BPS eq. and 1 loop Feynman diagram calculations by thinking of small perturbation around a known solution. i.e.

$$(i\,\Gamma^{\mu}\dot{x}^{\mu}+\Gamma^{i}\theta^{i}|\dot{x}|)(\varepsilon_{0}+\Gamma^{\mu}x^{\mu}\varepsilon_{1})=0$$

$$\diamond=\frac{g^4N^2}{2^7\pi^4}\int\varepsilon_{123}\frac{\theta_1\cdot\theta_3-\dot{x}_1\cdot\dot{x}_3}{x_{13}^2}\frac{x_{32}\cdot\dot{x}_2}{x_{32}^2}\log\left(\frac{x_{21}^2}{x_{31}^2}\right)=0$$

under

$$\begin{cases} x^{\mu} & \to x^{\mu}_{(0)}+\delta\ x^{\mu}_{(1)}+\cdots \\ \theta^{i} & \to \theta^{i}_{(0)}+\delta\ \theta^{i}_{(1)}+\cdots \end{cases}$$

tried to get x-dep. of θ for each order of δ in both side, but not yet finished.

2. get the general solutions for BPS eq.

$$(i\not{\dot{x}} + |\dot{x}|\not{\theta})(\varepsilon_0 + \not{x}\varepsilon_1) = 0 \Leftrightarrow (1 + i\frac{\not{\dot{x}}}{|\dot{x}|}\not{\theta})(i\not{\dot{x}}\varepsilon_0 + |\dot{x}|\not{\theta}\not{x}\varepsilon_1) = 0$$

then BPS eq. is

$$Mv = 0, \quad \begin{cases} M & \equiv 1 + i\frac{\not{\dot{x}}}{|\dot{x}|}\not{\theta} \\ v & \equiv i\not{\dot{x}}\varepsilon_0 + |\dot{x}|\not{\theta}\not{x}\varepsilon_1 \end{cases}$$

constructing v with 0-eigen vector v_0 of M $\rightarrow \theta = \theta(x^{\mu}(t))$

2. has been solved by [Dymarsky, Pestun]

$$\rightarrow \begin{cases} \text{1. check} \\ \text{2. discuss about } \diamondsuit = 0 \end{cases}$$